

3D Object Model Reconstruction Based on Polywogs Wavelet Network Parametrization

Mohamed Othmani, Yassine Khelifi

Abstract—This paper presents a technique for compact three dimensional (3D) object model reconstruction using wavelet networks. It consists to transform an input surface vertices into signals, and uses wavelet network parameters for signal approximations. To prove this, we use a wavelet network architecture founded on several mother wavelet families. POLYNomials WindOwed with Gaussians (POLYWOG) wavelet families are used to maximize the probability to select the best wavelets which ensure the good generalization of the network. To achieve a better reconstruction, the network is trained several iterations to optimize the wavelet network parameters until the error criterion is small enough. Experimental results will shown that our proposed technique can effectively reconstruct an irregular 3D object models when using the optimized wavelet network parameters. We will prove that an accurateness reconstruction depends on the best choice of the mother wavelets.

Keywords—3D object, optimization, parametrization, Polywog wavelets, reconstruction, wavelet networks.

I. INTRODUCTION

PARAMETERIZATION is an important component in various computer graphics and geometry processing applications, such as surface fitting, 3D modeling, medical visualization, mesh compression, remeshing and 3D face recognition [7], [14]. However, surfaces represented by 3D meshes may contain noise or some unrequired details which add a complexity to the 3D object geometry and topology. Therefore, it has great practical significance to find a new technique with better filtering precision and noise suppression. Wavelet networks can be treated as a universal tool for anti-noise signal processing and image reconstruction [2], [8], [16]. The most used approach in the community of signal processing is frequency spectra construction using decompositions in databases functions [12], [6]. Wavelet networks combine some of the useful classification properties of Neural Networks with the localization and feature extraction properties of wavelets [10]. Wavelet networks replace the global sigmoidal activation units of the classic feedforward Neural Networks with wavelets, while preserving the network's universal approximation property. In recent years, wavelet networks combining the best features of time and frequency methods are rapidly becoming a new approach for anti-noise signal processing and Multidimensional data reconstruction [4], [11], [15]. They have the benefit capability to adapt their parameters to changes in the environment and

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are able to provide instantaneously an estimation of the output values for input values. Wavelet networks already have been proven to perform better than scalar wavelets in applications like image reconstruction, 1D and 2D data approximation and reconstruction [3], [8] and 3d modeling [9]. The technique that we will use to reconstruct an object based on wavelet networks parameterization. The general idea can be stated as reconstructing a model of an object by approximating a finite set of points in the space belonging to it.

II. WAVELET NETWORK ARCHITECTURE

The wavelet network architecture proposed by Zhang and Benveniste [10], provides a link between the neural network and the wavelet decomposition. A wavelet network with one output y , N_i inputs x_1, x_2, \dots, x_{N_i} and N wavelets can be parameterized as follows:

$$f(x) = \sum_{i=1}^N \omega_i \psi_i(x) \quad (1)$$

where the network parameters are $\omega_i \in \mathbb{R}$, $d_i \in \mathbb{R}^*$ and $t_i \in \mathbb{R}^d$, correspond respectively to the wavelet coefficient, dilation parameter and translation parameter. Wavelets ψ_i , are dilated and translated versions of a single function ψ termed the "mother wavelet": $\mathbb{R}^d \rightarrow \mathbb{R}$:

$$\psi_i = \psi(d_i(x - t_i)) \quad (2)$$

with

$$\psi(d_i(x - t_i)) = \frac{1}{\sqrt{d_i}} \psi\left(\frac{x - t_i}{d_i}\right) \quad (3)$$

The wavelet network contains only one hidden layer. In total, it is composed of three layers: input layer, hidden layer and output layer. The input layer consists of simple vectors whose entries are equal to outputs: these are the values of input data. The hidden layer contains neurons wavelet type or activation functions. The output layer contains the network output values. Between the hidden layer and output layer there are the connection weights. These are used to calculate the network output, which is a linear combination of wavelets in the hidden layer, weighted by the connection weights. The wavelet network architecture is illustrated in Fig. 1.

The proposed wavelet network architecture shown in Fig. 2 is an extension of our previous works [8] applied to multidimensional signal inputs. The classic uses dilation and translation versions of only one mother wavelet, besides the proposed version constructs the network by the implementation of several mother wavelets in the hidden layer. The objective is to maximize the potentiality to select the best wavelets that make a good approximation to the signal.

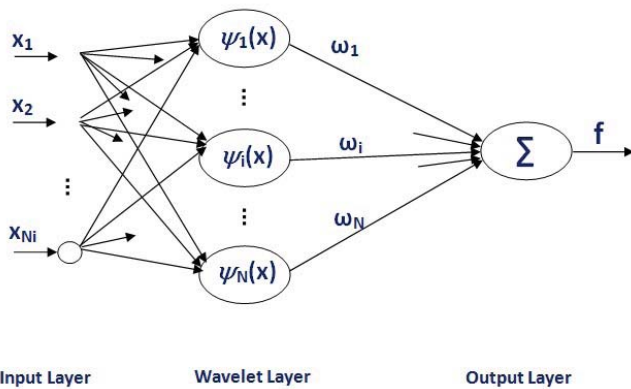


Fig. 1 The classic wavelet networks architecture

This wavelet network architecture with one output f , can be expressed by the following equation:

$$f(x) = \sum_{j=1}^M \sum_{i=1}^{N_{Mw}} \omega_i^j \psi_i^j(x) + \sum_{k=1}^{N_i} a_k x_k + b \quad (4)$$

$x = [x_1, x_2, \dots, x_{N_i}]^T$ is the vector of inputs. Functions ψ_i^j , are dilated and translated versions of a several mother wavelets ψ^j . The number of selected wavelets for each mother wavelet family depends on the wavelet family and the choice of the mother wavelet. Concerning the other parameters: $a \in \mathbb{R}^d$, $b \in \mathbb{R}$ are the linear coefficients and bias term. This wavelet network architecture can be viewed as a network with an input vectors of N_i components, a hidden layer that is constituted of N_{Mw} wavelets of M Polywogs mother wavelets; each belongs to a wavelet family of $N_l (l = [1, \dots, M])$ size and a linear output neuron.

III. POLYWOGS WAVELET NETWORK PARAMETERIZATION

As already presented in the previous paragraph, there are four main parameters to adjust in a the proposed network: the activation functions type (mother wavelet) for each wavelet, the structural parameters (translation, dilation) of the mother wavelet and the connections weights between the hidden wavelet layer and the output layer. Changing one of these parameters causes a change in network behavior. The wavelet network advantage is that when the wavelet parameters are determined, the connection weights parameter becomes simple because the dependence of the network weights is linear. It can also use an additional parameter: bias. The network expression, according to these parameters, is presented by (4).

A. Activation Functions

Several choices of the wavelets are available. Best known wavelets are certainly those which are the Haar system in the orthogonal wavelet context. The Haar system functions are not differentiable, it is not possible to apply the algorithms to estimate parameters, such as wavelet networks. Therefore, to construct the networks we used POLYNomials WindOwed with Gaussians (POLYWOG) wavelet families. These functions are differentiable and have the universal approximation proprieties.

B. Translation and Dilation

The variation pitch of the parameters of translation in time t and d dilation depend on each other, and are defined by:

$$d = d_0^{-m}, \quad t = kt_0 d_0^{-m}, \quad d_0 > 1, t_0 > 0, \quad m, k \in \mathbb{N} \quad (5)$$

In our network, the parameters d_0 and t_0 are defined as follows : $d_0 = 2$ and $t_0 = 1$. So, each wavelet are dilated by a factor of 2^{-m} and translated by a factor of $2^{-m}k$.

C. Connection Weights

The weights are generally determined by the method of least squares. The performance criterion, the most used, is a criterion for the correction error in the sense of least squares. The problem is to find a weight parameters w such the sum of squared errors defined by SSE (6) is minimum.

$$SSE(w) = \sum_{n=1}^N \left\{ \sum_{j=0}^M \sum_{i=1}^{N_l} \omega_i^j \psi_i^j(x_n) + \sum_{k=0}^{N_i} a_k x_k + b - y_k^n \right\}^2 \quad (6)$$

where y_k^n the k^{th} output target value for the index n .

D. Bias

The term independent bias can be added to the model. Therefore, it is an important parameter which cheek a stabilizing role for the network: it serves to offset the difference between the mean value of network output(on all training points) and the mean value of target values. If the wavelet network is considered without bias, the general expression of the outputs, for any vector $x = (x_1, x_2, \dots, x_N)$ space entry is:

$$f(x) = \sum_{j=1}^M \sum_{i=1}^{N_l} \omega_i^j \psi_i^j(x_n) \quad (7)$$

In our approach, we added the bias to stabilize the network, so the outputs become :

$$f(x) = \sum_{j=1}^M \sum_{i=1}^{N_l} \omega_i^j \psi_i^j(x_n) + \bar{f} \quad (8)$$

Supposing that $\bar{f} = (\omega_1^0, \dots, \omega_M^0)$. However, the term bias \bar{f} can be inserted into the summation by adding a basic function fictitious, ψ_0^0 , which is not a wavelet, but the function constant 1:

$$f(x) = \sum_{j=0}^M \sum_{i=1}^{N_l} \omega_i^j \psi_i^j(x_n) \quad (9)$$

with w the weight vector, x_n the training set value of index n . This term adds a component to be determined. Knowing the expression output of the network, the Sum of Squared Errors (SSE) between these outputs and target values can be calculated as:

$$E(w) = \frac{1}{2} \sum_{n=1}^N \left\{ \sum_{j=0}^M \sum_{i=1}^{N_l} \omega_i^j \psi_i^j(x_n) - y_k^n \right\}^2 \quad (10)$$

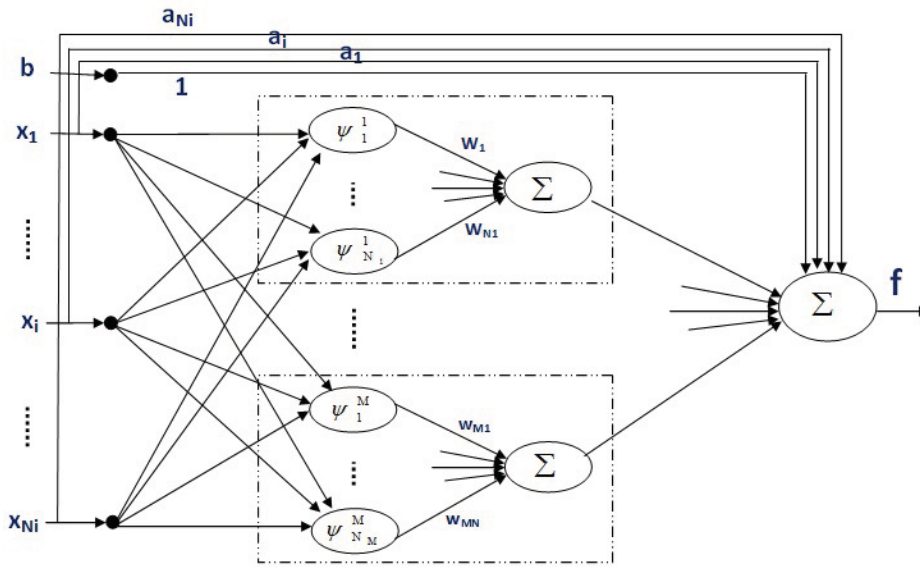


Fig. 2 The proposed wavelet networks architecture

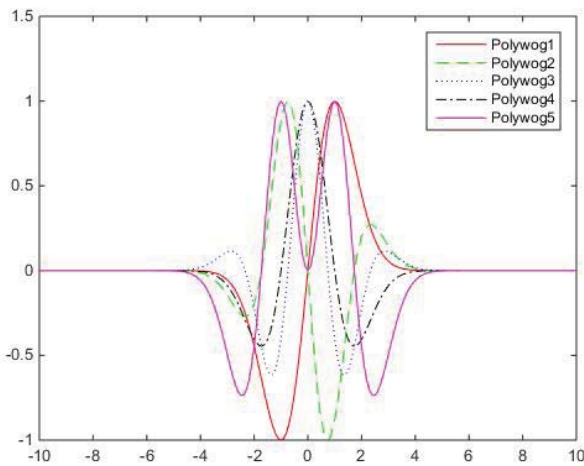


Fig. 3 The Polywog wavelets

where y_k^n the k^{th} output target value for the index n . The expression of the error can now be derived from the k^{th} component of the vector bias w_k^0

$$\frac{\partial E}{\partial w_k^0} = \sum_{n=1}^N \left\{ \sum_{j=1}^M \sum_{i=1}^{N_i} \omega_i^j \psi_i^j(x_n) + w_k^0 - y_k^n \right\} \quad (11)$$

The error is minimum for:

$$\frac{\partial E}{\partial w_k^0} = 0 \Rightarrow w_k^0 = \bar{y}_k - \sum_{n=1}^N \left\{ \sum_{j=1}^M \sum_{i=1}^{N_i} \omega_i^j \overline{\psi_i^j(x_n)} \right\} \quad (12)$$

with $\bar{y}_k = \frac{1}{N} \sum_{n=1}^N y_k$ and with $\overline{\psi_i^j(x_n)} = \frac{1}{N} \sum_{n=1}^N \psi_i^j(x_n)$ that is used to explain the bias value equal to the difference between the mean target values and the mean output values of the network.

IV. RECONSTRUCTION PROCEDURE

The reconstruction procedure has several steps to achieve:

- We generate a set of vertices $O = \{v = (x_v, y_v, z_v) \in \mathbb{R}^3\}$ with coordinates: x_v, y_v and z_v .
- We associate to every coordinate x_v, y_v and z_v an one dimensional signal which interpolate all vertices of the object O . Therefore, these signals become the wavelet network inputs.
- We initialize the wavelet network parameters by creation of a library which composed of several Polywogs mother wavelets.
- We select the best wavelet from this library using Orthogonal Least Squares (OLS) to construct the network hidden layer.
- we apply an optimization method to update the wavelet network parameters.
- We calculate the new position V of each vertex v of the object O using a wavelet network parameters to reconstruct object. Therefore, the 3D objects reconstruction can be transformed as an approximation of these signals using wavelet network.

A. Network Initialization

The first step in the reconstruction process consists of creation of the wavelets family library. This library is generated by a several Polywog mother wavelets family and it will be used to construct the network. The library elements, generated by distributing the parameters on a dyadic grid, are grouped in levels on the basic of the dilation and translation parameters. This choice presents the advantage not only to enrich the library, but also to get an efficiency performance for a given wavelet number. The inconvenience introduced by this choice concerns the size of the library. A wavelet library having several wavelets families is more voluminous than the one that possesses the same wavelet mother. It implies a

more elevated calculation cost during the process of selection. Nevertheless, using a selection algorithms, in the selection of wavelets is often shorter than the training of the dilations and translations; the supplementary cost introduced by different versions can be therefore acceptable. Let x be the variable, t_i the translation parameter and d_i the dilation parameter. The wavelet ψ_i^j of the mother wavelet family ψ^j is defined as:

$$\psi_i^j = \psi^j(d_i(x - t_i)) \quad (13)$$

wavelet library W , generated from the mother wavelet families, is defined as:

$$= \{\psi^1(d_i(x - t_i)), \dots, \psi^j(d_i(x - t_i)), \dots, \psi^M(d_i(x - t_i))\} \quad (14)$$

B. Wavelet Selection Method

The library size is usually excessively large, subset model selection is necessary. Optimal subset selection techniques are computationally prohibitive and impractical. The practical method is the forward selection, and the OLS procedure [13] which is an efficient implementation of this subset selection procedure. For the best selection, we used the OLS procedure for subset model selection [8]. Let us note the original W library has $L = M * N$ columns. To create a parsimonious model which has K significant parameters, we are actually trying to pick K columns from the input library W to form the optimal output wavelet network f . The objective of this algorithm is to select columns from the input library sequentially. The column that provides the best combination with the output f to model the signal Y will be picked to form the new output f . The OLS procedure automatically uses the orthogonalization techniques to eliminates duplication of information in the library. The orthogonalization is made very efficient by employing orthogonalization of the modified Gram-Shmidt [5].

C. Parameter Optimization

This phase is used to construct the wavelet network by optimizing its parameters, namely, their weight and their structural parameters (dilations, translations) to identify the relationship between the signal data (input - output). At each iteration, the optimization algorithm adjusts the network parameters to minimize the error between the input data and those are obtained by propagation through the wavelet network. Therefore, to estimate the network parameters, we need an optimization algorithm such as "back-propagation". In general, choosing a method of selection being made, it remains to choose the optimization algorithm to optimize the parameters. The optimization method adopted to adjust the proposed network parameters is based on Levenberg-Marquardt algorithm [1]. This is very well suited to wavelet network training where the performance index is the mean squared error. We define the mean square error of training (MSE) as:

$$MSE(\omega_i, t_i, d_i) = \frac{1}{N_i} \sum_{k=1}^{N_i} (Y(k) - f(x_k))^2 \quad (15)$$



Fig. 4 The original Feline and Happy Buddha objects

where $Y(k)$ which is the desired output corresponding to the signal data example k , and $f(x_k)$ is the wavelet network output corresponding to the example k .

D. Approximation Phase

After the optimization phase, the wavelet network is constructed using an optimal parameters. These parameters make an efficient approximated for each input signals with a good filtering precision and noise suppression. Therefore, the 3d object will be reconstructed by these wavelet network parameters and the generated object vertex will be defined as follows:

$$O_N = \{V_N = (x_N, y_N, z_N)^T \in \mathbb{R}^3\} \quad (16)$$

where x_N , y_N and z_N are the approximated signals data.

V. EXPERIMENTAL RESULTS

To evaluate the performance of this proposed technique, we used a wavelet network whose library consists of several Polywog wavelets. The Polywog wavelets are defined as a mother wavelets and used to construct the wavelet networks. As a performance index, we adopt the Mean of Square Errors (MSE) and the Normalized Square Root of the Mean Square Error (NSRMSE). The normalization is made over the empirical deviation of the output patterns. The MSE and the NSRMSE are defined as:

$$MSE = \frac{1}{N_i} \sum_{k=1}^{N_i} (O_N(x_{Nk}, y_{Nk}, z_{Nk}) - O(x_k, y_k, z_k))^2 \quad (17)$$

$$NSRMSE = \sqrt{\frac{\sum_{k=1}^{N_i} (O_N(x_{Nk}, y_{Nk}, z_{Nk}) - O(x_k, y_k, z_k))^2}{\sum_{k=1}^{N_i} (\bar{O} - O(x_k, y_k, z_k))^2}} \quad (18)$$

where O is the original object, k is the number of observations and $\bar{O} = \sum_{k=1}^{N_i} O(x_k, y_k, z_k)$.

For the simulation, we used an irregular high-dimensional objects: "Feline" and "Happy Buddha" (Fig. 4) with a special form, which represent the various deformations and folds. So, it is not easy to model the details of these objects. If we successful to reconstruct these objects we can say that the proposed technique is reliable and efficient to reconstruct a 3D object independent to its forms.

The 3D object used as input to the network is provided by VRML files. So, the vertices and their corresponding normal

TABLE I
 MSE AND NSRMSE FOR THE OBJECTS USING POLYWOGS WAVELET NETWORKS

Wavelet number	Feline		H Buddha	
	300	350	300	350
Polywog1	50	57	60	71
Polywog2	55	62	70	64
Polywog3	57	65	43	55
Polywog4	70	87	56	82
Polywog5	68	79	71	78
MSE	4.1684e-5	8.2285e-7	7.6317e-08	7.1521e-09
NSRMSE	8.8511e-3	1.2435e-3	3.6539e-3	1.1185e-3

TABLE II
 MSE AND NSRMSE FOR THE FELINE OBJECT USING CLASSIC WAVELET NETWORKS WITH 350 WAVELETS

Wavelets	Polywog1	Polywog2	Polywog3	Polywog4
Feline				
MSE	3.02484e-3	7.30474e-3	1.95822e-3	3.29988e-3
NSRMSE	7.53982e-2	3.7052e-2	6.06653e-2	7.87514e-2
H Buddha				
MSE	3.05864e-05	7.08267e-06	2.86787e-06	2.0829e-06
NSRMSE	7.31504e-2	3.52007e-2	2.23992e-2	6.03653e-2

values of a 3D object model given by the VRML are used to create a training set. We proved that the Polywogs wavelet network is adequate for reconstruction complicated objects. The only unknown variable is the number of hidden wavelets. There is no known efficient, fail proof way of determining this number for the specific problem.

In these simulation results, each subset input pattern contains 1000 points. Table I gives the mean square error and the final normalized square root of the Mean Square Error after 100 training iterations using the proposed technique constructed with 300 and 350 wavelets in hidden layer and based on Polywog mother wavelets. For example, to reconstruct the "Feline" object, which is composed of 49919 vertex, using a wavelet network composed of 350 wavelets we obtained a MSE of 8.2285e-7 and a NSRMSE of 1.2435e-3. From this table we see clearly that increasing wavelet number in hidden layer increase the approximation capacity. But increasing the wavelet number increases time cost and algorithm complexity. Also to perform these criterions (MSE and NSRMSE) we can increase the iterations number in the training stage, but in the same way as increasing wavelet number, time cost and algorithm complexity increase considerably. To compare the performances of the proposed technique we proceed by using the wavelet network composed of Polywog1, Polywog2, Polywog3, Polywog4 and Polywog5 wavelets against the classic wavelet network composed of only one mother wavelet, we intend to reconstruct "Feline" and "Happy Buddha" objects using classic wavelet networks in the same condition as the object examples using the proposed technique with 350 Polywog wavelets in hidden layer. Table II represent the MSE and NSRMSE of the object reconstructions. The number of wavelets selected to construct the wavelet network is very important and depends on the form of the 3d object. for example, to reconstruct the "Happy Buddha" object, which is composed of 32328 vertex, we used 350 wavelets. The Polywog wavelet network is constructed with 71 wavelets from Polywog1, 64 wavelets from Polywog2, 55



Fig. 5 The Results of 3D reconstruction with the following number of POLYWOG wavelets ((A):300 wavelets, (B):350 wavelets)

wavelets from Polywog3, 82 wavelets from Polywog4 and 78 wavelets from Polywog5. The selected number for each mother wavelets is generated automatical during the reconstruction procedure phases.

When comparing Tables I and II we can say that the performances obtained in term of MSE and NSRMSE using the new technique based on Polywogs wavelet networks are often very better than the one obtained with the classic. This shows that the proposed procedure brings effectively a good reconstruction capacity using the several mother wavelets.

When comparing Figs. 5 and 6 which are obtained with the proposed architecture and the other one which is obtained with classic, we can say that the new technique present the advantage of constructing the network by several mother wavelets. For that we used a Polywog mother wavelets family to achieve an accurateness performance and to improve the reconstruction quality. Based on these experiment results, We can conclude that the 3D objects reconstruction by wavelet networks is not reliable with the classic wavelet network. It cannot reconstruct the object form clearly, however, the proposed technique achieves a good precision of reconstruction in terms of error and in term of representation without noise.

VI. CONCLUSION

This paper presents our technique for the 3D object reconstruction using Polywog wavelet network parameters. To achieve an accurateness reconstruction, we used an initial and selected phase to select best wavelets for every mother wavelet. The network is trained several iterations to optimize the wavelet network parameters, until the error criterion is small enough. Some simulations treated to compare the capacity of reconstruction using the proposed and the classic wavelet networks. We deduce from these simulations that the reconstruction quality depends a lot on the choice of the activation function (wavelet) used in hidden layer and on their localization. We prove that the new technique is able to model the 3D objects with an efficiency reconstruction.

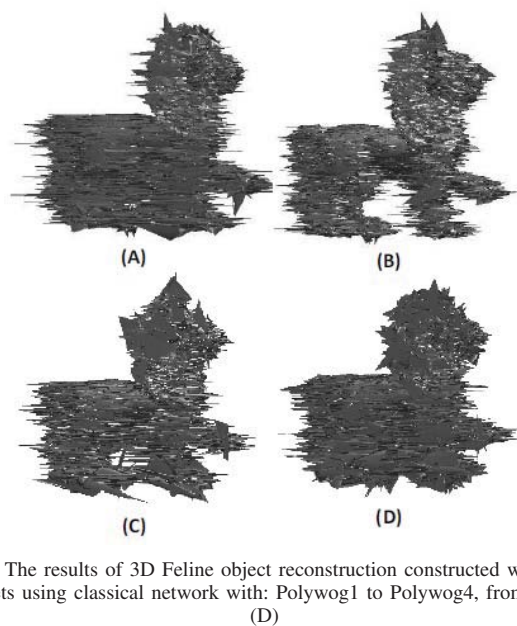


Fig. 6 The results of 3D Feline object reconstruction constructed with 350 wavelets using classical network with: Polywog1 to Polywog4, from (A) to (D)

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