World Academy of Science, Engineering and Technology
International Journal of Computer and Systems Engineering
Vol:10, No:6, 2016

Application of the Total Least Squares Estimation Method for an Aircraft Aerodynamic Model Identification

Zaouche Mohamed, Amini Mohamed, Foughali Khaled, Aitkaid Souhila, Bouchiha Nihad Sarah

***Abstract*—**The aerodynamic coefficients are important in the evaluation of an aircraft performance and stability-control characteristics. These coefficients also can be used in the automatic flight control systems and mathematical model of flight simulator. The study of the aerodynamic aspect of flying systems is a reserved domain and inaccessible for the developers. Doing tests in a wind tunnel to extract aerodynamic forces and moments requires a specific and expensive means. Besides, the glaring lack of published documentation in this field of study makes the aerodynamic coefficients determination complicated. This work is devoted to the identification of an aerodynamic model, by using an aircraft in virtual simulated environment. We deal with the identification of the system, we present an environment framework based on Software In the Loop (SIL) methodology and we use Microsoft™ Flight Simulator (FS-2004) as the environment for plane simulation. We propose The Total Least Squares Estimation technique (TLSE) to identify the aerodynamic parameters, which are unknown, variable, classified and used in the expression of the piloting law. In this paper, we define each aerodynamic coefficient as the mean of its numerical values. All other variations are considered as modeling uncertainties that will be compensated by the robustness of the piloting control.

***Keywords*—**Aircraft aerodynamic model, Microsoft flight simulator, MQ-1 Predator, total least squares estimation, piloting the aircraft.

I. Introduction

In literature, the aeronautic system identification is widely explored. Particularly in avionics, in this field, the aerodynamic coefficients identification from the flight data was successfully done by the physical model interlude [1], [2].

The parameters of an aircraft aerodynamic model are variable because the aerodynamic coefficients changes. The techniques employed to obtain the different values of the aerodynamic coefficients derivatives are based on the four information sources:

1. Handbook methods
2. Computational fluid dynamics
3. Wind tunnel
4. Flight testing

In the first two sources, the aerodynamic coefficients derivatives are based on theoretical calculations. In wind tunnel experiments, the aerodynamic derivatives are more or less directly measurable through systematically varying one model state at a time and observing the resulting changes in the aerodynamic forces and moments obtained from the wind tunnel balance measurements.

Unfortunately, the straightforward procedure applied in wind tunnel experiments does not carry over to flight testing. In free flight, the aircraft is not attached to a balance and the aerodynamic forces and moments cannot be measured directly. Measuring all aircraft state elements, which is relatively simple in wind tunnel testing, becomes a non-trivial problem in flight testing. Certain state elements may be too cumbersome to be measured directly. Moreover, systematically changing one state variable at a time is not feasible (for example, changing the elevator deflection angle δ_e during flight, will immediately result in a pitch rate and angle-of-attack response as well). As a result of these difficulties, the extraction of aerodynamic derivatives from flight data is a complicated matter which is most efficiently solved with the application of the system identification techniques [3].

In this paper, we have to identify the aerodynamic aircraft model. The dynamic model of this system is nonlinear, MIMO and coupled. It's composed by six states variables: the roll, pitch and yaw rate p, q, r around body axes in $\left(rd/s\right)$ and the airspeed components u, v, w along the body axes in $\left(m/s\right)$. The first three components are provided by the gyro meter, after their processing we can use them in the control algorithms, in the opposite, the last three components are unknown which forces us to estimate them.

II. Problem Statement

Through a methodology based on the confrontation between the real and the simulated world, the main objective of the present work is to identify the aerodynamic model of an aircraft flying in a virtual environment (Fig. 1). To achieve this objective, we use Flight Simulator FS2004 as simulated world environment, coupled to a hardware and software development platform. Flight Simulator FS2004 is dovelopped by Microsoft and it has a worthy simulated aircrafts library. To identify the aerodynamic aircraft model, we propose the following approach:

- Implementation of a real time interface between the flight simulator FS2004 and the real time Windows target module of Simulink/Matlab.

M. Zaouche is with the Technology Department, Centre de Recherche et Développement Réghaia, Algeria (phone: 00213551391389, e-mail: zaouchemohamed@yahoo.fr).

M. Amini, K. Foughali, S. Aitkaid and N. S. Bouchiha are researchers at Centre de Recherche et Développement Réghaia.

- Description and analysis of the aerodynamic model.
- Development and implementation of the identification techniques based on TLSE for the identification of the aerodynamic parameters.
- Flight tests.

We choose the MQ-1 Predator airplane which is used in reconnaissance or attack.

Fig. 1 Real trajectory

Fig. 2 Aircraft and environment visualization

III. Characteristics of the Unmanned Aerial Vehicle MQ-1 Predator

Airwrench tool gives access to flight dynamic characteristics (mudpond.org/Air Wrench/main.htm). This tool allows creating and tuning flight dynamics files description of simulated planes models. This software uses aerodynamics formulas and equations described on the Mudpond Flight Dynamics Workbook. It calculates aerodynamic coefficients based on the physical characteristics and performance of the aircraft (Table I).

IV. Implementation of a Real-Time Interface between Microsoft Flight Simulator and the "Real Time Windows Target" Module of Simulink/MATLAB

We design our Software to interface the simulated aircraft in Flight Simulator environment (read and/or write many sensors, actuators data and parameters).

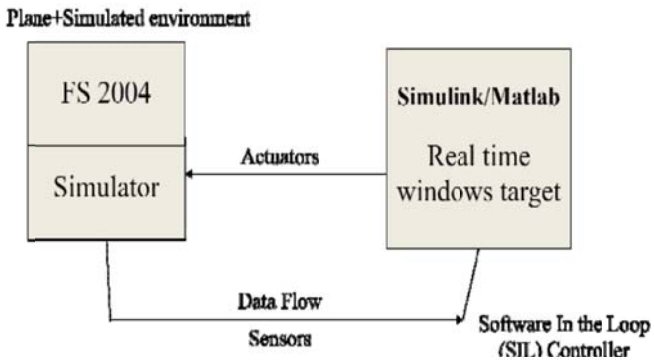

Fig. 3 Block diagram of the software environment design

TABLE I
FS2004 Aircraft Simulated Characteristics MQ-1 Predator

Dimensions	Fixed Pitch propeller	Moments of inertia
Length: 11.88 M	Prop Diameter: 1.92 M	Pitch: 1800.00
Wingspan: 14.84 M	Prop Gear Ratio: 1.00	Roll: 3700.00
Wing Surface Area: 11.43 M2	Tip Velocity: 1.478 Mach	Yaw: 1800.00
Wing Root Chord: 1.55 M	Prop Blades: 2	Cross: 0.00
Aspect Ratio: 19.28	Beta Fixed Pitch: 20.00deg	
Taper Ratio: 0.10	Prop Efficiency: 0.870	
	Design Altitude: 1524.0 M	

We communicate with FS2004 by using a dynamic link library called FSUIPC.dll (Flight Simulator Universal Inter Process Communication). This library created by Peter Dowson is downloadable from his website [20] and can be installed by being copied in the directory (module) of FS2004. It allows external applications to read and write in and from Microsoft Flight Simulator MSFS by exploiting a mechanism for IPC (Inter-Process Communication) using a buffer of 64 Kb. The organization of this buffer is explained in the documentation given with FSUIPC, from which Table II is taken.

To read or write a variable, we need to know its address in the table, its format and the necessary conversions. For example, the indicated air speed is read as a signed long S32 at the address 0x02BC.

TABLE II
FLIGHT PARAMETERS IN THE BUFFER FSUIPC

Adress	Name	Var.Type	Size (octet)	Usage
6010	Latitude (λ)	FLT64	8	Degree
6018	Longitude (μ)	FLT64	8	Degree
6020	Altitude (h)	FLT64	8	Meter
057C	Bank angle (ϕ)	S32	4	Degree
0578	Elevation angle (θ)	S32	4	Degree
0578	Head angle (ψ)	U32	4	Degree
30B0	Rotation rate (p)	FLT64	8	rad/s
30A8	Rotation rate (q)	FLT64	8	rad/s
30B8	Rotation rate (r)	FLT64	8	rad/s
3060	Acceleration (a_x)	FLT64	8	ft/s^2
3068	Acceleration (a_y)	FLT64	8	ft/s^2
3070	Acceleration (a_z)	FLT64	8	ft/s^2
0842	Vertical speed (Vz)	S16	2	meter/min
02BC	Speed IAS (V)	S32	4	Knot*128
2ED0	Incidence (α)	FLT64	8	Radian
2ED8	Incidence (β)	FLT64	8	Radian
0BB2	Elevator deflection (δe)	S16	2	-16383 to +16383
0BB6	Aileron deflection (δa)	S16	2	-16383 to +16383
0BBA	Rudder deflection (δr)	S16	2	-16383 to +16383
088C	Thrust control (δx)	S16	2	-16383 to +16383

V. IDENTIFICATION PROCEDURE

Structuring the Aerodynamic Data Base Aerodynamic Models

The aerodynamic data are expressed in terms of three forces (drag X, lift Z, side force Y) and three moments (pitching moment M, rolling moment L, yawing moment N) that act on the aircraft. Next, the effect of dynamic pressure $Q = \frac{1}{2}\rho V^2$ and aircraft size (expressed in terms of wing area S, and mean aerodynamic chord $\bar{c}$ or wing span b) is eliminated through working with dimensionless aerodynamic forces and moments coefficients $C_X \cdots C_n$ [8], [9], [13]–[15].

$$C_X = \frac{X}{\frac{1}{2}\rho V^2 S}, C_Z = \frac{Z}{\frac{1}{2}\rho V^2 S}, C_Y = \frac{Y}{\frac{1}{2}\rho V^2 S} \tag{1}$$

$$C_l = \frac{L}{\frac{1}{2}\rho V^2 Sb}, C_m = \frac{M}{\frac{1}{2}\rho V^2 S\bar{c}}, C_n = \frac{N}{\frac{1}{2}\rho V^2 Sb} \tag{2}$$

where; V : true airspeed m/s, ρ : air density Kg/m^3.

The aerodynamic coefficients $C_X \cdots C_n$, in (1) and (2) are functions of the time histories of the aircraft state, i.e. angle-of-attack α, angle-of-sideslip β, the aircraft rotation rates p, q, r, as well as the control surface deflections δ_e, δ_r, δ_r. The functional relationship between the aerodynamic coefficients and the state variables are expressed in terms of Taylor's series expansions about a reference state. A representative example is [7], [8], [14], [15], [18], [19]:

$$\begin{cases} C_X = C_{X_0} + C_{X_\alpha}\alpha + C_{X\alpha^2}\alpha^2 + C_{Xq}\frac{q\bar{c}}{V} + C_{X\delta_e}\delta_e + C_{X_0F_p}F_p \\ C_Y = C_{Y_0} + C_{Y\beta}\beta + C_{Yp}\frac{pb}{2V} + C_{Yr}\frac{rb}{2V} + C_{Y\delta_a}\delta_a + C_{Y\delta_r}\delta_r + C_{Y\beta F_p}\beta F_p \\ C_Z = C_{Z_0} + C_{Z\alpha}\alpha + C_{Zq}\frac{q\bar{c}}{V} + C_{Z\delta_e}\delta_e + C_{Z\alpha F}\alpha F_p \\ C_l = C_{l_0} + C_{l\beta}\beta + C_{lp}\frac{pb}{2V} + C_{lr}\frac{rb}{2V} + C_{l\delta_a}\delta_a + C_{l\delta_r}\delta_r + C_{lF_p}\delta_1 F_p \\ C_m = C_{m_0} + C_{m\alpha}\alpha + C_{mq}\frac{q\bar{c}}{V} + C_{m\delta_e}\delta_e + C_{m\alpha F}\alpha F_p \\ C_n = C_{n_0} + C_{n\beta}\beta + C_{np}\frac{pb}{2V} + C_{nr}\frac{rb}{2V} + C_{n\delta_a}\delta_a + C_{n\delta_r}\delta_r + C_{nF_p}\delta F_p \end{cases} \tag{3}$$

where,

$$C_{Z\alpha} = \frac{\partial C_Z}{\partial \alpha}, C_{Y\beta} = \frac{\partial C_Y}{\partial \beta}$$

$$\delta_1 = (y_{cg} - y_{jet}),\ \delta = (z_{cg} - z_{jet})$$

x_{jet}, y_{jet} and z_{jet} are the positions of the specific force sensors.

TABLE III
AERODYNAMIC COEFFICIENTS

C_X :	Axial force coefficient (body frame)
C_{X0} :	Drag at zero angle of attack and sideslip
$C_{X\alpha}$:	Drag due to the angle of attack. Angle of attack will increase the area facing the wind and therefore increase the drag
C_Y :	Side force (body frame)
$C_{Y\beta}$:	Cross-wind force due to the sideslip. Sideslip will expose one side of the fuselage to the wind causing a net force in the Y direction
C_Z :	Normal force coefficient (body frame)
C_{Z0} :	Lift at zero angle of attack. This will be non-zero for asymmetric airfoils
$C_{Z\alpha}$:	Lift due to the increased angle of attack. The airfoil will generate greater lift as the angle of attack increases until the wing stalls
C_l, C_m, C_n	Roll, pitch, yaw moment coefficients;
C_{lp} :	Damping from angular velocity. Mainly caused by the main wing when rolling
$C_{l\delta_a}$:	Roll moment due to aileron control surface deflection
$C_{l\beta}$:	Restoring roll moment due to the sideslip. Mainly caused by the dihedral angle of the wing and the vertical stabilizer
$C_{m\alpha}$:	Restoring pitch moment due to the angle of attack. The tail horizontal surface will be the main contributor to this moment as it will hit the wind with an angle and thus generate the restoring moment
C_{mq} :	Damping from angular velocity. Mainly caused by the vertical stabilizer when yawing
$C_{m\delta_e}$:	Pitch moment due to the elevator surface deflection
$C_{n\beta}$:	Pitch damping coefficient
C_{nr} :	Restoring yaw moment due to theside slip. Vertical stabilizer at the tail will generate a restoring moment because of the exposed area while side slipping
$C_{n\delta_r}$	Yaw moment due to the rudder control surface deflection

There is a verification model that matches the aerodynamic coefficients to the measurements provided by the sensors of the aircraft. In this case, the actual time series of the aerodynamic coefficients are not directly obtained from the simulator, but calculated based on typical aircraft instrumentation. The instruments are in this case, assumed to provide noise and bias free perfect measurements. This does not mean that systematic errors are not included and simulations will demonstrate how the corrections affect the results. Table III shows the used aircraft sensors.

TABLE IV
SENSORS USED FOR OBTAINED MEASUREMENTS

Sensor	Measured value	Description
Accelerometer	a_x, a_y, a_z	Acceleration in body coordinates.
Rate gyro	p, q, r	Angular velocity in body co- ordinates.
Alpha vane	α	Angle of attack sensor.
Beta vane	β	Measures the flank angle.
Pitot-static tube	Q	Measures dynamic pressure.
Aileron sensor	δ_a	Aileron deflection angle.
Elevator sensor	δ_e	Elevator deflection angle.
Rudder sensor	δ_r	Rudder deflection angle.

$$\frac{m\left(a_x\cos\alpha\cos\beta+a_y\sin\beta+a_z\sin\alpha\cos\beta\right)-F_p\cos\alpha_m\cos\beta_m}{\frac{1}{2}\rho SV^2}=$$
$$C_{X_0}+C_{X_\alpha}\alpha+C_{X\alpha^2}\alpha^2+C_{X_Q}\frac{q\bar{c}}{V}+C_{X_{\delta_e}}\delta_e$$
$$\frac{m\left(-a_x\sin\alpha+a_z\cos\alpha\right)-F_p\sin\alpha_m}{\frac{1}{2}\rho SV^2}=C_{Z_0}+C_{Z_\alpha}\alpha+C_{Z_Q}\frac{q\bar{c}}{V}+C_{Z_{\delta_e}}\delta_e$$
$$\frac{m\left(-a_x\cos\alpha\sin\beta+a_y\cos\beta-a_z\sin\alpha\sin\beta\right)+F_p\cos\alpha_m\sin\beta_m}{\frac{1}{2}\rho SV^2}=$$
$$C_{Y_0}+C_{Y_\beta}\beta+C_{Y_P}\frac{pb}{2V}+C_{Y_R}\frac{rb}{2V}+C_{Y_{\delta_a}}\delta_a+C_{Y\delta_r}\delta_r$$
$$\frac{\dot{p}I_{xx}+q.p\left(I_{zz}-I_{yy}\right)-\left(p.q+\dot{r}\right)I_{xz}}{\frac{1}{2}\rho SbV^2}=$$
$$C_{l_0}+C_{l_\beta}\beta+C_{lp}\frac{p.b}{2V}+C_{lr}\frac{r.b}{2V}+C_{l\delta_a}\delta_a+C_{l\delta_r}\delta_r$$
$$\frac{\dot{q}I_{yy}+r.p\left(I_{xx}-I_{zz}\right)+\left(p^2-r^2\right)I_{xz}-\Delta F_P}{\frac{1}{2}\rho S\bar{c}V^2} \tag{4}$$
$$=C_{m_0}+C_{m\alpha}\alpha+C_{mq}\frac{q.\bar{c}}{V}+C_{m\delta_E}\delta_e$$
$$\frac{\dot{r}I_{zz}+q.p\left(I_{yy}-I_{xx}\right)+\left(r.q-\dot{p}\right)I_{xz}}{\frac{1}{2}\rho SbV^2}$$
$$=C_{n_0}+C_{n_\beta}\beta+C_{np}\frac{p.b}{2V}+C_{nr}\frac{r.b}{2V}+C_{n\delta_a}\delta_a+C_{n\delta_r}\delta_r$$

a_x, a_y, a_z are the linear acceleration components.

In *Airwrench*, the aircraft engine position has a pitch and a yaw offset orientation angles. In the case of our Unmanned Aerial Vehicle "UAV" (MQ-1 Predator), the pitch setting is $\alpha_m = 20\text{ degree} = 0{,}349\text{ radian}$, and the yaw setting is $\beta_m = 0$. The engine propulsion force is written in the body frame reference [5]:

$$F=F_p\begin{pmatrix}\cos\beta_m\cos\alpha_m\\ \sin\beta_m\\ \cos\beta_m\sin\alpha_m\end{pmatrix}\sigma_t=\frac{K_m\rho}{V_a} \tag{5}$$

The modulus of the aerodynamic velocity is represented by V_a with K_m as a constant and σ_t representing the position of the throttle, between 0 and 1 inclusive. This verification model allows a linear formulation of the aerodynamic coefficients identification problem presented by the (9).

Measurement vector Y is defined as:

$$Y_1 = \frac{m(a_x \cos\alpha\cos\beta + a_y \sin\beta + a_z \sin\alpha\cos\beta) - F_p \cos\alpha_m \cos\beta_m}{0.5\rho S V^2}$$
$$Y_2 = \frac{m(-a_x \sin\alpha + a_z \cos\alpha) - F_p \sin\alpha_m}{0.5\rho S V^2}$$
$$Y_3 = \frac{m(-a_x \cos\alpha\sin\beta + a_y \cos\beta - a_z \sin\alpha\sin\beta) + F_p \cos\alpha_m \sin\beta_m}{0.5\rho S V^2}$$
$$Y_4 = \frac{\dot{p} I_{xx} + q.p(I_{zz} - I_{yy}) - (p.q + \dot{r}) I_{xz}}{0.5\rho S b V^2}$$
$$Y_5 = \frac{\dot{q} I_{yy} + r.p(I_{xx} - I_{zz}) + (p^2 - r^2) I_{xz} - \alpha.\delta F_p}{0.5\rho S \bar{c} V^2}$$
$$Y_6 = \frac{\dot{r} I_{zz} + q.p(I_{yy} - I_{xx}) + (r.q - \dot{p}) I_{xz}}{0.5\rho S b V^2}$$
$$Y = [Y_1 \quad Y_2 \quad Y_3 \quad Y_4 \quad Y_5 \quad Y_6]^T \tag{6}$$

A level of (4) and (6), we note that there is no sensors which provide the angular accelerations $\dot{p}$, $\dot{q}$ and $\dot{r}$. We used a differentiator presented by [5]–[7], [12]:

$$\dot{z}_i = z_{i-1} + \lambda_i |z_1 - f_i(t)|^{\frac{(n-i)}{n}} sign(z_1 - f_i(t)) \tag{7}$$

where, $i = 1,2,3$, $\lambda_i \succ 0$, $i = 1,2,3$, $z_1 = [p \quad q \quad r]^T$.

$$f_1(t) = \frac{-I_{zz}}{\Delta}\left[-I_{xz} pq + (I_{yy} - I_{zz}) qr + C_{l_2} p + C_{l_3} q\right] - \frac{I_{xz}}{\Delta}\left[(I_{xx} - I_{yy}) pq - I_{xz} qr - C_{n_2} p - C_{n_3} r\right] - \frac{1}{\Delta}\left[I_{zz}\left(C_{l_5}\beta + C_{l_1}\dot{\beta} + C_{n7}\right) - I_{xz}\left(C_{n_6}\beta + C_{n_1}\dot{\beta}\right) + C_{n_7}\right]$$

$$f_2(t) = \frac{1}{I_{yy}}(I_{zz} - I_{xx}) pr + I_{xz}(r^2 - p^2) + C_{m_5}\alpha + C_{m_1}\dot{\alpha} + C_{m_4}$$

$$f_3(t) = \frac{-I_{xz}}{\Delta}\left[I_{xz} pq - I_{xx}(I_{yy} - I_{zz}) qr - C_{l_2} p - C_{l_3} r\right] - \frac{I_{xx}}{\Delta}\left[(I_{yy} - I_{xx}) pq + I_{xz} qr + C_{n_2} p + C_{n_3} r\right] - \frac{1}{\Delta}\left[-I_{xz}\left(C_{l_5}\beta + C_{l_1}\dot{\beta} + C_{l7}\right) + I_{xx}\left(C_{n_5}\beta + C_{n_1}\dot{\beta}\right) + C_{n_7}\right] \tag{8}$$

$$\Delta = I_{xz}^2 - I_{xx} I_{zz}$$

The modified aerodynamic coefficients are presented in Table V [7].

Analyzes Tests of Flight

The estimation of the aerodynamic coefficients derivatives of an aircraft requires the data processing of flight. Consequently, these data are recorded in real-time and treated. Between the flights, measured variables can be traced to make sure that at least the sensors answered the movements of the aircraft. However, the estimated aerodynamic coefficients derivatives start as soon as the test routine of flight is finished. This after flight procedure of analysis of data does not modify the flight test results.

TABLE V
EXPRESSION OF THE MODIFIED AERODYNAMIC COEFFICIENTS

$C_{n1} = \frac{QSb^2 C_{n\dot{\beta}}}{2V}$	$C_{n2} = \frac{QSb^2 C_{np}}{2V}$	$C_{n3} = \frac{QSb^2 C_{nr}}{2V}$
$C_{n6} = QSbC_{n\beta}$	$C_{n7} = QSbC_{n0}$	$C_{l1} = \frac{QSb^2 C_{l\dot{\beta}}}{2V}$
$C_{l2} = \frac{QSb^2 C_{lp}}{2V}$	$C_{l3} = \frac{QSb^2 C_{lr}}{2V}$	$C_{l4} = QSbC_{l\delta a}$
$C_{l5} = QSbC_{l\beta}$	$C_{l7} = QSbC_{l0}$	$C_{m1} = \frac{QSc^2 C_{m\dot{\alpha}}}{V}$
$C_{m2} = \frac{QSc^2 C_{mq}}{V}$	$C_{m3} = \frac{QScC_{m\delta e}}{I_{yy}}$	$C_{m4} = QSbC_{m0}$
$C_{m5} = QScC_{m\alpha}$		

VI. TOTAL LEAST-SQUARES METHOD

A. Introduction

The Total Least Squares (TLS) method has been devised as a more global fitting technique than the ordinary least squares technique for solving over determined sets of linear equations $A\underline{\Theta} \approx \underline{Y}$ when errors occur in all data. This method, introduced into numerical analysis by Golub and Van Loan [4], is strongly based on the Singular Value Decomposition (SVD). If the errors in the measurements A and $\underline{Y}$ are uncorrelated with a zero mean and an equal variance, TLS is able to compute a strongly consistent estimate of the true solution of the corresponding unperturbed set $A_0\underline{\Theta} = Y_0$. In the statistical literature, these coefficients are called the parameters of a classical errors-in-variables model [17].

TLS is an extension of the usual Least Squares method: it allows dealing also with uncertainties on the sensitivity matrix. In this paper the TLS method is analyzed with a robust use of the SVD decomposition technique, which gives a clear understanding of the sense of the problems and provides a solution expressed in closed form in the cases where a solution exists. We discuss its relations with the LS problem [16], [17] and give the expression for the parameters governing the stability of the solutions. At the end we present the algorithm for computing $\underline{\Theta}_{TLS}$, solution of the estimated aerodynamic coefficients problem.

B. Algorithm

The following theorem gives conditions for the existence and uniqueness of a TLS solution.

Theorem 1[4], [10], [11] (Solution of the classical TLS problem):

Let; $C := [A \quad Y] = U\Sigma V^T$, where $\Sigma = diag(\sigma_1 \quad \cdots \quad \sigma_{n+d})$ is the Singular Value Decomposition (SVD) of C with $\sigma_1 \geq \cdots \geq \sigma_{n+d}$ are the singular values of C.The partitioning is defined as:

$$V := \begin{array}{c} \quad n \qquad d \\ \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \begin{matrix} n \\ d \end{matrix} \end{array} \text{ and } \Sigma := \begin{array}{c} \quad n \qquad d \\ \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{matrix} n \\ d \end{matrix} \end{array} \tag{9}$$

A TLS solution exists if and only if V_{22} is non-singular. In addition, it is unique if and only if $\sigma_n \neq \sigma_{n+1}$.

In the case when TLS solution exists and is unique, it is given by:

$$\hat{\Theta}_{TLS} = -V_{12}V_{22}^{-1} \tag{10}$$

and the corresponding TLS correction matrix is:

$$\Delta C_{TLS} = [\Delta A_{TLS} \quad \Delta Y_{TLS}] = -U.diag(0, \Sigma_2)V^T \tag{11}$$

In the generic case when a unique TLS solution $\hat{\Theta}_{TLS}$ exists, it is computed from the d right singular vectors corresponding to the smallest singular values by normalization. This gives Algorithm 1 as a basic algorithm for solving the classical TLS problem. Note that TLS correction matrix ΔC_{TLS} is such that TLS data approximation

$$\hat{C}_{TLS} = C + \Delta C_{TLS} = U.diag(\Sigma_1, 0)V^T \tag{12}$$

is the best rank-n approximation of C.

Algorithm 1 Basic total least squares algorithm
Input: $A \in R^{m\times n}, \underline{Y} \in R^{m\times d}$

1: Compute the singular value decomposition $[A \quad Y] = U\Sigma V^T$
2: **if** V_{22} is not singular **then**
3: Set $\hat{\Theta}_{TLS} = -V_{12}V_{22}^{-1}$
4: **else**
5: Output a message that the problem (TLS) has no solution and stop.
6: **end if**
7: **Output:** $\hat{\Theta}_{TLS}$ a total least squares solution of $A\underline{\Theta} \approx \underline{Y}$

VII. Application of the TLSE Method for MQ-1 Predator

A. Problem Formulation

The equations can be represented using vector and matrix notation,

$$\underline{Y} = A\underline{\Theta} \tag{13}$$

where Y is (6×1) the dimensional vector of the variable-to-be-explained, A is the (6×37) dimensional matrix of explanatory variables, and $\underline{\Theta}$ is the (37×1) dimensional vector of system parameters where,

$$Y = [C_X \quad C_Y \quad C_Z \quad C_l \quad C_m \quad C_n]^T_{6\times 1} \tag{14}$$

$$A = [A_1 \quad A_2 \quad A_3]_{6x37} \tag{15}$$

$$A_1 = \begin{bmatrix} \alpha & 0 & 0 & 0 & 0 & 0 & \alpha^2 & \frac{q\bar{c}}{V} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & \frac{pb}{2V} & \frac{rb}{2V} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{q\bar{c}}{V} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{pb}{2V} & \frac{rb}{2V} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{q\bar{c}}{V} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{pb}{2V} & \frac{rb}{2V} \end{bmatrix}_{6x16}$$

$$A_2 = \begin{bmatrix} \delta_e & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_a & \delta_r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_e & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_a & \delta_r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_e & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta_a & \delta_r \end{bmatrix}_{6.9}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & F_p & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & \beta F_p & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \alpha F_p & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \delta_1 F_p & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \alpha F_p & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & \delta F_p \end{bmatrix}_{6\times 12}$$

$$\underline{\Theta} = [\underline{\Theta}_1 \quad \underline{\Theta}_2 \quad \underline{\Theta}_3 \quad \underline{\Theta}_4]^T_{37\times 1} \tag{16}$$

$$\underline{\Theta}_1 = \left[C_{X\alpha} \quad C_{Y\beta} \quad C_{Z\alpha} \quad C_{l\beta} \quad C_{m\alpha} \quad C_{n\beta} \quad C_{X\alpha^2} \quad C_{Xq} \quad C_{Yp} \quad C_{Yr} \quad C_{Zq} \right]$$

$$\underline{\Theta}_2 = \left[C_{lp} \; C_{lr} \; C_{mq} \; C_{np} \; C_{nr} \; C_{X\delta_e} \; C_{Y\delta_a} \; C_{Y\delta_r} \; C_{Z\delta_e} \; C_{l\delta_a} \right.$$

$$\underline{\Theta}_2 = \left[C_{m\delta_e} \quad C_{n\delta_a} \quad C_{n\delta_r} \quad C_{X_0} \quad C_{Y_0} \quad C_{Z_0} \quad C_{l_0} \quad C_{m_0} \quad C_{n_0} \quad C_{XF_p} \quad C_{Y\beta F_p} \quad C_{Z\alpha F} \right]$$

$$\underline{\Theta}_4 = \left[C_{lF_p} \quad C_{m\alpha F} \quad C_{nF_p} \right]$$

This is accomplished through rewriting the linear model of (3) as,

$$\left[A \quad \left| \underline{Y} \right. \right] \begin{bmatrix} \underline{\Theta} \\ -1 \end{bmatrix} = 0 \tag{17}$$

Both the vector of the variable-to-be-explained Y and certain columns of the matrix of explanatory variables A stem from measurements which are subject to measurement errors. Under these circumstances, a clear distinction between true values and measured data must be made,

$$\left[A_m \quad \left| \underline{Y}_m \right. \right] = \left[A_0 \quad \left| \underline{Y}_0 \right. \right] + \left[\Delta A_0 \quad \left| \Delta \underline{Y}_0 \right. \right] \tag{18}$$

where, index m is used to indicate the measurements, index 0 is used to indicate the true values, and prefix A is used to indicate measurement errors.

Notice that the linear relation (13) is valid for the true data but will, in general, not be valid for the measured data,

$$\begin{aligned} \left[A_0 \quad \left| Y_0 \right. \right] \begin{bmatrix} \underline{\Theta}_0 \\ -1 \end{bmatrix} &= 0 \\ \left[A_m \quad \left| \underline{Y}_m \right. \right] \begin{bmatrix} \underline{\Theta}_m \\ -1 \end{bmatrix} &\neq 0 \end{aligned} \tag{19}$$

Define the Singular Value Decomposition of the compound data matrix according to Theorem l as:

$$\left[A_m \quad \left| \underline{Y}_m \right. \right] = U \begin{bmatrix} \sigma_1 & & & & & \\ & \sigma_2 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & \sigma_{n+1} \\ -- & -- & -- & -- & -- & -- \\ & & & 0 & & \end{bmatrix} V^T \tag{20}$$

The last and smallest singular value σ_{n+1} indicates the Frobenius norm distance of matrix $\left[A_m \quad \left| \underline{Y}_m \right. \right]$ to the nearest rank deficient matrix. With this knowledge, the estimates of the (most probable) minimum norm data correction matrix and the (most probable) corrected data matrix are,

$$\left[\Delta \hat{A} \quad \Delta \left| \underline{\hat{y}} \right. \right] = U \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & 0 & \\ & & & & & \sigma_{n+1} \\ -- & -- & -- & -- & -- & -- \\ & & & 0 & & \end{bmatrix} V^T \tag{21}$$

$$\left[\hat{A} \left| \underline{\hat{Y}} \right. \right] = U \begin{bmatrix} \sigma_1 & & & & & \\ & \sigma_2 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \sigma_n & \\ & & & & & 0 \\ -- & -- & -- & -- & -- & -- \\ & & & 0 & & \end{bmatrix} V^T \tag{22}$$

An estimated of the (most probable) extended and transformed parameter vector $\left[\underline{\hat{\Theta}}^T \quad \left| \underline{-1} \right. \right]^T$ must satisfy,

$$\left[\hat{A} \quad \left| \underline{\hat{Y}} \right. \right]^T . \begin{bmatrix} \underline{\hat{\Theta}} \\ -1 \end{bmatrix} = 0 \tag{23}$$

or,

$$\begin{bmatrix} \underline{\hat{\Theta}} \\ -1 \end{bmatrix} = \lambda . \ker \left\{ \left[\hat{A} \quad \left| \underline{\hat{y}} \right. \right] \right\} \tag{24}$$

in which λ is a scalar multiplier used to make the last element of $\underline{v}_{n+1}$ equal to -1. The kernel of matrix $\left[\hat{A} \quad \left| \underline{\hat{Y}} \right. \right]$ equals the last right singular vector $\underline{v}_{n+1}$.

B. Simulation Results

We note that the aircraft's takeoff operation is executed manually (by keyboard and / or joystick). Then we make the real-time acquisition of sensor response signals developed by our code. The piloting controls are sent by using the PPJoy (virtual joystick).

Several flight tests were conducted by changing the simulation parameters (season, time of day, weather). We present some recorded signals taken from the Inertial Measurement Unit (IMU).

We present some results of aerodynamic coefficients derivatives. They are function of the time and their values are around the intrinsic values.

The mean values of aerodynamic coefficients derivatives are given in Table VI.

TABLE VI
ESTIMATED AERODYNAMIC DERIVATIVES VALUES

$C_{X_0} = -0.0314$	$C_{X\alpha} = 8.619e-05$	$C_{X\alpha^2} = -2.366e-07$
$C_{Xq} = 132e-06$	$C_{X\delta_e} = -0.0314$	$C_{X_0F_p} = -0.01012$
$C_{Y_0} = -37.82$	$C_{Y\beta} = -3.23$	$C_{Yp} = 0.0036$
$C_{Yr} = -0.93$	$C_{Y\delta_a} = -37.82$	$C_{Y\delta_r} = -37.82$
$C_{Y\beta F_p} = -1.041$	$C_{Z_0} = 0.0982$	$C_{Z\alpha} = 0.051$
$C_{Zq} = 0.078$	$C_{Z\delta_e} = -18.52$	$C_{Z\alpha F} = 0.01638$
$C_{l_0} = -2.523$	$C_{l\beta} = -0.2154$	$C_{lp} = 24e-05$
$C_{lr} = -0.0622$	$C_{l\delta_a} = -2.523$	$C_{l\delta_r} = -2.523$
$C_{lF_p} = -0.8129$	$C_{m_0} = -16.7$	$C_{m\alpha} = 0.04585$
$C_{mq} = 0.07$	$C_{m\delta_e} = -16.7$	$C_{m\alpha F} = 0.01477$
$C_{n_0} = -3.104$	$C_{n\beta} = -0.265$	$C_{np} = 3e-05$
$C_{nr} = -0.07652$	$C_{n\delta_a} = -3.104$	$C_{n\delta_r} = -3.104$
$C_{nF_p} = -1$		

VIII. CONCLUSION

In the frame of this paper, an identification procedure based on free flight measurements was developed for the aerodynamic coefficients determination and tested for a piloting application of a UAV. Moreover, to increase the probability that the coefficients define the system's aerodynamics over the entire range of test conditions and to improve the accuracy of the estimated coefficients, a multiple fit strategy was considered. This approach provides a common set of aerodynamic coefficients that are determined from multiple data series simultaneously analyzed, and gives a more complete spectrum of the system's motion.

We have presented TLSE applied to the aerodynamic identification problem. This method is based on the use of the SDV decomposition. It has the interesting propriety of giving the best approximation of the augmented measurement matrix, by another matrix with the same dimension, but with a lesser range, in the sense of the least squares.

In addition to the dimension reducing propriety, the SDV has the advantage of being able to estimate the invert of any matrix, whether it is square or rectangular, and most of all, whether it is singular or not.

The SDV interpretation key is the weights distribution exam (singular values). The decreasing order of those weights allows us to say that the first modes contain the main proprieties of the considered data, more exactly, they are the modes that will catch the major part of the global variance of the data.

The obtained results $C_X \cdots C_{n\delta_r}$ by TLSE, are defined as the mean values of those aerodynamic coefficients derivatives. All parametric variations $\Delta C_X \cdots \Delta C_{n\delta_r}$ will be compensated by the robustness proprieties of the piloting law to be elaborated.

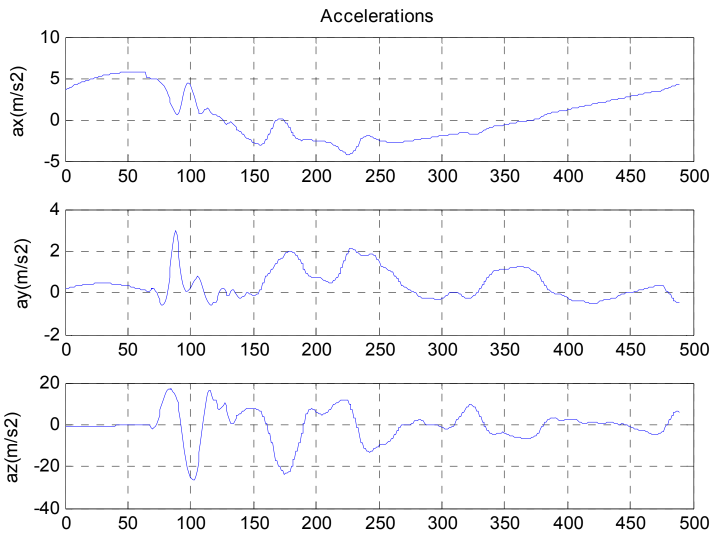

Fig. 4 Accelerations (m/s^2)

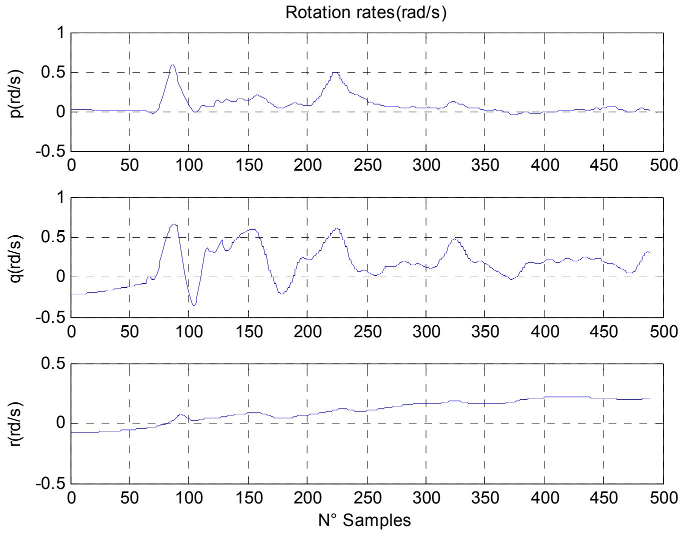

Fig. 5 Rotation rates (rd/s)

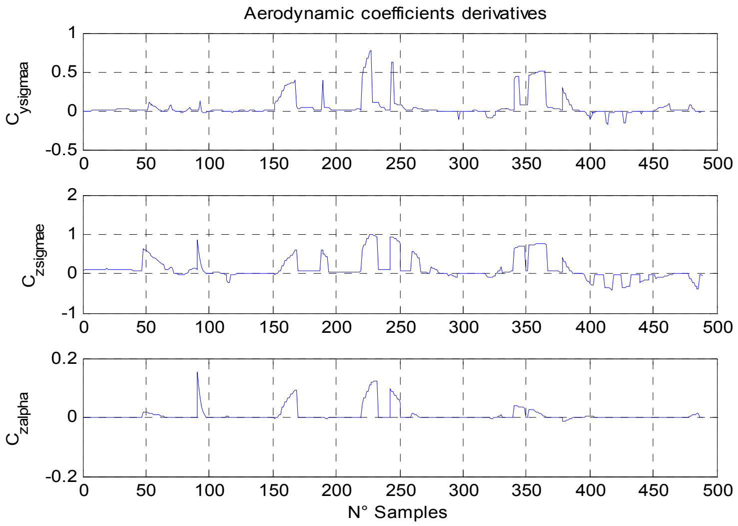

Fig. 6 Aerodynamic coefficients derivatives

REFERENCES

[1] R. Jategaonkar, «*Flight vehicle system identification: a time domain methodology*», American Institute of Aerodynamics and Astronautics, 2006.

[2] V. Klein, and E. Morelli, «*Aircraft system identification: theory and practice*», American Institute of Aerodynamics and Astronautics, 2006.

[3] M.Laban, «*On-Line Aircraft Aerodynamic Model Identification*», PHD Dissertation, Technische Universiteit Delf, May 1994.

[4] Gene H. Golub, Charles F. Van Loan, «*Matrix computations*», 4th Edition, The Johns Hopkins University Press Baltimore, chap. 8, 2013.

[5] Jean-Luc Boiffier: « *The dynamics of flight: The equations* », John Wiley and Sons,pp. 92, 1998.

[6] M. Zaouche, A. Beloula, R. louali, S. Bouaziz, M. Hamerlain «*Adaptive differentiators via second order sliding mode for a fixed wing aircraft*», Computer Modeling in Engineering and Sciences, Vol.104, No.3, 2015.

[7] M. Zaouche, « *Identification et commande robust d'un engine aérodynamique Volant à 03 axes*», Thèse doctorat, Ecole Militaire Polytechnique, Algiers, 2015.

[8] Jan Roskam, «*Airplane flight dynamics and automatic flight controls, part I*», DARcorporation, 2001, chap. 3.

[9] John D. Anderson «*Aircraft performance and design*», McGraw-Hill, 1999, pp. 57–70.

[10] Ivan Markovsky, Sabine Van Huffel, «*Overview of total least-squares methods*», Signal Processing, Science direct, 2007.

[11] Sabine Van Huffel, JoosVandewell, «*Analysis and properties of the generalized total least $AX \approx B$ when some or all columns squares problem, In A are subject to error*», SIAM J. Matrix Anal. Appl.,Vol. 10, No. 3, pp. 294-315, July 1989.

[12] D. Allerton, «*Principles of flight simulation*», Aerospace series, Wiley, chap.3, 2004.

[13] E. Torenbeek, H. Wittenberg, «*Flight physics: Essentials of Aeronautical Disciplines and Technology, with Historical Notes*», Springer, chap. 4, 2002.

[14] R.C. Nelson, «*Flight stability and automatic control*», McGraw-Hill Book Company, 1989, pp. 34-111.

[15] S.M. Calhoun, «*Six degree-of-freedom modeling of an uninhabited aerial vehical*», Master thesis, Russ college of engineering and technology of Ohio University, 2006.

[16] Arthur Gelb, «*Applied optimal estimation*», Massachusetts Institute of Technology, Cambridge~Massachusetts, and London, England, chap.2, 2001.

[17] H. Muhammad, H. P.Thien, T. Mulyanto, «*Total Least Squares Estimation of Aerodynamic Parameter of Micro Coaxial Helicopter from Flight Data*», International Journal of Basic & Applied Sciences IJBAS-IJENS Vol: 12 No: 02, 2012.

[18] J. C. Hoff, «*Initial evaluation of the modified step-wise regression procedure to estimate aircraft stability and control parameters from flight test data*», College of Aeronautics Reports, Cranfield University, BedfordshireMK43 0AL, UK, 1993.

[19] A.Jacek Goszczyński, «*Practical aspects of identification of the aerodynamic characteristics*», Journal of theoretical and applied mechanics 44, 1, pp. 31-50, Warsaw 2006.

[20] P. Dowson, "FSUIPC: Application interfacing module for Microsoft Flight Simulator", August 30 2012, Available online at: www.schiratti.com/dowson.html 2012.

[21] Zaouche Mohammed, Amini Mohammed, Foughali Khaled, Hamissi Aicha, Aktouf Mohand Arezki, Boureghda Ilyes, "*Application of the Piloting Law Based on Adaptive Differentiators via Second Order Sliding Mode for a Fixed Wing Aircraft*", World Academy of Science, Engineering and Technology, International Journal of Mechanical, Aerospace, Industrial, Mechatronic and Manufacturing Engineering Vol:10, No:6,pp.998-1008, 2016.