

# Mecano-Reliability Approach Applied to a Water Storage Tank Placed on Ground

Amar Aliche, Hocine Hammoum, Karima Bouzelha, Arezki Ben Abderrahmane

**Abstract**—Traditionally, the dimensioning of storage tanks is conducted with a deterministic approach based on partial coefficients of safety. These coefficients are applied to take into account the uncertainties related to hazards on properties of materials used and applied loads. However, the use of these safety factors in the design process does not assure an optimal and reliable solution and can sometimes lead to a lack of robustness of the structure. The reliability theory based on a probabilistic formulation of constructions safety can respond in an adapted manner. It allows constructing a modelling in which uncertain data are represented by random variables, and therefore allows a better appreciation of safety margins with confidence indicators. The work presented in this paper consists of a mecano-reliability analysis of a concrete storage tank placed on ground. The classical method of Monte Carlo simulation is used to evaluate the failure probability of concrete tank by considering the seismic acceleration as random variable.

**Keywords**—Reliability approach, storage tanks, Monte Carlo simulation, seismic acceleration.

## I. INTRODUCTION

IN engineering offices of structures design, civil engineers use the deterministic methods as recommended by dimensioning codes [1], [2]. These methods based on limit states theory aim to obtain a sufficiently safe dimensioning by relying on partial coefficients of safety. In the field of fluid storage tanks, the best known deterministic model is the one developed by Housner [3]. This deterministic dimensioning often leads to a deliberately pessimistic margin to possible failure modes and causes the unjustified over-dimensioning, which leads to additional costs.

Faced with the need of designing more reliable and economical structures, a new methodology based on the probability theory is developed. This probabilistic method takes into account uncertainties of parameters and involved phenomena in the design of structures. The interested reader can consult the references below [4]-[7]. Thus, we note a growing interest of the scientific community of civil engineering for the application of probabilistic approaches in the design and analysis of the structures stability. We can cite the works [8], [9] in the structural reliability of dams and [10] in the reliability to fatigue of off shores structures.

Few studies have been published on the use of probabilistic

approaches in the field of storage tanks. We cite researches of Berahman et al. [11] which are focused on the evaluation of the seismic vulnerability of steel storage tanks, by developing a probabilistic model of failure modes. Sani et al. [12], being based on the FORM approximation method, studied the failure of an underground concrete tank in rectangular form, considering three failure modes (bending, shear and torsion). As for concrete elevated tanks, Möller et al. [13], being based on a probabilistic method, developed a simple methodology to design the circular section of the supporting system of reinforced concrete elevated tanks, with a probability of tolerable failure.

In the present work, we develop a probabilistic approach to analyze the failure functions related to the overall stability of a concrete storage tank, placed on ground. The considered state functions are those related to sliding and overturning at the ultimate limit state and those related to the verification of stresses (tension and compression) in the wall and sloshing effect of free surface water at limit state of service. To take the uncertainties into account, two types of variables are considered: hydraulic load which is a function of time and seismic acceleration which is considered as a random variable. For the need of reliability analysis, Monte Carlo simulation method is used. Also, the MATLAB<sup>®</sup> software is used for generating random draws.

## II. MECHANICAL MODEL

To analyze the stability of the tank and its hydrodynamic behavior (effect of the liquid on the walls under seismic action), Housner [3] decomposes the liquid into a passive action causing pulse efforts and active action causing effort oscillation [14], [15]. The pulse efforts come from of that a portion of the liquid mass, called passive mass, reacts by inertia to the translation of tank walls. Its equivalent mechanical system is obtained by considering a mass  $M_i$ , rigidly connected to the tank at a height  $h_i$ , as it exerts on walls the same horizontal forces that the equivalent water mass (Fig. 1). As for oscillation efforts, they come from that another part of the liquid mass, said active mass which is set in oscillation motion under the seismic action. Its mechanical equivalent is obtained by considering a mass  $M_0$  that is retained by springs with stiffness  $K_0$  at  $h_0$  level, whose horizontal oscillations exert the same vibratory efforts that the active mass of the liquid.

## III. RELIABILITY ANALYSIS OF THE STORAGE TANK

The storage tank failure can be caused by loss of stability at the ultimate limit state and by loss of strength at the limit state

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of service. Thus it is appropriate to define the different limit state functions  $G(\{X\})$  that are dictated by the physical causes of the failure, as they are given by [4] in (1):

$$G(\{X\}) = R(\{X\}) - S(\{X\}) \quad (1)$$

where;  $R(\{X\})$ : strength of the structure,  $S(\{X\})$ : active loading,  $\{X\}$ : random vector constituted by random variables  $x_i$ . Thus, the reliability analysis consists to calculate the failure probability of the structure defined by (2):

$$P_f = P(G(\{X\}) \leq 0) \quad (2)$$

which is given by the integral of the joint function of the probability density  $f_x(x)$  of the random vector  $\{X\}$  as follows:

$$P_f = \int_{D_f} f_x(x) dx \quad (3)$$

$D_f$  is the ruin domain defined by:  $D_f = \{x \in R / G(x) \leq 0\}$ .

Under seismic action effect, five different failure modes (limit states) are considered in the case of a circular concrete tank placed on the ground.

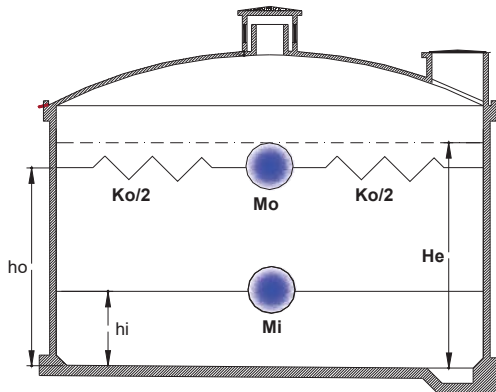


Fig. 1 Equivalent mechanical system

#### A. Overturning

The ultimate limit state of the overall stability of the tank to overturning consists to verify that the stabilizer moment of the structure is greater than the overturning moment. The performance function  $G_1$  associated with this limit state is given by (4):

$$G_1: M_s - M_r \quad (4)$$

#### B. Sliding

The overall stability of the tank under the seismic effect action to the ultimate limit state can also be lost by sliding. The Sliding resistance is calculated assuming that the failure occurs in the soil and not at the interface of foundation-soil. For this failure mode, the corresponding limit state function is given by (5):

$$G_2: N_u \cdot \tan \varphi + C \cdot A - F_h \quad (5)$$

where;  $N_u$ : vertical component of the ultimate loads,  $C$ : cohesion of the foundation soil,  $\varphi$ : internal angle of friction,  $A$ : area of the foundation part in contact with the ground,  $F_h$ : resultant of the horizontal seismic forces.

#### C. Tensile Stress in Steels

The limit state function of failure by loss of steels tensile strength, corresponding to the occurrence of cracks in the cylindrical wall of the tank, is given by (6):

$$G_3: \bar{\sigma}_{st} - \sigma_{st} \quad (6)$$

Considering that cracks are very harmful, the limit tensile stress  $\bar{\sigma}_{st}$ , given by [16], is written as:

$$\bar{\sigma}_{st} = 0,80 \cdot \min \left\{ \frac{2}{3} \cdot f_c ; \max \left( \frac{f_c}{2} ; 90 \sqrt{\eta \cdot f_{tj}} \right) \right\} \quad (7)$$

#### D. Compression Stress in Concrete

The test of justification in this limit state consists of verifying the concrete compression stress  $\sigma_{bc}$  at the limit state of service in the cylindrical wall. The function of limit state is given by (8):

$$G_4: \bar{\sigma}_{bc} - \sigma_{bc} \quad (8)$$

The compression ultimate stress  $\bar{\sigma}_{bc}$  is given by [16] as a function of the thickness ( $e$ ) of the tank wall by (9)

$$\bar{\sigma}_{bc} = \min \left\{ \frac{e + 0,55}{3} \cdot f_{c28} ; \frac{130 \cdot e}{D_{int}} \cdot f_{c28}^{1/3} ; 0,60 \cdot f_{c28} \right\} \quad (9)$$

#### E. Sloshing

In a partially filled tank, seismic action sets a portion of the fluid in motion. A freeboard must be provided to prevent damage of the dome due to wave effect, or to prevent liquid overflow when the tank has no rigid roof. According to [2], the expression of the wave peak can be assessed by (10):

$$d_{max} = 0,84 \frac{S_{ai}}{g} R \quad (10)$$

The limit state function corresponding to failure mode by sloshing is given by (11):

$$G_5: H - d_{max} \quad (11)$$

where,  $H$  is the height between the water free level and the cover dome.

#### IV. PRACTICAL APPLICATION

##### A. Tank Presentation

As a practical application, we consider a circular concrete tank with capacity 200 m<sup>3</sup>, placed on ground. This tank is implanted on a loose soil, considered as site type S<sub>3</sub>, according to [1]. Geometrical characteristics of the tank are illustrated in Fig. 2.

For the reliability analysis of the tank, we became interested on two variables, designating the external loads, which are the seismic loading and the hydraulic loading.

The seismic acceleration imposed on the tank, taking into account its interaction with the ground, is obtained from the dimensioning spectrum as a function of the seismic zone and the period T according to [1], as shown in Fig. 3.

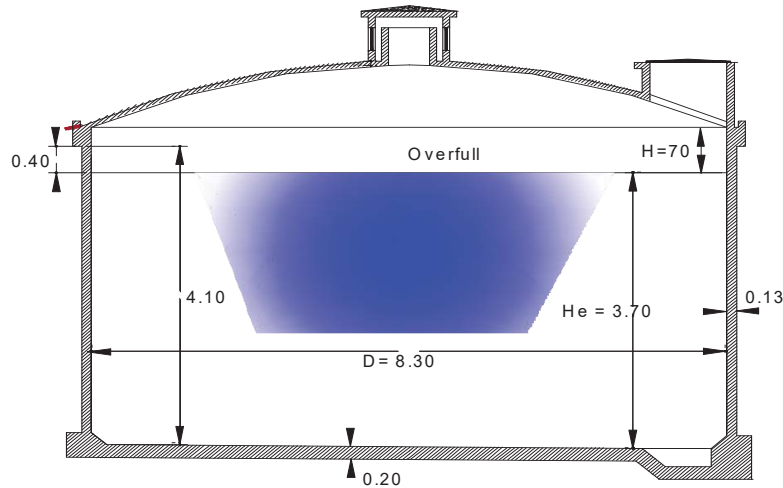


Fig. 2 View and cross section of the tank placed on ground

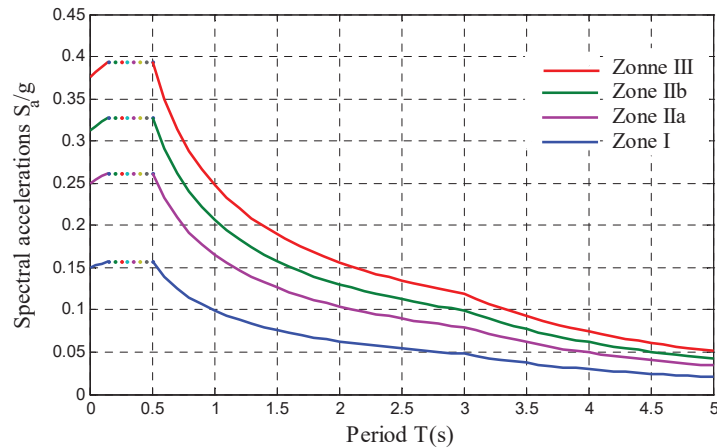


Fig. 3 Response spectra

To determine the distribution type of seismic acceleration, which is considered as the random variable, a statistical analysis was conducted on a database containing 45 accelerograms. These latter are registered during the earthquake of 21 May 2003 in Boumerdes (Algeria) by different seismographs installed by the National Earthquake Engineering Research Center of Algiers in the central zone of Algeria. We give in Fig. 4 an example of the accelerogram registered in site of Kheddara Dam (50 km East of Algiers), and used for statistical analysis. The histogram of peak acceleration seismic, on which we have superposed the Gumbel distribution law is given in Fig. 5.

Hydraulic loading represents the useful height of water in

the tank. It varies during the day depending on the distribution of the adduction flow at the tank entry and the hourly consumption flow, related to the importance of the served agglomeration [17]. For the case of a less important city, the tank reaches a theoretical maximum volume of (10. Qh) (Fig. 6). Qh is an average hourly flow of distribution. The hourly variation of the water height H<sub>e</sub>(t) in the tank can be put as a function of the tank capacity V(t) and the internal cross section Ω of the tank container, as given in the relation (12):

$$H_e(t) = \frac{V(t)}{\Omega} \quad (12)$$

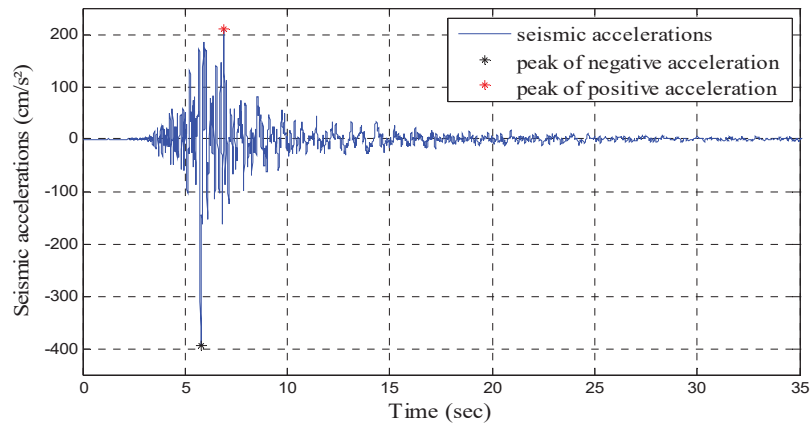


Fig. 4 Example of accelerogram (KeddaraDam)

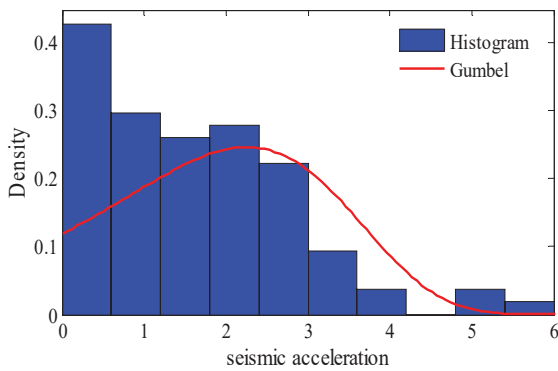


Fig. 5 Generation of the random variable

**B. Evaluation of Failure Probability of the Tank**

The analytical evaluation of the probability failure of storage tank by using (3) is very difficult or impossible. For this reason, we use classical simulation method of Monte Carlo. This old and intuitive technique assesses the failure

probability  $P_f$  by succession independent random draws. It consists to reach a significant number  $N_t$  of random variables according to their joint distribution law. Thus, a ruin indicator  $I_G$  is used to define the failure state of the system for the given state function  $G$  [4]. This ruin indicator  $I_G$  is given by (13):

$$I_{G \leq 0} = \begin{cases} 1 & \text{si } G \leq 0 \\ 0 & \text{si } G > 0 \end{cases} \quad (13)$$

For each mode of ruin, the probability of failure  $P_f$  is given by (14) [4]:

$$P_f = \frac{\sum_{i=1}^{N_t} I_{G \leq 0}}{N_t} \quad (14)$$

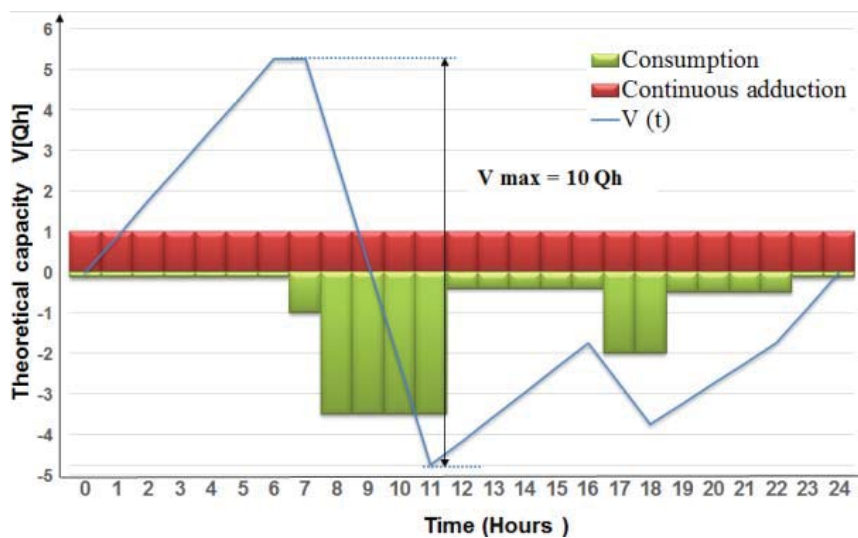


Fig. 6 Theoretical capacity in continuous water supply

C. Results and Discussion

The coefficient of variation  $C_v$ , representing the distribution of the random variable realizations (seismic acceleration) is taken directly from the parameters of statistical analysis, according to the average  $\mu$  and standard deviation  $\sigma$ . Its value is 0.664. As for the number of random draw  $N_t$ , convergence tests have shown that the stability of failure probability value  $P_f$  is obtained from one number of simulations equal to  $2 \times 10^5$  (Fig. 7). So, the number of  $10^6$  is used for different simulations.

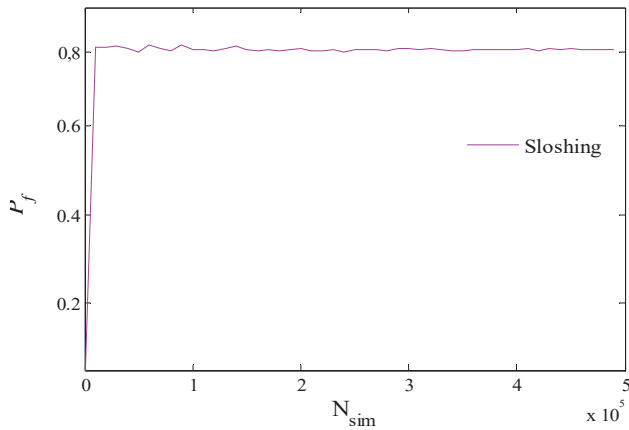


Fig. 7 Convergence and stability of failure probability  $P_f$  as a function of draws number

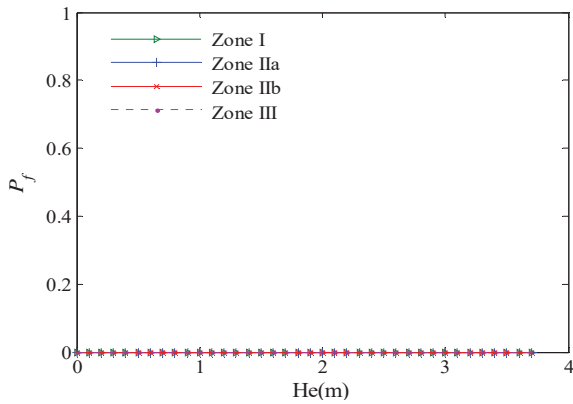


Fig. 8 Failure mode by overturning

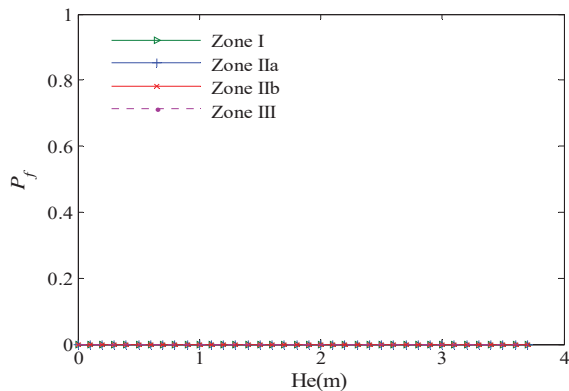


Fig. 9 Failure mode by sliding

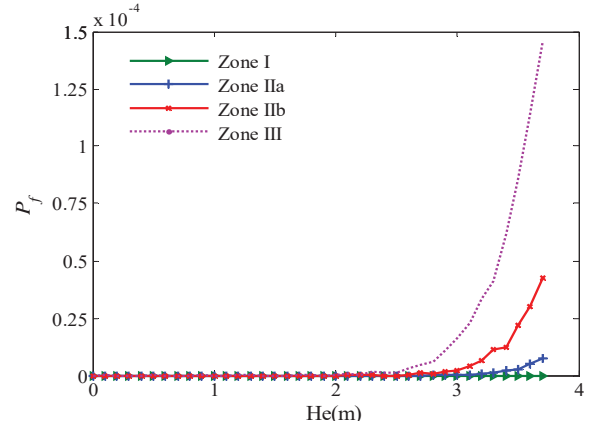


Fig.10 Failure mode by crushing concrete

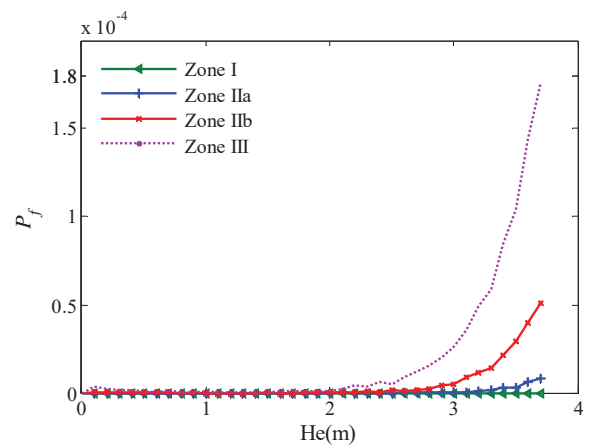


Fig. 11 Failure mode by steels traction

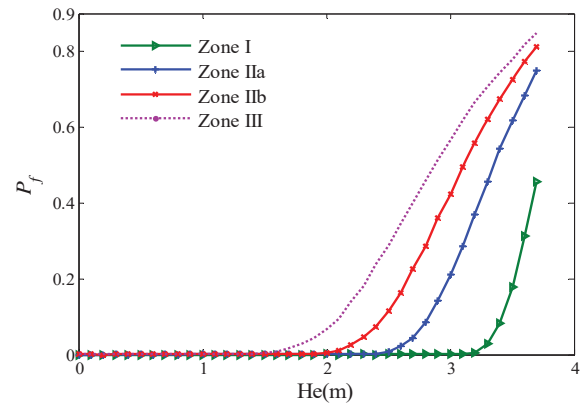


Fig. 12 Failure mode by sloshing

Figs. 7-12 show the evolution of the failure probability  $P_f$  for each ruin mode considered as function of the water level in the tank and the different seismic zones. We note for the failure mode by overturning (Fig. 8) and by sliding (Fig. 9) that the failure probability  $P_f$  is null, and thus the tank is stable. As for the failure modes by crushing of concrete by compression (Fig. 10) and by steels ruin by tensile (Fig. 11), the failure probability increases with the water height in the tank and the seismic zone. However, the values of  $P_f$  still

under  $10^{-3}$ , which is the admissible value for civil engineering structures [18]. The failure mode by sloshing represents, in this case, the most critical state function. The results reported in Fig. 12 show that starting from a water height upper to 40% of the useful height of water, the values of  $P_f$  largely exceed the allowable value ( $P_f \gg 10^{-3}$ ) and that for different seismic zones. The tank enters into the failure domain.

The evolution of the failure probability  $P_f$  at different times of the day is shown in (Figs. 13–17) for different ruin modes considered and different seismic zones. The curves show that  $P_f$  evolves in the time slot between 1 a.m. to 8 a.m., where the drinking water network is the least requested by subscribers. The tank stocks more than it can distribute. We clearly note that the failure mode by sloshing is critical (Fig. 17). The values of the probability reach peaks which largely exceed admissible limits between 6 a.m. and 7 a.m., corresponding to the maximum water level in the tank.

### V. CONCLUSION

The analysis of seismic reliability of a concrete tank placed on the ground, which is the subject of this article, is conducted with a Gumbel distribution law of seismic acceleration, and a coefficient of variation  $Cv=0.664$ . The MATLAB<sup>®</sup> software is used to generate random draws where the number of simulations is  $10^6$ . Calculation results of the failure probability showed that the structure is stable to different considered ruin modes, except the failure mode by sloshing and that for the time slot between 1 a.m. and 7 a.m.. During this time slot, where the tank stores the maximum capacity, the cover dome can be damaged under the wave's effect created by the seismic action. We can conclude that the Algerian seismic code must more consider this problem of sloshing by proposing a relation that allows estimating a practical height of freeboard.

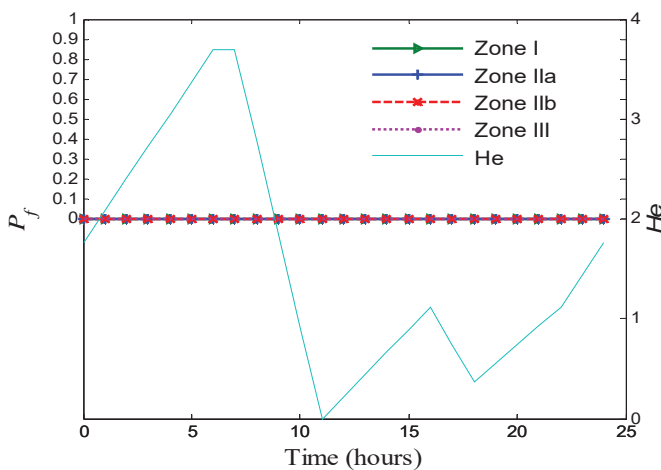


Fig. 13 Failure mode by overturning

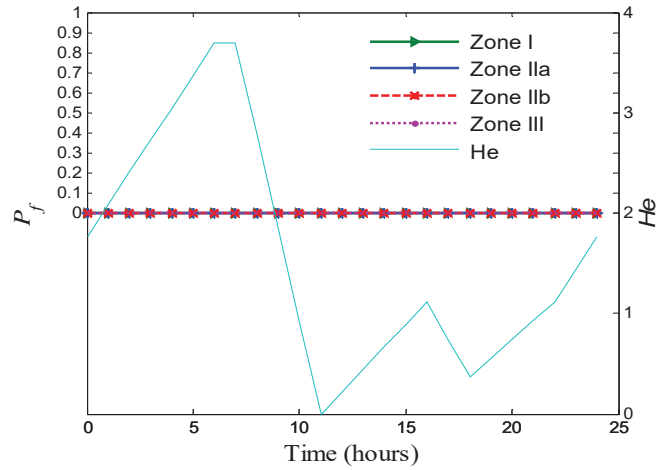


Fig. 14 Failure mode by sliding

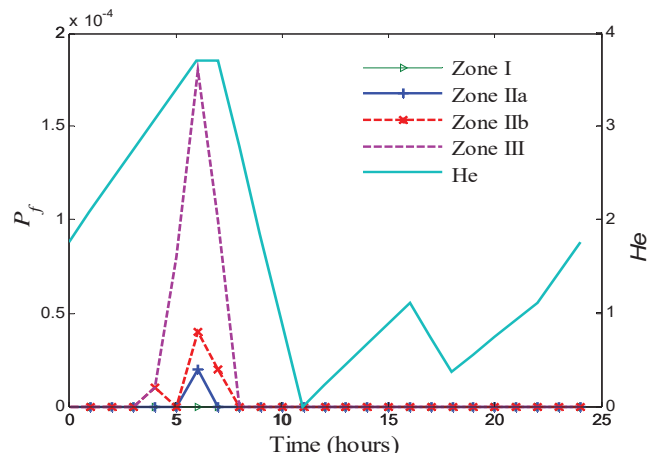


Fig. 15 Failure mode by crushing concrete

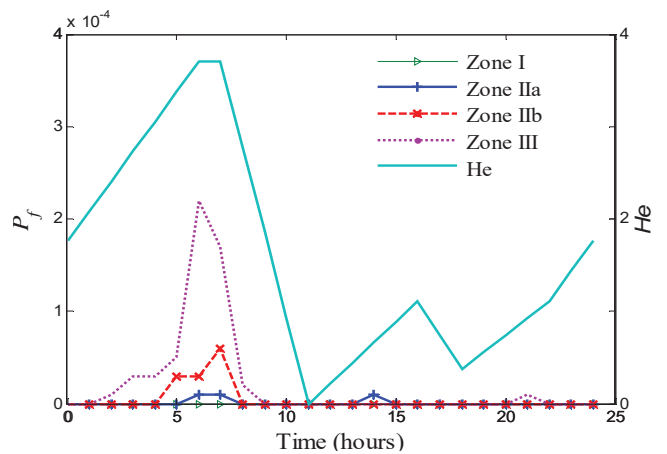


Fig. 16 Failure mode by steels traction

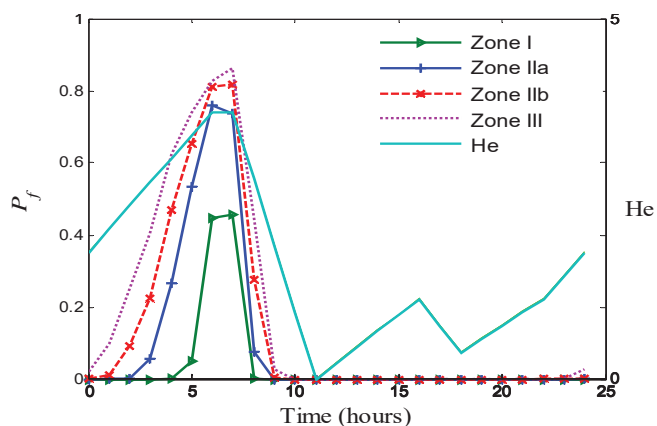


Fig. 17 Failure mode by sloshing

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