Black Box Model and Evolutionary Fuzzy Control Methods of Coupled-Tank System

S. Yaman, S. Rostami

Abstract—In this study, a black box modeling of the coupled-tank system is obtained by using fuzzy sets. The derived model is tested via adaptive neuro fuzzy inference system (ANFIS). In order to achieve a better control performance, the parameters of three different controller types, classical proportional integral controller (PID), fuzzy PID and function tuner method, are tuned by one of the evolutionary computation method, genetic algorithm. All tuned controllers are applied to the fuzzy model of the coupled-tank experimental setup and analyzed under the different reference input values. According to the results, it is seen that function tuner method demonstrates better robust control performance and guarantees the closed loop stability.

Keywords—Function tuner method, fuzzy modeling, fuzzy PID controller, genetic algorithm.

I. INTRODUCTION

Since it was first introduced by Zadeh [1] in 1965, fuzzy set theory has been widely used in many industrial applications and demonstrated satisfying results. After first implementation of fuzzy logic controller by Mamdani [2] on a steam engine, it has prevalently been used in processes, specifically where system dynamics are either complex or include high nonlinearity. Due to their capability to cope with complex and highly nonlinear structures, the fuzzy logic controllers are found in many different application areas.

The fuzzy logic sets are defined based on the application of IF-THEN rules. It can exist either as defined by Mamdani [2] or by Takagi-Sugeno [3]. Defining the rule base is the key of success of fuzzy logic controllers to handle with large complexities. It should be mentioned that, to be able to maintain a satisfactory control performance, modeling the system is crucial. However, numerical computational errors and sensor sensitivities can cause obtaining faulty system models. To overcome this problem, using fuzzy models is quite beneficial to obtain correct model of unknown systems as well as achieving substantial control performance, which is explained in [4] and [5] in detail.

Modeling has been always one of the critical topics among control system studies. Obtaining an accurate model of the systems with high nonlinearity or complexity can be real complex and it can increase computationally burden as discussed in [6]. Fuzzy modeling can be applied to the systems as black box modeling and it is fair to say that, it has satisfactory results on modeling, as well. Using only input and output data of a system, it is possible to obtain a correct model of the system.

Although fuzzy logic controller (FLC) is successful compared to classical control methods, the design procedure is depending on the human experience and knowledge and it limits the control rules. To be able to overcome this drawback, evolutionary FLC is proposed as in [7] and [8]. The basic principle of the evolutionary optimization algorithms is based on to obtain optimized parameters by Darwinian natural selection as given in [9]. In literature, different types of evolutionary algorithms can be found, such as, evolutionary programming and genetic algorithm as given in [10].

In last decades, many control theories have been developed, proportional–integral–derivative (PID) controllers are still widely used in many industrial applications as discussed in [11]. This control type has three parameters as, proportional, integral and derivative. The structure of a PID controller is quite simple: The input of the controller is the control error and the output is the sum of three terms, the proportional, integral term and the derivative term which are referred as being proportional to the error, the integral of the error and the derivative of the error, respectively. PID control is accepted as the best controller in control system applications and by choosing suitable parameters for specific works, its success is incontestable as analyzed in [11] and [12]. While using PID structure, one may keep the third parameter of the controller as zero and apply the controller in that way. In other words, PID controller can be applied as PD (Proportional-Derivative) or PI (Proportional-Integral) as well. The proportional, integral and derivative parameters affect the system in terms of steady state error, rising time, settling time and overshoot. In order to surpass the steady state error, PI controller usually preferred, since the integral effect of this controller as explained in [11]. However, efficiency of this controller is affected by the load disturbances and parameter alteration. Yet, this concern about PI controller can be solved by using fuzzy structure.

Tuning the parameters of PID controller has been a widely researched topic since it first evolved either in classical type or fuzzy type. In literature there exist tuning strategies for the adaptation of scaling factors of fuzzy controllers as demonstrated in [13]-[16]. As given in [17], a parameter adaptive PID-type FLC using a peak observer has been evolved to tune the input and output scaling factors regarding to the integral and derivative parameters of the controller. Correlatively, a function tuner has been presented to tune the parameters of the PID-type FLC through functions in [18]. In
this study experimental results are given for classical PID controller, fuzzy PID controller and the function tuner method. All parameters have been computed by genetic algorithm including additional parameters of function tuner method.

The rest of the paper is organized as follows, first the fuzzy modeling of an unknown system is obtained through input and output data, and evolutionary control methods are applied as evolutionary classical PID controller, evolutionary fuzzy PID controller and evolutionary function tuner method, respectively.

II. PID CONTROLLER

A. Classical PID Controller

PID controller is the most commonly used controller due to its simple structure and it is easy to use. Its success based on the fact that, attempting to minimize the error by modifying the control signal. It can be said that using the PID controller is the best way to handle with an unknown system. Yet, to be able to maintain good performance, parameters of the PID controller must be tuned, based on the desired criteria of the system. The algorithm and the transfer function of PID controller is given in (1) and (2), where $T_i$ and $T_d$ are derivative and integral time constants, respectively. More specifically, when the proportional parameter of PID controller $K_p$ increases, the steady state error decreases; however, P control by itself can never achieve eliminating the steady state error as good as PI control do. Using PI controller is the best way to get rid of the steady state error, yet, it causes system to have a slow response. In applications where speed is essential, then PI controller may not be the best choice.

$$u(t) = K \left( e(t) + \frac{1}{T_i} \int e(t) dt + T_d \frac{de(t)}{dt} \right)$$

For all those reasons, PID controller is the best controller to maintain a good performance in terms of both speed and error elimination. It also has to be mentioned that if the parameters of PID controller are tuned compatible with the nature of the system, the performance will be satisfactory.

$$PID(s) = K \left( 1 + \frac{1}{sT_i} + sT_d \right)$$

B. Fuzzy PID Controller

In many literature studies and industrial applications, based on fuzzy sets, fuzzy PID controller is one of the most commonly used controller type since its simple structure and error elimination success. As a result of having fuzzy nature, in fuzzy PID controller, the input signals are defined as error and the derivative of the error and they are assigned to language expressions as small, medium and so on. Those language expression is multiplied by scaling factors and fuzzy values are obtained. In this study, these scaling factors are found by using genetic algorithms. After applying the fuzzy inference process (fuzzification) and defuzzification methods, a crisp value, as control signal, can be achieved.

### TABLE I

<table>
<thead>
<tr>
<th>de/e</th>
<th>S</th>
<th>SM</th>
<th>M</th>
<th>LM</th>
<th>L</th>
</tr>
</thead>
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<td>-0.7</td>
<td>-0.5</td>
<td>-0.3</td>
<td>0</td>
</tr>
<tr>
<td>SM</td>
<td>0.7</td>
<td>-0.4</td>
<td>-0.2</td>
<td>0</td>
<td>0.3</td>
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<tr>
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<td>0.2</td>
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<tr>
<td>LM</td>
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<td>0.2</td>
<td>0.4</td>
<td>0.7</td>
</tr>
<tr>
<td>L</td>
<td>0</td>
<td>0.3</td>
<td>0.5</td>
<td>0.7</td>
<td>1</td>
</tr>
</tbody>
</table>

III. FUNCTION TUNING METHOD

In order to improve the performance of the fuzzy PID controller, some self-tuning mechanisms have been proposed in literature. In this study, the function tuner method (FTM) is provided which is simply based on the idea that, without changing the fuzzy rules and scaling factors, just changing the membership functions (MF), the steady-state response can be improved [19]. In other words, FTM changed the parameters used in the adaptive method given in [18]. In parameter adaptive method the parameters are given as:

$$\beta = \delta \beta_i$$

$$K_d = \frac{K_d}{\delta}$$

The FTM provides a more feasible rule base than the conventional fuzzy PID controller. Since the conventional fuzzy PID controller needs three inputs and three dimensions, designing the rule base is more difficult than FTM [18]. The $K_d$ and $\beta$ values have been changed as given in (3) and (4), where $K_d$ and $\beta_i$ are the initial values of $K_d$ and $\beta_i$.

![Fig. 1 Closed-loop control structure for parameter adaptive PID-type FLC via function tuner [19]](image-url)
Finally, self-tuning scalar factors can be defined as given in following equations, where \( \beta_t \) and \( K_{\beta} \) are the initial values of the scaling factors.

\[
\beta(t) = \beta_t f(e(t)) \quad (7)
\]

\[
K_{\beta}(t) = K_{\beta} g(e(t)) \quad (8)
\]

In order to explain more specifically the principle of this method, with the change of error, the \( \beta(e(t)) \) decreases, since the error relation of the objective function \( f(e(t)) \). If the error is zero, \( f(e(t)) \) will be equal to the parameter \( \alpha_1 \). On the other hand, in steady state, the objective function \( g(e(t)) \) will be equal to \( b_1 + b_2 \). As a consequence, in this method, adjusting the \( \beta(e(t)) \) and \( K_{\beta}(e(t)) \) based on error is aimed.

In this structure, five symmetrical uniformly distributed triangular MF are chosen for the inputs of the fuzzy controller, which are error and change of error. The outputs of the fuzzy controller have been chosen as Singleton. The rule base for the fuzzy controller is as given in Table I where S, SM, M, LM, L refer “small”, “small-medium”, “medium”, “large-medium” and “large” respectively. The statement “de” refers the change of the error.

### IV. SIMULATION RESULTS

As mentioned before, for all three different controller types, genetic algorithm is used to tune the controller parameters. Detailed computations are given in [20], [21].

#### A. Modeling

For simulations, QUANSER Coupled-Tank experimental setup is used. The system's basic operating principle is as follows:

1) The water in the reservoir at the bottom of the system is transferred to tank 1 with the help of pumps and hoses located above the engine. The water transferred to tank 1 is transferred indirectly to tank 2 due to the hole at the bottom of the tank 1.

2) Depending on the configuration used herein, the water level in tank 1 or tank 2 is used to maintain the reference value. For this purpose, in both water tanks pressure sensors that determine the water level and used as feedback signal are used.

3) At the same time, to make the reference tracking problem little more difficult there is one disruptive tap connected to the tank 1. In necessity this tap is opened and disturbance from the outside state simulation is performed. In this study data is provided by input-output relation of tank 1. Components of the system are given in Fig. 2 and nomenclature is given in Table II.

\[
G_{\text{continuous}} = \frac{K}{\tau s + 1} \quad (9)
\]

In experimental setup of the coupled-tank system, tank 1 was used to gather the data and modeled as first order system. The continuous time and discrete time models with sampling time 0.1 of the system are given in (9) and (10), respectively, where \( K \) is 1 and \( \tau \) is 5.2. Such a system is stable since its unique pole (system of order one) is located on the left-hand-side of the s-plane. By not having any pole at the origin of the s-plane, \( G(s) \) is of type zero. It also should be noted that the model is linearized along equilibrium points, the experimental data, however, is gathered from real tank system.

\[
\frac{G_{\text{discrete}}}{z - 0.981} = 0.01905 \quad (10)
\]

### Table II

<table>
<thead>
<tr>
<th>ID #</th>
<th>Description</th>
<th>ID #</th>
<th>Description</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>Coupled-Tank overall frame</td>
<td>2</td>
<td>Tank 1</td>
</tr>
<tr>
<td>3</td>
<td>Tank 2</td>
<td>4</td>
<td>Main Water Basin</td>
</tr>
<tr>
<td>5</td>
<td>Pump</td>
<td>6</td>
<td>Flexible Tubing</td>
</tr>
<tr>
<td>7</td>
<td>Quick-Connect Inlet Orifice (Out1)</td>
<td>8</td>
<td>Quick-Connect Inlet Orifice (Out2)</td>
</tr>
<tr>
<td>9</td>
<td>Quick-Connect Coupling and Hose (Out1)</td>
<td>10</td>
<td>Quick-Connect Coupling and Hose (Out2)</td>
</tr>
<tr>
<td>11</td>
<td>Small Outlet Insert</td>
<td>12</td>
<td>Medium Outlet Insert</td>
</tr>
<tr>
<td>13</td>
<td>Large Outlet Insert</td>
<td>14</td>
<td>Plain Outlet Insert</td>
</tr>
<tr>
<td>15</td>
<td>Disturbance Tap</td>
<td>16</td>
<td>Flow Splitter</td>
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<tr>
<td>17</td>
<td>Pressure Sensor</td>
<td>18</td>
<td>Calibration and Signal</td>
</tr>
<tr>
<td>19</td>
<td>Pump Motor 4-Pin DIN Connector</td>
<td>20</td>
<td>Conditioning Circuit Board</td>
</tr>
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<td>21</td>
<td>Tank Level Scale (cm)</td>
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<td>Pressure Sensors Cable 6-Pin-Mini-DIN Connector</td>
</tr>
</tbody>
</table>

![Coupled-tank experimental setup](image)

Fig. 2 (a) Coupled-tank experimental setup front view (b) Coupled-tank experimental setup back view

It was assumed that the system is totally unknown. The input-output data of the unknown system is used and a fuzzy
model is obtained by using adaptive neuro fuzzy inference system (ANFIS). Since it is assuming that we have no information about the system, we have to examine different inputs to get the most accurate model which can anticipates the future output with the least error. So that, the input-output set given in (11) is defined to ANFIS to decide on the best inputs, in terms of having minimum root mean square error (RMSE), to the fuzzy model and as a result, the selection of best four input-output set among those eleven values, is expected. In Fig. 4, it can be seen the results of selected values as input set.

\[
\begin{bmatrix}
y(k-1), y(k-2), y(k-3), y(k-4), y(k-5), \\
u(k), u(k-1), u(k-2), u(k-3), u(k-4), u(k-5)
\end{bmatrix}
\]  

(11)

The best values are selected as \(y(k-1), y(k-2), u(k-1), u(k-2)\). These four inputs and the output \(y(k)\) is applied to ANFIS again. The data is split as 70% training data and 30% testing data and applied Takagi-Sugeno type fuzzy modeling structure. Three MF are defined and they are selected as Gaussians. After 500 epochs, the simulation resulted in error with 0.004 which is enough to continue on other steps.

In order to validate the obtained model and the real system, step responses of the real system and the obtained fuzzy model are compared. From Figs. 5 and 6, it can be seen the results of modeling steps. The model is also compared with the real data with same step inputs which is given in Fig. 7.
B. Control

In this section, three different control techniques such as evolutionary classical PID (ECPID), evolutionary fuzzy PID (EFPID) and evolutionary function tuner method (EFTM) have been performed on the fuzzy model of the system, by applying references with different amplitudes as 7, 9 and 12 respectively. Control signals of the regarding cases are also provided to make accurate observations on performances.

1) Step Input 7

From the graphical results, it is fair to say that EFTM has substantial success in terms of overshoot. It has comparatively slow response than ECPID controller and EFPID controller, however, this is result is expected since it has a trade-off between its parameters. FTM provides a robust, yet slow performance, which is not an issue in chemical or process applications such as coupled-tank system.

2) Step Input 9

3) Step Input 12
part of the study, three different control methods, classical PID controller, fuzzy PID controller and fuzzy PID type controller with self-tuning scaling factors, are utilized as evolutionary structures, genetic search algorithm, with the cost function of integral time squared error (ITSE) and compared in three different reference input cases, 7, 9, and 12, respectively. In the fuzzy PID-type controller with self-tuning scaling factors, two sorts of triangular MF are applied uniformly distributed and modified triangular MF. Evolutionary algorithm (EA) is used to tune the parameters of the classical PID controller, fuzzy PID controller and FTM. From the test results it is seen that, EA based FTM has demonstrated successful results for the closed loop operation of the coupled-tank experiment system.

REFERENCES

V. CONCLUSION
Having no information on the dynamic model of the system, the accurate fuzzy model of the system is obtained by only using input and output data of the system. In order to maintain this task, ANFIS software is used to train a fuzzy model. After testing the model, satisfying results are obtained. As second