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Peeling Behavior of Thin Elastic Films Bonded to Rigid Substrate of Random Surface Topology

Ravinu Garg, Naresh V. Datla

***Abstract*—**We study the fracture mechanics of peeling of thin films perfectly bonded to a rigid substrate of any random surface topology using an analytical formulation. A generalized theoretical model has been developed to determine the peel strength of thin elastic films. It is demonstrated that an improvement in the peel strength can be achieved by modifying the surface characteristics of the rigid substrate. Characterization study has been performed to analyze the effect of different parameters on effective peel force from the rigid surface. Different surface profiles such as circular and sinusoidal has been considered to demonstrate the bonding characteristics of film-substrate interface. Condition for the instability in the debonding of the film is analyzed, where the localized self-debonding arises depending upon the film and surface characteristics. This study is towards improved adhesion strength of thin films to rigid substrate using different textured surfaces.

***Keywords*—**Debonding, fracture mechanics, surface topology, thin film adhesion.

I. Introduction

THIN film-substrate systems are ubiquitous in various modern advanced engineering materials. Wide application of these systems motivated the research towards the in-depth understanding of adhesion and peeling behavior of thin films. In the past few decades, various analytical models have been proposed to define the peeling mechanism of thin films over the rigid substrates. The first theoretical peel model was presented by Rivlin [1]. Rivlin's model was limited to inelastic thin films, which was later modified by Kendall [2], by including elastic term in the analysis of peel mechanism. Later based on the pioneering work of Kendall, the model was extended to include various other aspects such as large deformation of thin films [3] and plastic yielding of film [4]. Reliability of film-substrate system is based on interfacial bonding between the film and rigid substrate. In recent studies various research efforts have been dedicated to enhance the interfacial bonding strength of film – substrate interfaces using numerous techniques such as introduction of elastic heterogeneity in thin films [5], modifying surface topology [6], and bioinspired hierarchical thin films [7].

Most of the theoretical models were constrained to rigid flat substrates. Recently Chen et. al., provided an analytical framework to analyze the peeling behavior of thin films from corrugated surfaces [8]. However, their analysis was limited to sinusoidal profiles of the corrugated surface. In this study, work of Chen et. al. was further extended for surfaces of random geometric profiles. Analytical framework has been provided to determine the peel strength of film – substrate system with defined random topographies of substrate. The developed framework was applied for sinusoidal and circular substrate profiles to analyze peel instabilities.

Naresh. V. Datla is with the Department of Mechanical Engineering, Indian Institute of Technology Delhi, New Delhi, 110016, India (phone: +91-11-2659 6071; e-mail: datla@mech.iitd.ac.in).

Ravinu Garg is masters student with the Department of Mechanical Engineering, Indian Institute of Technology Delhi, New Delhi, 110016, India (e-mail: ravinu09@gmail.com).

II. Theoretical Model of Thin Film Peeling from a Random Surface Topology

Consider the quasi-static peeling of thin elastic film from a rigid surface of random surface topology as shown in Fig. 1. The film with thickness h and elastic modulus E is perfectly bonded to the rigid substrate up to length l and the remaining length $L-l$ is being peeled off by peeling force F at a constant peel angle θ_F from the free end of the film. The unbonded length $L-l$ is assumed to be long enough so that the steady state can be achieved and the angle θ_L at the extreme end is equal to peel angle θ_F. Therefore, we have boundary conditions

$$\theta(L) = \theta_F \, , \theta'(L) = 0 \tag{1}$$

The random profile of the substrate is assumed to follow an arbitrary function $y = f(x)$. The profile of the film is defined by a function of $\theta(s)$, where s is the arc-length of the film from the origin and θ is tangential angle of the film at each point with respect to the horizontal plane.

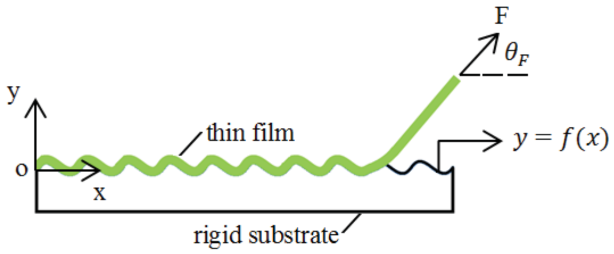

Fig. 1 Schematic diagram of peeling of thin elastic film from rigid substrate of $y = f(x)$ profile

The potential energy V of the film-substrate system shown in Fig. 1 can be expressed as,

$$V = \int_0^L \frac{D}{2}\theta'^2 ds + \int_l^L \frac{1}{2} E\varepsilon^2 h ds - \vec{F}.\vec{u}_F - \int_0^L F\varepsilon ds - \int_0^l \Delta\gamma ds \tag{2}$$

where D is the bending stiffness, $D = Eh^3/12$ of the film, ε is the elastic strain, $\varepsilon = F.\cos(\theta - \theta_F)/Eh$, γ is the interfacial adhesion energy per unit length. The first term on the right hand side of (2) is the bending elastic energy of bonded and unbonded part of the film. The second term on the right hand side is the strain energy stored due to extension of film. The third term corresponds to the potential energy of the external applied force due to displacement $\overrightarrow{u_F}$ of the free end of the film.

The displacement $\overrightarrow{u_F}$ can be expressed as [5]

$$\overrightarrow{u_F} = \int_0^L \begin{pmatrix} cos\theta - cos\theta_F \\ sin\theta - sin\theta_F \end{pmatrix} ds \tag{3}$$

U_f is measured relative to a reference point at

$$\begin{pmatrix} Lcos\theta_F \\ Lsin\theta_F \end{pmatrix} \tag{4}$$

The fourth term on the right hand side of (2) is the potential energy of the external work due to the extension of the film. The last term corresponds to the interfacial adhesion energy.

The first term of the right hand side of (2) is combination of elastic bending energy of bonded and unbonded part of the film. The elastic bending energy of perfectly bonded part is governed by the profile of rigid substrate and can be evaluated as,

$$U(l) = \int_0^l \frac{D}{2}\theta'^2 ds = \int_0^{x(l)} \frac{D}{2}\left(\frac{\partial^2 y}{\partial x^2}\right)^2 dx \tag{5}$$

Substituting (2) and (4), into (1) yields,

$$V = \int_0^l \frac{D}{2}\theta'^2 ds + U(l) + \int_l^L \frac{F^2}{2Eh} cos^2(\theta - \theta_F) ds - \int_0^L F[cos(\theta - \theta_F) - 1] ds - \int_l^L \frac{F^2}{Eh}\cos(\theta - \theta_F)\, ds + \int_0^l \Delta\gamma ds \tag{6}$$

In order to find the equilibrium profile shape and peel front position, principal of minimum potential energy is used. Let the first variation of the potential energy in (5) with respect to Q and l equal to zero.

$$\delta V = \delta V_1 + \delta V_2 = 0 \tag{7}$$

where,

$$\delta V_1 = -\int_l^L \left[D\theta'' + \frac{F^2}{Eh}\cos(\theta - \theta_F) . \sin(\theta - \theta_F) - Fsin(\theta - \theta_F) - \frac{F^2}{Eh}\cos(\theta_L - \theta_F) . \sin(\theta - \theta_F)\right] \delta\theta ds + D\theta'\delta\theta|_l^L \tag{8}$$

and

$$\delta V_2 = \left[\frac{1}{2} D\theta'^2 - \frac{F^2}{2Eh} cos^2(\theta - \theta_F) + \frac{F^2}{Eh}\cos(\theta_L - \theta_F)\right]\Big|_{s=l} \delta l + \frac{dU}{dl}\delta l - \Delta\gamma\delta l \tag{9}$$

Here $dU/dl = dU/dx \,.\, dx/dl$. At the peel front θ can be defined as

$$\theta(l) = \arctan\left(\frac{dy}{dx}\right) \tag{10}$$

Substituting boundary conditions of (1) and (10) into (8) and (9), we get

$$D\theta'' + \frac{F^2}{2Eh} cos(\theta - \theta_F). sin(\theta - \theta_F) - Fsin(\theta - \theta_F) - \frac{F^2}{Eh}\cos(\theta - \theta_F) . sin(\theta - \theta_F) = 0 \tag{11}$$

and

$$\frac{1}{2} D\theta_l'^2 - \frac{F^2}{2Eh} cos^2(\theta_F - \varphi) + \frac{F^2}{Eh}\cos(\theta_F - \varphi) + \frac{D}{2}\left(\frac{\partial^2 y}{\partial x^2}\right)^2 \frac{1}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} = \Delta\gamma \tag{12}$$

Multiplying (11) with θ' and integrating it from l to L. Simplify (11) using (12), we get

$$\frac{F^2}{2Eh} + F\left[1 - cos\left(\theta_F - tan^{-1}\left(\frac{dy}{dx}\right)\right)\right] - \Delta\gamma + \frac{D}{2}\left(\frac{\partial^2 y}{\partial x^2}\right)^2 \frac{1}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} = 0 \tag{13}$$

Substituting $\arctan(dy/dx) = \varphi$ into (13) and solve,

$$\frac{F^2}{2Eh} + F[1 - \cos(\theta_F - \varphi)] - \Delta\gamma + \frac{D}{2} sec^3\varphi \left(\frac{\partial\varphi}{\partial x}\right)^2 = 0 \tag{14}$$

III. Results and Discussion

Effect of different surface profiles on the peel strength of thin film is investigated in this section. In this study, overall peel angle $\theta_F = 90°$ is considered for each profile. However, local peel angle $(\theta_F - \varphi)$ is continuously varying for curved surfaces, which is defined by the profile of the surface, as shown in Fig. 2. In case of horizontal and inclined flat surfaces, local peel angle is $(\theta_F - \varphi)$ and $(\partial\varphi/\partial x) = 0$. Therefore (14) reduces to Kendall's equation, and peel force remains constant for a particular peel angle.

$$\frac{F^2}{2Eh} + F[1 - \cos(\theta_F)] - \Delta\gamma = 0 \tag{15}$$

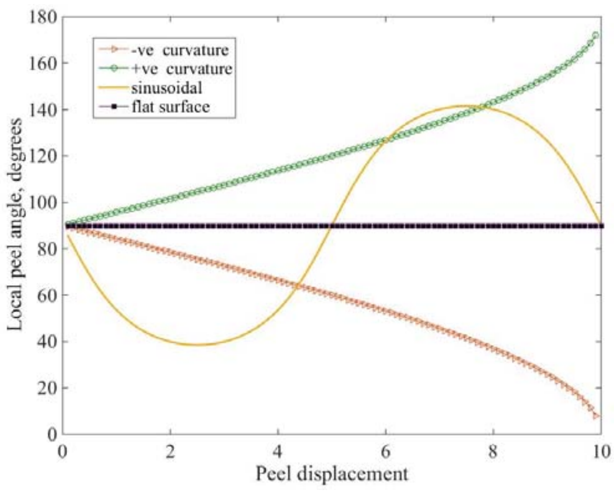

Fig. 2 Variation of local peel angle as the peel front propagates on different surface profiles

Figs. 3-7 show the non-dimensional peel off force as a function of peel displacement from the surfaces having different profiles. It can be seen that as the peel front propagates, peel force varies depending upon the position of peel front and local peel angle, which is varying as the peel front propagates.

For sinusoidal profile, peel force is periodic in nature and the effective peel force is function of amplitude of sinusoidal profile of substrate. It can be noted that effective peel force increases as the amplitude increases. However, peel force become negative at certain locations for the larger amplitudes, which is consistent with the results of [8]. Negative peel force indicates the condition of instability, where the self debonding takes place.

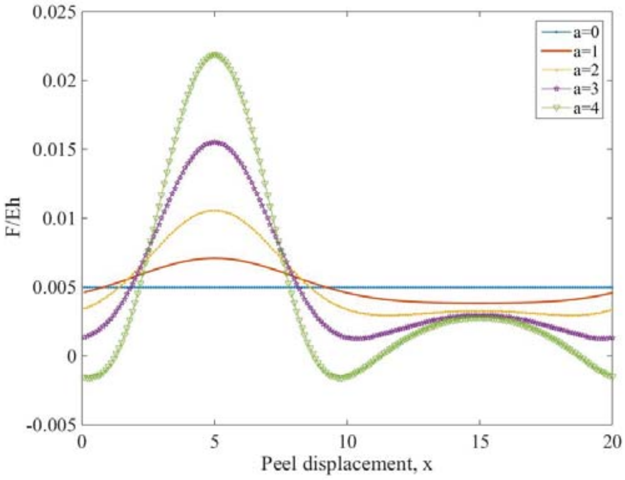

Fig. 3 The non-dimensional peel off force along the peel displacement from sinusoidal surface for different amplitudes

Figs. 4 and 5 show the non-dimensional peel off force as a function of peel displacement from circular profiles with negative and positive curvatures. For negative curvatures, the peel force is monotonically decreasing. The rate of decrease in the peel force is a function of magnitude of curvature as shown in Fig. 4. For lower curvatures, the peel force reduces at rapid rate and reaches to zero, thereafter self debonding occurs, which arises the condition for instability in the system. It can be noted that by increasing the magnitude of curvature, the event of instability can be postponed. From Fig. 4, it can be concluded that improvement in the peel force cannot be achieved by introducing negative curvature profile on the rigid substrate.

For circular profiles with positive radius of curvature, initially the peel force is monotonically increases and then start decreasing and reduces to zero, as shown in Fig. 5. Improvement in the peel force can be achieved up to certain peel distance.

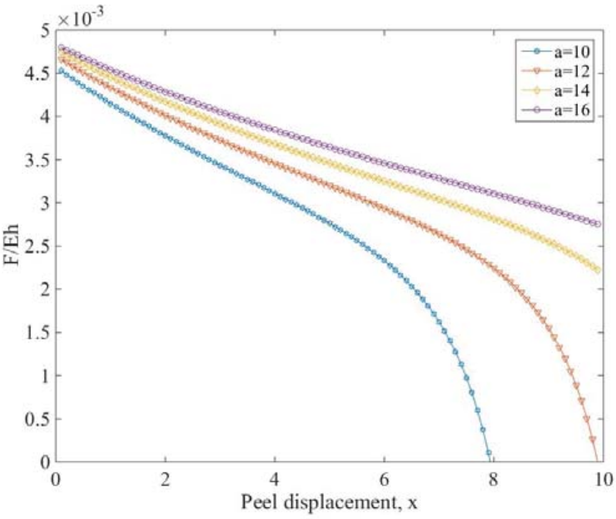

Fig. 4 The non-dimensional peel off force along the peel displacement from circular profile having different radius of negative curvatures

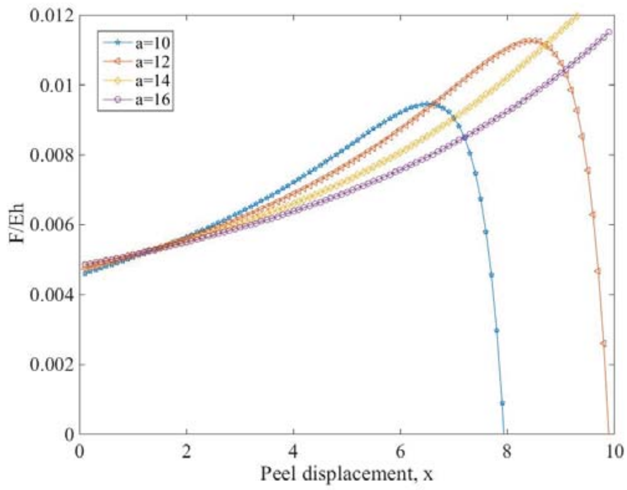

Fig. 5 The non-dimensional peel off force along the peel displacement from circular profile for different radius of positive curvatures

IV. Conclusion

A generalized analytical model has been developed to analyze the peeling behavior of elastic thin film from rigid surface of random surface topography. Different profiles have been considered to demonstrate the effect of surface profile of substrate on peel force. Results were consistent with the already available solutions of sinusoidal profile. Condition of self debonding was analyzed for different cases. It was shown that enhancement in the peel force cannot be achieved for the circular profiles with negative curvature. However, for positive curvatures, improvement in the peel force can be achieved up to certain peel displacements.

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