Reliability Levels of Reinforced Concrete Bridges Obtained by Mixing Approaches

Adrián D. García-Soto, Alejandro Hernández-Martínez, Jesús G. Valdés-Vázquez, Reyna A. Vizguerra-Alvarez

***Abstract*—**Reinforced concrete bridges designed by code are intended to achieve target reliability levels adequate for the geographical environment where the code is applicable. Several methods can be used to estimate such reliability levels. Many of them require the establishment of an explicit limit state function (LSF). When such LSF is not available as a close-form expression, the simulation techniques are often employed. The simulation methods are computing intensive and time consuming. Note that if the reliability of real bridges designed by code is of interest, numerical schemes, the finite element method (FEM) or computational mechanics could be required. In these cases, it can be quite difficult (or impossible) to establish a close-form of the LSF, and the simulation techniques may be necessary to compute reliability levels. To overcome the need for a large number of simulations when no explicit LSF is available, the point estimate method (PEM) could be considered as an alternative. It has the advantage that only the probabilistic moments of the random variables are required. However, in the PEM, fitting of the resulting moments of the LSF to a probability density function (PDF) is needed. In the present study, a very simple alternative which allows the assessment of the reliability levels when no explicit LSF is available and without the need of extensive simulations is employed. The alternative includes the use of the PEM, and its applicability is shown by assessing reliability levels of reinforced concrete bridges in Mexico when a numerical scheme is required. Comparisons with results by using the Monte Carlo simulation (MCS) technique are included. To overcome the problem of approximating the probabilistic moments from the PEM to a PDF, a well-known distribution is employed. The approach mixes the PEM and other classic reliability method (first order reliability method, FORM). The results in the present study are in good agreement whit those computed with the MCS. Therefore, the alternative of mixing the reliability methods is a very valuable option to determine reliability levels when no close form of the LSF is available, or if numerical schemes, the FEM or computational mechanics are employed.

***Keywords*—**Structural reliability, reinforced concrete bridges, mixing approaches, point estimate method, Monte Carlo simulation.

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I. Introduction

BRIDGES are key strategic structures in any country, because they connect different regions and enable the establishment of transportation nets to exchange good and services among other important matters. They are also critical infrastructure in emergency situations. When there is need for a shorter, safer, or faster way to communicate people and regions through highways systems, to build bridges is often unavoidable, especially in countries with complex and irregular geographic areas (like Mexico and many other countries around the world). These strategic structures are designed by code regulations to ensure certain safety levels that should ideally be quantitatively determined (e.g. in terms of the reliability index or probability of failure), and that should be related to the loads and materials of the region under consideration, since loads and material properties differ for different regions. This emphasizes the relevance of developing codes with load and resistance factors calibrated for the societal and geographical conditions where the code is applicable. The target reliability levels are associated to safety levels deemed adequate by the society and, even though they are notional in nature because the probability of failure of real structures is difficult if not impossible to determine, they are a useful tool to compare safety levels among different structures and for code developers to establish adequate load and resistance factors for design [1]. Among many of the available methods to quantify the reliability indices (assumption of Gaussian distributions, Taylor series expansion, first or second order reliability methods (FORM or SORM), etc.), it is very often required to define an ultimate or serviceability LSF explicitly. In case the ultimate limit state is implicit, direct integration or MCS approaches are an alternative; however, these approaches are expensive from computing resources and time standpoints, but may be necessary if, for instance, the FEM, a numerical method, or computational mechanics are involved in obtaining the probabilities of failure of structural systems, including bridges to be designed by code regulations. As an option to avoid a large number of simulations for implicit LSFs (which could be especially critical when the FEM or computational mechanics are used) the PEM can be employed [2]-[11]. For this method, a large number of simulations are not mandatory; in fact, only a few point estimates need to be determined to evaluate the moments of the LSF of interest; moreover, the whole probabilistic characterization or the random variables may not necessarily be determined, but just the first few non-crossed probabilistic moments. The PEM was originally developed several decades ago [2]-[4], and improved afterwards especially in the 90s [6].

It was still investigated later on [7]-[11], and is an important currently used method in several fields including geotechnical and environmental engineering, where it is a relevant tool for assessing reliability levels [7]-[8]. One of the disadvantages of the PEM is that the LSF is determined only in terms of the probabilistic moments; thus, some assumption or technique is necessary to compute the probability of failure (e.g. by assuming or fitting the LSF statistics to a PDF.

An option to compute reliability indices as a mixture of the PEM and other well-known approach is presented herein. This proposal can be useful to overcome the use of non-explicit LSFs for assessing probabilities of failure using only a few point estimates. A case study for safety levels of reinforced concrete bridges designed by Mexican regulations and where a numerical method could be required is presented. Also, comparisons by using simulation technique and only the PEM are carried out. For mixing approaches (idea originally developed in a previous study [12] for other regulations, structures, statistics and scope) and overcome the problem of approximating the probabilistic moments from the PEM to a PDF, the Normal distribution is employed. In fact, the essence of mixing approaches shown in [12] employs a different PDF (Lognormal) which exhibits better results and is not presented here since the study is unpublished. The results in the present study does not differ significantly whit those computed with the MCS and they could be a very useful tool for reliability assessment of bridge structures, especially if the original deterministic approach is based on finite element techniques, computational fluid mechanics (CDF) or simply a numerical method (i.e. when non-explicit LSFs can only implicitly be established).

II. Field Information on Live Loads and Concrete Strength

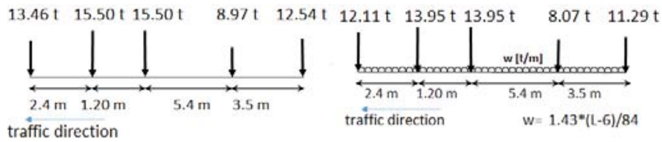
To assess the reliability levels of reinforced concrete bridges, information from field surveys and tests is considered. The most relevant information for traffic load and load effects comes from weigh-in-motion (WIM) surveys carried out in a Mexican Highway and the computing of the load effects using such database; details can be found in previous works [13], [14]. Fig. 1 is included to show the maximum daily gross truck weight obtained from the referred WIM data. It can be observed that the daily average is around 1000 kN (aprox. 100 tonne). These weights are larger than those reported in other countries [15], [16], pointing out the need for developing code formats and load and resistance factors consistent with the demands of the region of code applicability and the selected target reliability.

A couple of ongoing projects to get field information from concrete core tests to evaluate the compressive strength and Young modulus (PRODEP), and more field information on traffic loads and other parameters from different types of surveys (CONACYT) are aimed at investigating the reliability levels of existing bridges in Mexico. The projects are sponsored by the Ministry of Education of Mexico (SEP, for its initials in Spanish) through the grant for the improvement of teaching for new full-time Professors (PRODEP, for its initials in Spanish), and by the National Research Council of Science and Technology (CONACYT, for its initials in Spanish). However, these projects are currently being carried out and, even though the surveys and tests will be carried out soon, the information is still not available and will be included in future studies. It is expected that tools like that developed in [12] and use in the present study will be helpful for obtaining the reliability levels of bridges located in Guanajuato, Mexico, once the information become available.

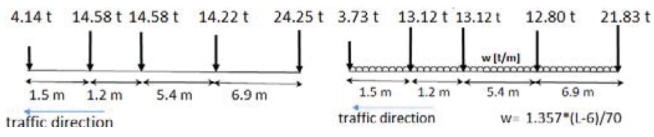
Fig. 1 Maximum daily gross truck weight from WIM data in Mexico (modified from [14])

III. Considered Codes, Models and LSF

The Mexican highway bridge design regulations are one of the used references in Mexico for bridge design, however the code neither contain information on statistics of load and strengths, nor on the implicit reliability levels by using such regulations. Therefore, live load models and statistics reported in studies aimed at improving the current models in the Mexican regulations are considered. For instance, Fig. 2 shows models proposed for bridge design in Mexico and in Guanajuato state [13], [14].



(a)



(b)

Fig. 2 (a) Proposed live load model for bridge design in Mexico, (b) Proposed live load model for bridge design in Guanajuato; units are in tonnes and meters

Since reinforced concrete bridges are considered in this study, the Mexico City Code (NTC04) for reinforced concrete structures [17] is employed to state the capacity of the concrete bridges.

Only the bending moment is considered to establish the ultimate LSF and corresponding random variables in this study. They are based on an equation stipulated in the NTC04. For tension-controlled reinforced concrete beams (RCB) the following equation determines the flexure capacity [17]

$$M_R = F_R b d^2 f_c'' q(1-0.5q) \quad (1)$$

where

$$q = \frac{A_s}{bd}\frac{f_y}{f_c''} \quad (2)$$

F_R is the strength factor (equal to 0.9), b denotes the beam width, d the effective depth, f_c'' is a uniform compressive stress, f_y is the yielding reinforcement strength and A_s the reinforcement area. Note that the uniform compressive stress is given by

$$f_c'' = (0.85)(0.8)\, f_c' \quad (3)$$

where f_c' is the compressive stress; the reduction multiplying by 0.8 accounts for uncertainty associated to the resistance of the concrete once it has been cast [17], and it will likely disappear in oncoming versions of the code [18]. It is assumed that multiplying by 0.85 the section compressive stress can be equivalently considered uniform [17].

Based on the previous information (3) can be rewritten as

$$M_R = \phi\left(A_s f_y d - \frac{0.5 A_s^2 f_y^2}{(0.85 \times 0.8 \times f_c')b}\right) \quad (4)$$

where ϕ is the strength factor.

Equation (4) allows for a direct appreciation of the random variables to be employed in the reliability assessment. For the present study concrete with nominal f_c' = 250kg/cm² (24.52 Mpa) is considered.

So far we have dealt only with the bending capacity of a bridge concrete beam. To establish the LSF not only the capacity, but also the demand must be dealt with. To incorporate the demand in the LSF the basic load combination case of dead load effect (*D*) plus live load effect (*L*) [17] is assumed. This leads to the following LSF

$$g = B\left(A_s f_y d - \frac{0.5 A_s^2 f_y^2}{(0.85 \times 0.8 \times f_c')b}\right) - D - L \quad (5)$$

where *B* is a random variable which envisages the modelling error.

Note also that reinforced concrete and simply supported bridges are the more common types for bridges in Guanajuato and this is possibly the case for bridges in whole Mexico [14]; this can be observed in Fig. 3. Therefore, the present study is focused on simply supported beams.

Note that if instead of (1), a numerical scheme based on the equilibrium and the commonly considered hypothesis for concrete behavior [17], [19] is used, the flexure capacity can be also determined. This is schematically shown in Fig. 4.

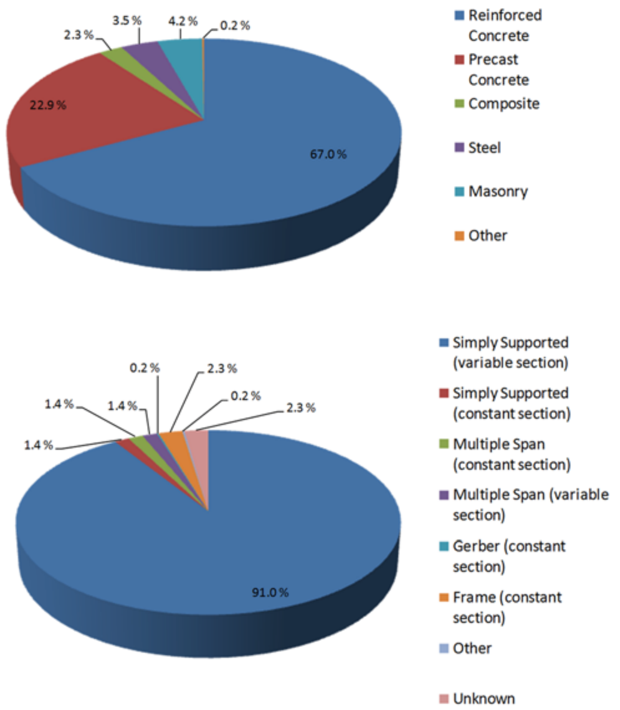


Fig. 3 Bridge type distribution in Guanajuato State, Mexico (modified from [14])

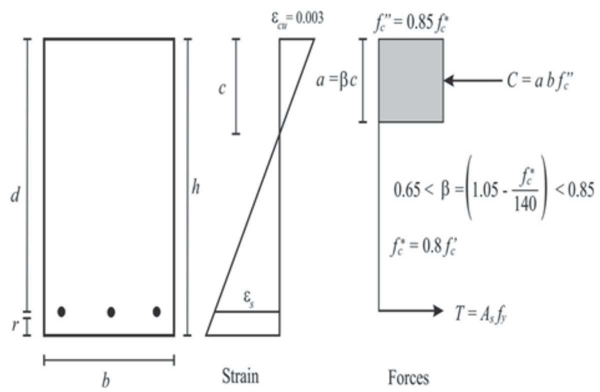


Fig. 4 Schematic representation of force equilibrium in a RCB section subjected to bending

Actually, such numerical scheme leads to the same results obtained by using (1) for tension-controlled beams. However, the numerical approach can be used for cases in which a close form expression like (1) cannot be formulated (e.g. beams with reinforcement in both, the tension and the compression zones; non-common beam sections, etc.). Actually, some codes require the use of a numerical approach in the first place [20]. In such cases an explicit LSF could be very difficult, when not impossible, to establish, and the PEM and mixing approaches could be more adequate, as it will be emphasized later.

TABLE I
PROBABILISTIC MODELS FOR RELIABILITY ANALYSES

Random variable	Mean	Coef. of variation	Distribution
B	1.10	0.10	Normal
b (mm)	354.2	0.014	Normal
d (mm)	796	0.012	Normal
f_c' *(Mpa)*	26.84	0.1636	Weibull
f_y *(Mpa)*	458.8	0.096	Lognormal
D (N.mm)	–	0.10	Normal
L (N.mm)	–	0.09	Gumbel

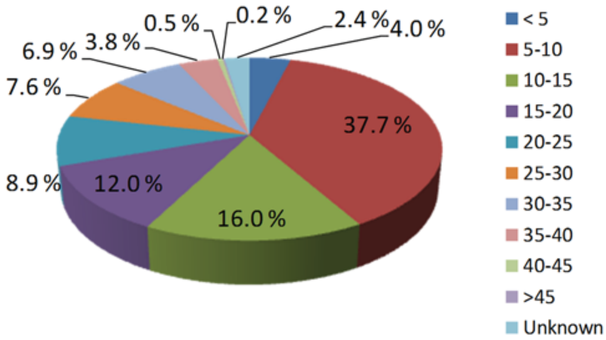


Fig. 5 Bridge span lengths (m) in Guanajuato State, Mexico (modified from [14])

IV. PROBABILITY MOMENTS AND DISTRIBUTION ASSIGNMENTS

It is required to assign PDFs to the random variables in (1) to carry out the reliability assessments, or at least the moments of such variables, depending on the reliability method to be performed. Note that independence is assumed among the random variables described herein.

For this study, *b* is considered Gaussian with mean to nominal value of 1.012 and coefficient of variation, *cov*, of 0.014. The effective depth *d* is also considered Gaussian, but with a mean to nominal value of 0.995, and *cov* of 0.012 [21]. According to the results of a previous study currently under review [22], which employed lab tests, for f'_c a Weibull PDF can be adopted, with a mean value of 26.84 Mpa and *cov* of 0.1636. Another random variable characterized as Gaussian is *B*, with mean and *cov* values of 1.10 and 0.10, respectively [23]. The reinforcement area, A_s, is considered deterministic. The yielding stress, f_y, is lognormally distributed with mean value equal to 458.8 Mpa and *cov* equal to 0.096 [22].

For the probabilistic models of the dead load effect (flexure), *D*, and the live load effect (flexure), *L*, the information from a previous study is adopted [13]. For *D* the mean value is to be determined as it will be explained later, and the *cov* is equal to 0.10. *L* is a Gumbel distributed random variable, and the designation of the mean value will also be explained later on. Note, however, that the *cov* is dependent on the span length with values around 0.09 to 0.13 considering multiple presence in one lane for span lengths ranging between 15 and 120 m [13]. Since the statistics for the Guanajuato bridges (Fig, 5) indicated that most of the state span bridges are between 5 to 30 m, a value of cov equal to 0.09, corresponding to a 15 m span length, is adopted for this study. Different span lengths lead to different reliability levels, but the selected case is deemed appropriate for the objective of the present work. Note that the *cov* for live load effect corresponds to those employed for computing the reliability index for one year [13].

The previous discussion results in the information for the reliability Analysis listed in Table I.

The mean to nominal values are employed to obtain the values of *b* and *d* in Table I from a nominal rectangular section with *b* = 350 mm, *d* = 800 mm, and A_s = 2800 mm^2. This section could be deemed representative of the bridges under study, but even if it is arbitrarily selected, it will not affect significantly the conclusions of this paper. To obtain the mean values of the load effects, undetermined in Table I, it is considered that the selected cross section just meets the code requirements, as implied by

$$1.3m_D + 1.95m_L = \phi\left(A_s m_{f_y} m_d - \frac{0.5A_s^2 m_{f_y}^2}{(0.85\times0.8\times m_{f_c'})m_b}\right) \quad (6)$$

where *m* denotes the mean values of the variables in the corresponding subscripts. This is based on (4) and on the Mexican Regulations for Bridge Design [24], which load factors for the basic combination of dead plus live load are adopted, except that a proposed live load model and strength factor (ϕ equal to 0.9 as in the NTC04) are adopted from other work [13], since a calibration task showed the adequacy of such code format [13]. In the left hand side of (6), the total flexure moment due to dead and live loads, can correspond to different fractions of each of the load effects. To account for this, the ratio of mean values of live and dead load effects, ζ, is included. For a certain value of $\zeta = m_V / m_D$, and using (6), the values of m_D and m_V can now be determined. The assumption referred above (i.e. the selected cross section just meets the code requirements) implicitly considers that lane factors and girder distribution factors have been properly taken into account. Other implied assumption is that not only the whole bridge load effects can be modeled as a Gumbel variable with certain probabilistic moments, but also the girders of the bridge superstructure, and that the element reliability instead of the system reliability is computed. Nevertheless, since bridges with simply supported spans are considered here, the element fail may imply the system fail, and thus the assessed reliability should not significantly differ from the system reliability. However, a proper treatment of the system reliability is out of the scope of the present work.

Although, the lane factors uncertainty is not studied here, an approach to probabilistically determine multiple lane factors for bride design has been proposed somewhere else [25] and could be developed and included in future studies. For the present study, only static live load effects are considered;

therefore, the above selected *cov* for *L* corresponds to the lane load case, in line with the assumption that for the lane load, the traffic jam scenario is assumed, and no significant dynamic effects are induced to the structure [16], [13].

As it was mentioned, the statistics and probabilistic models can be region-dependent. For instance, for Europe, probabilistic models are available in the literature [26]-[28]; in one of the reported models a *cov* equal to 0.15 is reported for bridge probabilistic analyses [28].

V. Reliability Analysis

The probabilistic models in the previous section are to be used for the reliability analyses in this one. The assessment of the reliability levels for RCB in this section are primarily aimed at briefly comparing the different approaches and discussing the adequacy of the different methods depending on the type of LSF and the available information, as well as the tools for deterministic analysis available to the engineer (e.g. FEM software). Nevertheless, the analyses will also be useful to estimate the implicit reliability levels of RCB designed by Mexican regulations and the proposed live load model and code format in a previous study [13].

In the following, first the reliability levels by using simulation technique are evaluated; then, the reliability assessment is performed by using the PEM; finally, a mixture of approaches which considers the PEM and the FORM is carried out. Along the reliability assessments, some brief comparisons and discussions are included.

A. Numerical Simulation

For the simulation a Monte Carlo technique is employed [30], [31]. The inverse cumulative distribution function (CDF) method [32] is used to obtain a total of 6×10^6 samples for each of the random variables in Table I, and an equal number of samples are obtained by using (5) for each set. Then, the probability of failure is evaluated as

$$p_f = \frac{N_f}{N_T} \tag{7}$$

where N_f is the number of times that evaluation of (5) leads to values ≤ 0; N_T is the total number of evaluated samples.

The corresponding reliability indices are obtained using $\beta = -\Phi^{-1}(p_f)$ [33] where $\Phi^{-1}(\cdot)$ denotes the inverse Normal distribution function. This is repeated for several ζ values. Results are shown in Fig. 6.

From Fig. 6 it can be observed that the annual reliability index is between around 3.5 and 4.0 for a range of ζ values. Note that in previous studies [13], a target reliability index for 1 year (β_T) equal to 3.75 was employed for the calibration of the proposed model and strength factor, considering the same load factors format from the Mexican regulations [24]; but the bridge capacity was not specifically stated for concrete bridges. The values in Fig. 6 are consistent with those in [13], and from results for different span lengths (not shown here), it can be said that, in average, the annual reliability levels also approximately correspond to an annual target reliability level of 3.75, which in turn correspond to a target reliability index of 3.5 for a service period of 75 years [29].

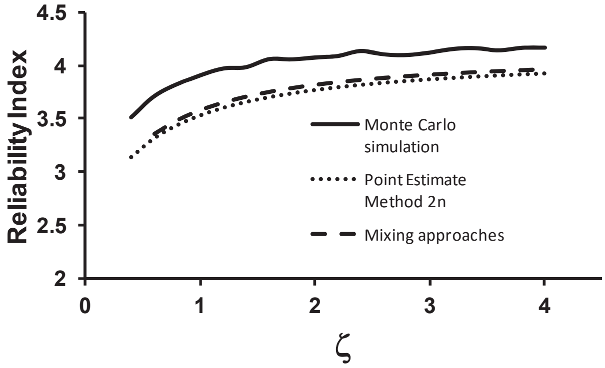


Fig. 6 Reliability levels using several approaches

Note that the MCS is very versatile, since it does not necessarily require a close form expression of the LSF like (5); instead, a numerical scheme by using equilibrium (Fig. 4) can be used with the same results. However, the MCS is computationally expensive. In such cases, the PEMs are a more efficient alternative.

B. Point Estimate Method (PEM)

In the PEMs the computing time is decreased compared to the MCS. Furthermore, they only need the moments of the random variables without the knowledge of the probability distribution functions; they also can be used when the LSF is only implicitly known. However, this approach has the shortcoming that the LSF resultant moments must be fitted to a probability distribution to obtain the reliability index. One of the PEM approaches referred before [6] is used, since it only requires 2n point estimates (i.e. 7×2 = 14 point estimates for the bridge under study). The results are shown in Fig. 6 assuming that the LSF is Gaussian. Note that the skewness coefficient is used in the PEM (a value of 0.301 for the Lognormal PDF, a value of 1.140 for the Gumbel PDF and a value of -0.480 for the Weibull PDF are used, respectively). Fig. 6 shows that the PEM could be an acceptable alternative, since the results are not extremely different to those from the MCS (often employed as a benchmark) and, as it was mentioned before, neither requires the whole probabilistic description of the random variables, nor the establishment of an explicit LSF.

C. Mixture of the PEM and the FORM

In this section a method which mixes the PEM with the well-known FORM [33] is used. This proposed approach [12] is appropriate when the LSF is implicit, and it is easy to implement, since it is based in the assumption that the capacity statistics (derived from the PEM) can be fitted to a known PDF; for this study the capacity is considered normally distributed. Once the bending capacity is completely characterized from a probabilistic point of view, the demand (in this case dead load effect plus live load effect) is taken as originally described (Section IV), and the FORM is performed (for three random variables, instead of seven). This is carried out and results are shown in Fig. 6. Fig. 6 shows that, for the case under study, the results are closer to those by using the MCS, but only slightly. Nevertheless, results from a previous study showed that the assumption of the flexure moment capacity as Lognormal leads to better results.

Note that if a numerical scheme (e.g. Fig. 4) is used, or the FEM or computational mechanics are employed (i.e. no explicit LSF is available), the alternative of mixing approaches can be a very valuable alternative.

VI. Conclusion

Reliability levels of reinforced concrete bridges in Guanajuato, Mexico, are investigated and some results by using different methods are reported.

For the MCS the annual reliability index is between around 3.5 and 4.0 for a range of ζ values. This is consistent with previous results. In average, the annual reliability levels approximately correspond to an annual target reliability level of 3.75 used for a calibration of the Mexican regulations, corresponding to a target reliability index of 3.5 for a service period of 75 years.

It is concluded that for the bridges considered in this study, PEM could be a fair alternative, since the results are not extremely different from those of the MCS; moreover, it neither requires the whole probabilistic description of the random variables, nor the establishment of an explicit LSF.

A method which mixes the PEM with the well-known FORM is also used. This approach is reported somewhere else with a different distribution assumption; in this study the capacity statistics are assumed to correspond to a Normal distribution. The results show that the method is a valuable alternative. Furthermore, if a numerical scheme, the FEM or computational mechanics are employed (i.e. when no explicit LSF is available), mixing approaches can be a pretty adequate alternative.

Acknowledgment

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