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Sampling Effects on Secondary Voltage Control of Microgrids Based on Network of Multiagent

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Abstract—This paper studies a secondary voltage control framework of the microgrids based on the consensus for a communication network of multiagent. The proposed control is designed by the communication network with one-way links. The communication network is modeled by a directed graph. At this time, the concept of sampling is considered as the communication constraint among each distributed generator in the microgrids. To analyze the sampling effects on the secondary voltage control of the microgrids, by using Lyapunov theory and some mathematical techniques, the sufficient condition for such problem will be established regarding linear matrix inequality (LMI). Finally, some simulation results are given to illustrate the necessity of the consideration of the sampling effects on the secondary voltage control of the microgrids.

Keywords—Microgrids, secondary control, multiagent, sampling, LMI.

I. Introduction

Network of multiagent have interested by the reason of their applications in many research fields such as robotics, power systems, and so on. Here, a prime concern is consensus, which means to attain an agreement regarding the state of all agents [1]. In this regard, in the power systems, microgrids are composed of various components such as distributed generators (DGs). Such DGs can be presented by a communication network. So, the consensus property is available to apply at the network consist of DGs in the microgrid. At this time, the primary control is applied to maintain the voltage stability, and the secondary control is applied to restore the voltage of DGs to their nominal value [2]. However, to the best of authors' knowledge, the network-induced constraints construct the consensus control has not been tackled in any other literature yet.

Motivated by the discussion above, this paper deals with the problem of a consensus problem in microgrids with sampling for the first time. Here, we consider the fixed and directed communication graph and synchronized sampling among the agents of the network. At this time, the sampling is considered as the communication constraint between each agent in network. In addition to this, because of the zero-order hold, continuous data are sampled before being used, a sampled-data appears discontinuous at sampling instants and continuous in other times [3]. To solve the problem mentioned above, by construction of a simple Lyapunov-Krasovskii functional and utilization of some mathematical techniques, the consensus criterion will be derived in Theorem 1. Through one example, it will be shown that the sampling on the secondary voltage control is the cause of the overshoot and the settling time.

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Notation: Throughout the paper, the used notations are standard. $\mathbb{R}$, $\mathbb{R}^n$, and $\mathbb{R}^{m\times n}$ denote, respectively, the sets of real numbers, n-vectors with the l_2-norm $\|\cdot\|$, and $m\times n$ matrices. I_n and 0 are $n\times n$ identity matrix and zero matrix of appropriate dimension. $X>0$ (<0) represents symmetric positive (negative) definite matrix. $X_\perp$ denotes a basis for the nullspace of X. $X_{[f(t)]}$ means the sum of a constant matrix X_1 and a linear matrix $f(t)X_2$ for all real scalars $f(t)$; *i.e.*, $X_{[f(t)]} = X_1 + f(t)X_2$.

II. Problem Statements

The interaction topology of a network of agents is represented using a directed graph (digraph) $\mathcal{G}=(\mathcal{V},\mathcal{E})$ with the set of nodes $\mathcal{V}=\{1,2,\ldots,N\}$ and the set of edges $\mathcal{E}=\{(i,j) : i,j\in\mathcal{V}\}\subset\mathcal{V}\times\mathcal{V}$. A degree of node i is denoted by $d_i=\sum_{j\in\mathcal{N}_i}a_{ij}$. A degree matrix of digraph $\mathcal{G}$ is a diagonal matrix defined as $\mathcal{D}=\begin{bmatrix} d_1 & & \\ & \ddots & \\ & & d_N\end{bmatrix}$. The Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$ is defined as $\mathcal{L}=\mathcal{D}-\mathcal{A}$. More details can be seen in [4].

Consider the following primary voltage control law of inverter-based DGs in the direct-quadratic (d-q) reference frame [5]:

$$\begin{aligned} v_i^d &= V_{n,i} - n_{Q,i}Q_i, \\ v_i^q &= 0,\ \ i=1,2,\ldots,N, \end{aligned} \tag{1}$$

where N is the number of DGs, the subscript i means the ith DG, v_i^d and v_i^q are the voltage amplitudes of the ith DG on the d-axis and q-axis generated by the primary control. Q_i and $n_{Q,i}$ are the measured reactive power at the ith DG terminal and the droop coefficient based on the reactive power rating of the ith DG. $V_{n,i}$ is the primary control reference.

Differentiating (1) leads to

$$\dot{v}_i^d = \dot{V}_{n,i} - n_{Q,i}\dot{Q}_i. \tag{2}$$

Let us define $\dot{V}_{n,i} - n_{Q,i}\dot{Q}_i$ as the consensus control u_i. Then, the secondary voltage control of a microgird is transformed to

$$\dot{v}_i^d(t) = u_i(t),\ \ i=1,2,\ldots,N, \tag{3}$$

Moreover, the reference for the voltage is given by

$$\dot{v}_r(t) = u_r(t). \tag{4}$$

Remark 1: The control $u_i(t)$ is used to investigate the consensus problem $\lim_{t\to\infty}(v_i^d(t) - v_r(t)) = 0$ for $i = 1, 2, \ldots, N$. This means that the secondary voltage control selects $V_{n,i}$ such that the terminal voltage amplitude of each DG synchronizes to the reference value.

The consensus control with sampled-data between each DG is constructed as follows:

$$u_i(t) = \sum_{j=1, j\neq i}^{N} a_{ij}[v_j^d(t_k) - v_i^d(t_k)] - b_i[v_i^d(t) - v_r(t)] \quad (5)$$

for all $t \in [t_k, t_{k+1})$, where a_{ij} and b_i are the connection weight defined as: $a_{ij} > 0$ if DG i is connected to DG j and $a_{ij} = 0$ otherwise, and where $b_i = 1$ if reference is connected to DG i and $b_i = 0$ otherwise. Moreover, the information flow between each DG is assumed to be generated by a zero-order holder with a sequence of sampling instants t_k satisfying $0 = t_0 < t_1 < \cdots < t_k < \cdots < \infty$. When the sampling interval is constant, $t_{k+1} - t_k = h_M$, where h_M is a known positive scalar. It should be noted that $0 \le t - t_k = h(t) \le h_M$ for $t \in [t_k, t_{k+1})$ and $\dot{h}(t) = 1$ for $t \neq t_k$.

By the Laplacian matrix $\mathcal{L} = [l_{ij}]_{N\times N}$ associated with the structure of the information flow satisfying $l_{ij} = -a_{ij}$ for $i \neq j$ and $l_{ii} = -\sum_{j=1,j\neq i}^{N} l_{ij}$, system (3) with the control (5) can be expressed in the matrix form

$$\dot{x}(t) = -\mathcal{B}x(t) - \mathcal{L}x(t - h(t)), \quad (6)$$

where

$$x(t) = \begin{bmatrix} v_1^d(t) - v_r(t) \\ \vdots \\ v_N^d(t) - v_r(t) \end{bmatrix} \in \mathbb{R}^N,\ \mathcal{B} = \begin{bmatrix} b_1 & & \\ & \ddots & \\ & & b_N \end{bmatrix}.$$

The aim of this paper is to analyse the sampling effect on the secondary voltage control from (6).

III. Main Results

In this section, the consensus condition is presented. For simplicity, some scalars and matrices are defined as follows:

$$\nu_1(t) = \frac{1}{h(t)}\int_{t-h(t)}^{t} x(s)ds,$$

$$\nu_2(t) = \frac{1}{h_M - h(t)}\int_{t-h_M}^{t-h(t)} x(s)ds,$$

$$\zeta(t) = [x^T(t), x^T(t-h(t)), x^T(t-h_M), \dot{x}^T(t), \nu_1^T(t), \nu_2^T(t)]^T,$$

$$\Theta = \begin{bmatrix} \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} & M \\ M^T & \begin{bmatrix} R & 0 \\ 0 & 3R \end{bmatrix} \end{bmatrix},$$

$$\Xi_{1[h(t)]} = \begin{bmatrix} e_1^T \\ h(t)e_5^T \\ +(h_M - h(t))e_6^T \end{bmatrix}^T P \begin{bmatrix} e_4^T \\ e_1^T - e_3^T \end{bmatrix} + \left(\begin{bmatrix} e_1^T \\ h(t)e_5^T \\ +(h_M - h(t))e_6^T \end{bmatrix}^T P \begin{bmatrix} e_4^T \\ e_1^T - e_3^T \end{bmatrix} \right)^T$$
$$+e_1 Q e_1^T - e_3 Q e_3^T + h_M^2 e_4 R e_4^T,$$

$$\Xi_2 = \begin{bmatrix} e_1^T - e_2^T \\ e_1^T + e_2^T - 2e_5^T \\ e_2^T - e_3^T \\ e_2^T + e_3^T - 2e_6^T \end{bmatrix}^T \Theta \begin{bmatrix} e_1^T - e_2^T \\ e_1^T + e_2^T - 2e_5^T \\ e_2^T - e_3^T \\ e_2^T + e_3^T - 2e_6^T \end{bmatrix},$$
$$\Xi_{[h(t)]} = \Xi_{1[h(t)]} - \Xi_2,$$
$$\Upsilon = -\mathcal{B}e_1^T - \mathcal{L}e_2^T - I_N e_4^T, \quad (7)$$

where $e_i \in \mathbb{R}^{6N\times 6N}$ for $i = 1, \ldots, 6$ are elementary matrices.

Now, the result is given by the following theorem:

Theorem 1: For given positive scalars h_M, all agents in the system (6) with the control (5) are consented to leader, if there exist positive definite matrices P, Q, R and any matrix M satisfying the following LMIs:

$$\Upsilon_\perp^T \Xi_i \Upsilon_\perp < 0, \quad i = 1, 2, \quad (8)$$
$$\Theta > 0, \quad (9)$$

where Ξ_i means the two vertices of $\Xi_{[h(t)]}$ with the bounds of $0 \le h(t) \le h_M$; *i.e.*, $\Xi_1 = \Xi_{[0]}$ and $\Xi_2 = \Xi_{[h_M]}$.

Proof: Consider the following Lyapunov-Krasovskii functional:

$$V = \begin{bmatrix} x(t) \\ \int_{t-h_M}^{t} x(s)ds \end{bmatrix}^T P \begin{bmatrix} x(t) \\ \int_{t-h_M}^{t} x(s)ds \end{bmatrix} + \int_{t-h_M}^{t} x^T(s)Qx(s)ds + h_M\int_{t-h_M}^{t}\int_{s}^{t} \dot{x}^T(u)R\dot{x}(u)duds. \quad (10)$$

Differentiating (10) leads to

$$\begin{aligned} \dot{V} &= 2\zeta^T(t)\begin{bmatrix} e_1^T \\ h(t)e_5^T \\ +(h_M - h(t))e_6^T \end{bmatrix}^T P \begin{bmatrix} e_4^T \\ e_1^T - e_3^T \end{bmatrix}\zeta(t) \\ &\quad +\zeta^T(t)(e_1 Q e_1^T - e_3 Q e_3^T)\zeta(t) \\ &\quad +h_M^2\zeta^T(t)e_4 R e_4^T\zeta(t) - h_M\int_{t-h_M}^{t} \dot{x}^T(s)R\dot{x}(s)ds \\ &= \zeta^T(t)\Xi_{1[h(t)]}\zeta(t) - h_M\int_{t-h_M}^{t} \dot{x}^T(s)R\dot{x}(s)ds. \quad (11) \end{aligned}$$

By applying Wirtinger-based inequality [6] and the reciprocal convex lemma [7], if (9) holds, then the above integral term is bounded as for any matrix M,

$$\begin{aligned} & h_M\int_{t-h_M}^{t} \dot{x}^T(s)R\dot{x}(s)ds \\ = {}& h_M\int_{t-h(t)}^{t} \dot{x}^T(s)R\dot{x}(s)ds + h_M\int_{t-h_M}^{t-h(t)} \dot{x}^T(s)R\dot{x}(s)ds \\ \ge {}& \frac{1}{\alpha(t)}\zeta^T(t)(e_1 - e_2)R(e_1 - e_2)^T\zeta(t) \\ & +\frac{3}{\alpha(t)}\zeta^T(t)(e_1 + e_2 - 2e_5)R(e_1 + e_2 - 2e_5)^T\zeta(t) \\ & +\frac{1}{\beta(t)}\zeta^T(t)(e_2 - e_3)R(e_2 - e_3)^T\zeta(t) \\ & +\frac{3}{\beta(t)}\zeta^T(t)(e_2 + e_3 - 2e_6)R(e_2 + e_3 - 2e_6)^T\zeta(t) \\ \ge {}& \zeta^T(t)\Xi_2\zeta(t), \quad (12) \end{aligned}$$

TABLE I
MAXIMUM INTERVAL

Method	h_M
Theorem 1	0.69

where $\alpha(k) = \frac{h(t)}{h_M}$ and $\beta(k) = 1 - \alpha(k)$.

Then, the $\dot{V}$ has an upper bound as

$$\dot{V}(t) \leq \zeta^T(t)\Xi_{[h(t)]}\zeta(t). \tag{13}$$

where $\Xi_{[h(t)]} = \Xi_{1[h(t)]} - \Xi_2$.

In succession, the consensus condition is obtained as follows:

$$\Xi_{[h(t)]} < 0 \tag{14}$$

subject to $\Upsilon\zeta(t) = 0$.

Finally, by Finsler's lemma [8], from the convexity on $h(t)$, if the LMIs (8) hold then the condition (14) is satisfied, which means that system (6) is asymptotically consensual. This completes our proof. ■

IV. ANALYSIS ON THE SAMPLING EFFECTS

In this section, an example is introduced to show the necessity about the sampling effect.

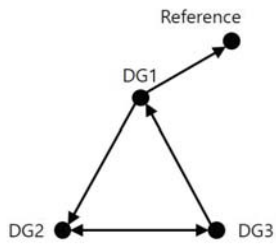

Fig. 1 3-DG Network

Consider the information flow consisting of a 3-DG network drawn by Fig. 1, and the corresponding matrices are presented as follows:

$$\mathcal{L} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \mathcal{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

From the following point of view, the sampling effect on the secondary voltage control in microgrids is analyzed as follows:

By applying Theorem 1, the maximum sampling interval considered by the consensus control (5) is listed in Table I. This means that through the control (5), the stability of the microgird consisting of a 3-DG network drawn by Fig. 1 can be guaranteed under the maximum sampling interval $h_M = 0.69$. In order to confirm the result of Table I, the simulation is drawn in Fig. 2. In details, when the sampling interval $h_M = 0.69$, the reference $v_r(t)$ is from 300 to 314 after $t = 1$ and the initial voltages of DGs $v_1^d(0) = 300$, $v_2^d(0) = 310$, and $v_3^d(0) = 290$, the control (5) ensure that the voltages of all DGs are synchronized to the reference value. At this time, the control trajectories are drawn by Fig. 3. For the case of non-sampling, the simulation is shown in Fig. 4. Compared with the case of sampling, there is the difference in the overshoot and the settling time.

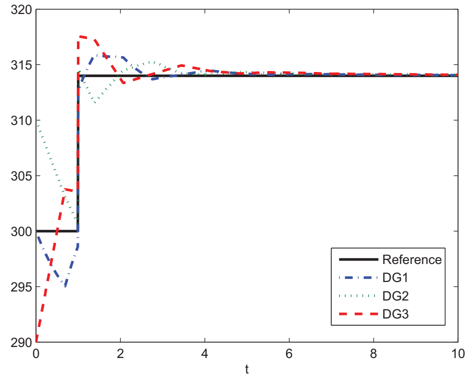

Fig. 2 Trajectories of all DGs under sampling with $h_M = 0.69$

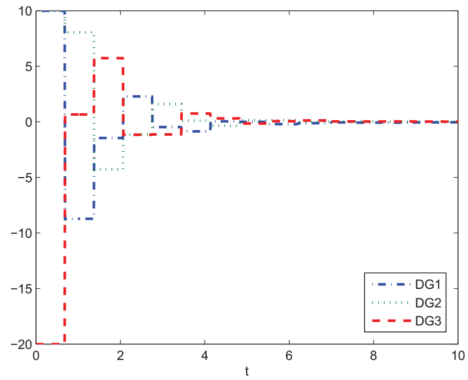

Fig. 3 Protocol trajectories of all DGs under sampling with $h_M = 0.69$

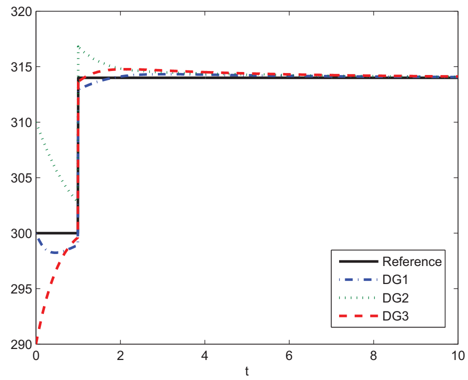

Fig. 4 Trajectories of all DGs under non-sampling

V. CONCLUSION

In this paper, the sampling effect on the secondary voltage control of microgrids has been investigated. To achieve this, by constructing simple Lyapunov-Krasovskii functional and using the network of multiagent, the sufficient condition for such problem have been derived in terms of LMIs. One numerical example has been given to show the necessity about the analysis on the sampling effect in microgrids.

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