World Academy of Science, Engineering and Technology
International Journal of Physical and Mathematical Sciences
Vol:10, No:1, 2016

Analysis of P, d and ^{3}He Elastically Scattered by ^{11}B Nuclei at Different Energies

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***Abstract*—**Elastic scattering of Protons and deuterons from ^{11}B nuclei at different p, d energies have been analyzed within the framework of optical model code (ECIS88). The elastic scattering of ^{3}He+^{11}B nuclear system at different ^{3}He energies have been analyzed using double folding model code (FRESCO). The real potential obtained from the folding model was supplemented by a phenomenological imaginary potential, and during the fitting process the real potential was normalized and the imaginary potential optimized. Volumetric integrals of the real and imaginary potential depths (J_R, J_W) have been calculated for ^{3}He+^{11}B system. The agreement between the experimental data and the theoretical calculations in the whole angular range is fairly good. Normalization factor N_r is calculated in the range between 0.70 and 1.236.

***Keywords*—**Elastic scattering, optical model parameters, double folding model, nuclear density distribution.

I. Introduction

WHEN we bombard a nucleus with a nucleon or with light ions like d, ^{3}He, α-particles, etc. various nuclear phenomena occur, including elastic scattering, inelastic scattering, nucleon transfer reactions and projectile fragmentation, depending on the projectile species and the bombarding energy. The simplest among these phenomena is the elastic scattering. Elastic scattering can provide valuable information about the interaction potential between two colliding nuclei [1]. From the phenomenological studies, it is clear that the major part of the nuclear interaction potential can be approximated by a Woods-Saxon form which gives a simple analytic expression, parameterized explicitly by the depth, the radius, and diffuseness of the potential well. In practice it is required to obtain the potential from the analysis of experimental data by varying their parameters to optimize the overall fit to the data, using appropriate OM codes [2]. The folding model has been used for years to calculate the nucleon-nucleus optical potential and inelastic form factors. It can be seen from the basic folding formulas that this model generates the first-order term of the microscopic optical potential is derived from Feshbach's theory of nuclear reactions [3]. The potential used in the double folding were obtained in a semi-microscopic way including nucleons exchange effects and the density dependence of the nucleon-nucleon interaction. A density dependent version of the M3Y Interaction (CDM3Y6) has been used in our calculation. The real part of optical potential obtained from the folding model analysis was supplemented by a phenomenological imaginary potential, and during the fitting process the real potential was normalized and the imaginary potential optimized. The basic inputs for a folding calculation are the nuclear densities of the colliding nuclei and the effective nucleon-nucleon (NN) interaction. A popular choice for the effective NN interaction has been one of the M3Y interactions which were designed to reproduce the G-matrix elements of the Reid [4] and Paris [5] NN potentials in an oscillator basis. The purpose of the present work is the extraction of reliable information about potential parameters for interaction of light charged particles (p and d) with ^{11}B nuclei from the optical model and ^{3}He+^{11}B nuclear system from double folding analysis of elastic scattering. We have calculated volume integrals for both real and imaginary potential depths for ^{3}He+^{11}B nuclear system. In the next section, we present the theoretical models used in our calculations.

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II. Theoretical Models

A. Optical Potential Parameters

Nuclear interaction potential between the two colliding nuclei replaced by the complex optical potential where it is real part represents the elastic scattering and the imaginary part corresponds to the absorption (loss of incident flux from elastic channel). Central optical potential can be written down in the form:

$$U_{op}(r) = V_C(r) - V(r) - iW_s(r) - V_{so}(r) \tag{1}$$

The first term is the Coulomb potential was assumed to be that between two uniform charge distributions with radii consistent with electron scattering.

$$V_C(r) = \frac{Z_P Z_T e^2}{2R_C}\left(3 - r^2/R_C\right) \text{For } r \le R_C$$

$$V_C(r) = \frac{Z_P Z_t e^2}{r} \quad \text{For} \quad r \succ R_C \tag{2}$$

The real part has the following form:

$$V(r) = V_o\left[1 + \exp\left(\frac{r - R_v}{a_v}\right)\right]^{-1} \tag{3}$$

The imaginary surface part has the following form:

$$W_S(r) = -4\ a_D W_D \frac{d}{dr}\left[1+\exp\left(\frac{r-R_D}{a_D}\right)\right]^{-1} \quad (4)$$

The real spin orbit potential has the following form:

$$V_{so}(r) = \frac{2}{h^2 r} V_{so}\left[-\frac{d}{dr}\left[\frac{1}{1+\exp\left(\frac{r-R_{SO}}{a_{so}}\right)}\right]\right] \quad (5)$$

B. Double-Folding Potential

The real part of the optical potential is calculated from a more fundamental basis by the folding method in which the NN interaction $V_{NN}(r)$ is folded into the densities of both the projectile and target nuclei [6].

$$V^{DF}(R) = N_r \int \rho_t(r_2)\rho_P(r_1) V_{NN}(r_{pt}) dr_P\, dr_t \quad (6)$$

where N_r is a free renormalization factor, $\rho_p(r_1)$ and $\rho_t(r_2)$ are the nuclear matter density distributions of both the projectile and target nuclei, respectively, and V_{NN} is the NN potential, $r_{pt}= R+r_2-r_1$. A popular choice for the effective NN-interaction has been one of the M3Y-interactions. In the present folding calculation, the effective NN-interaction is taken according to the form of radial shape of the M3Y-Paris interaction which is given in terms of three Yukawas [7] as:

$$V_D(S) = 11061.625\frac{\exp(-4S)}{4S} - 2537.5\frac{\exp(-2.5S)}{2.5S} \quad (7)$$

$$V_{EX}(S) = -1524.25\frac{\exp(-4S)}{4S} - 518.75\frac{\exp(-2.5S)}{2.5S} - 7.8474\frac{\exp(-0.7072S)}{0.7072S} \quad (8)$$

where $V_D(s)$ and $V_{EX}(s)$ are the direct and exchange component of the M3Y-Paris. The M3Y-Paris interactions are scaled by an explicit density dependent function $F(\rho)$:

$$F(\rho) = C\left[1+\alpha\exp(-\beta\rho) - \gamma\rho^n\right] \quad (9)$$

where ρ is the density of the surrounding nuclear medium in which the two nucleons are embedded and s is the inter-nucleon separation.

The parameters C, α, β, γ and n given in Table I were taken from [8] and [9].

TABLE I
PARAMETERS OF DENSITY –DEPENDENCE FUNCTION f(P)

Model	C	α	β $(fm)^3$	γ $(fm)^{3n}$	n
CDM3Y6	0.2658	3.8033	1.4099	4.0	1

The nuclear density distribution of ^{3}He was taken to be of Gaussian form [10]:

$$\rho(r) = \rho_O \exp\left(-C\ r^2\right) \quad (10)$$

where ρ_o= 0.20816 fm^{-3} and C= 0.53047 fm^{-2}; these parameters correspond to a root-mean-square (RMS) radius of ^{3}He of 1.68 fm.

The nuclear density distribution for ^{11}B was calculated using the harmonic oscillator model (HO), where ρ(r) was calculated from the relation [11]:

$$\rho(r) = \rho_O\left(1+\alpha\left(r^2/a^2\right)\right)\exp\left(-\left(r^2/a^2\right)\right) \quad (11)$$

where (a=1.69, α=0.811 and ρ_o= 0.1818 fm^{-3}) for ^{11}B.

III. RESULTS AND DISCUSSIONS

A. Optical Model Analysis of Protons Elastically Scattered on ^{11}B

Optical model analysis of the experimental data of the angular distribution of protons elastically scattered on ^{11}B at proton energies (0.543, 0.80, 1.2, 15.8, 17.35, 20 and 30.30 MeV) [12] is shown in Fig. 1. The present analysis was performed using the code ECIS88 where the following parameters were fixed r_c =1.15 fm, r_v=1.25 fm and r_D =1.15 fm. The optimal potential parameters used in the calculations are listed in Table II. As shown in Fig. 1, the agreement between theoretical predictions and experimental data is fairly good over the whole angular range which gives clear evidence about the pure potential character of protons elastic scattering on ^{11}B nuclei as well as the potential depth vary smoothly with proton energy.

The strength of parameters of real potential of proton with energy listed in Table II is represented by:

$$V_o = 54.89 - 0.58E_p$$

TABLE II
OPTICAL POTENTIAL PARAMETERS FOR PROTON ELASTIC SCATTERING ON ^{11}B

E_p MeV	V_o MeV	a_v Fm	W_D MeV	a_D fm	V_{so} MeV	r_{so} fm	a_{so} fm	X^2/N
0.543	56.72	0.65	1.067	0.633	5.466	1.15	0.70	4.13
0.800	53.85	0.75	2.015	0.747	7.02	1.15	0.50	0.483
1.2	52.57	0.87	6.15	0.60	15.00	1.15	0.50	0.963
15.8	45.049	0.71	8.00	0.70	7.80	1.15	0.65	1.954
17.35	50.99	0.594	18.27	0.285	8.25	1.10	0.57	1.063
20	36.63	0.67	3.45	1.01	8.25	1.15	0.57	8.98
30.30	38.66	0.587	3.70	1.00	8.25	1.10	0.60	2.977

B. Optical Model Analysis of Deuterons Elastically Scattered on ^{11}B

The optical model fitting of the experimental angular distributions of deuterons elastically scattered on ^{11}B at energies (5.5 MeV) [13], (11.8MeV) [14] and (27.7MeV) [15] is shown in Fig. 3. Analysis for d+^{11}B elastic scattering was performed using code ECIS88. Optical parameters were obtained as we fixed r_c=1.3 fm, r_v=1.25 fm, V_{so}=6.5MeV, r_{so}=0.9fm and a_{so}=0.6fm. The optimal potential parameters used in calculations are listed in Table III. As shown in Fig. 3, the agreement between theoretical predictions and experimental data is fairly good over the whole angular range which gives clear evidence about the pure potential character of deuterons elastic scattering on ^{11}B nuclei.

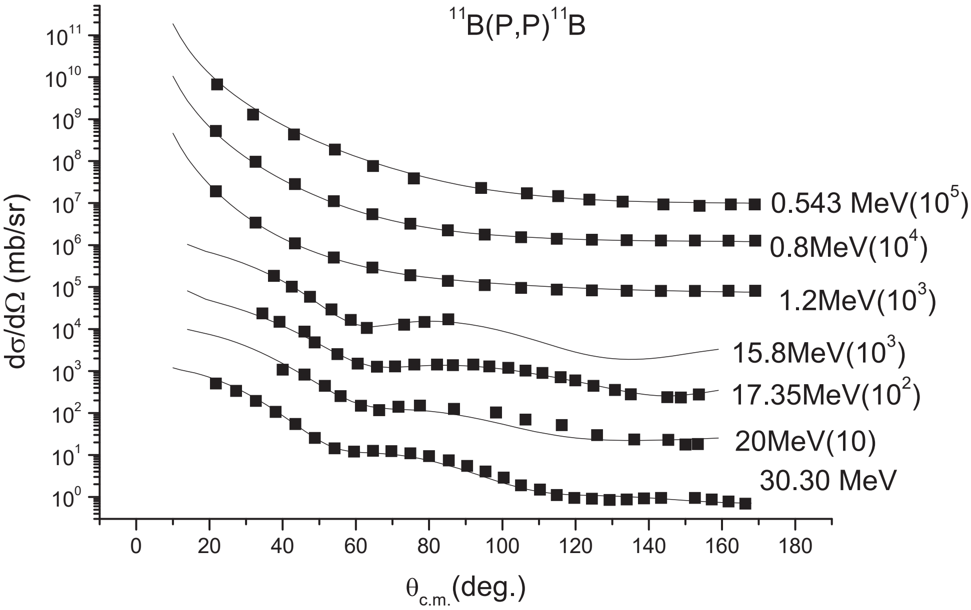

Fig. 1 Optical model fitting of experimental angular distributions of protons elastically scattered on ^{11}B at different energies

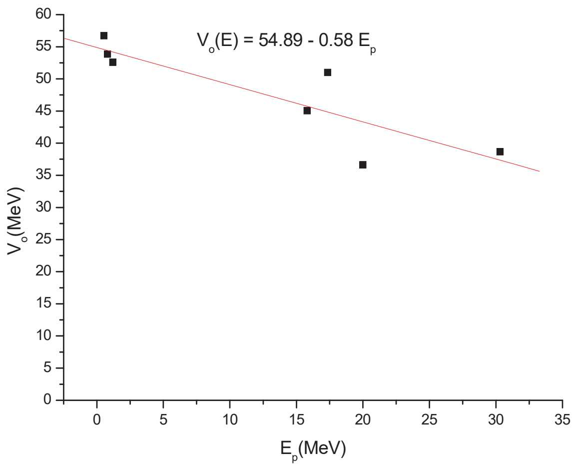

Fig. 2 Relation between real potential depth and proton energy

TABLE III
OPTICAL POTENTIAL PARAMETERS FOR DEUTERONS ELASTIC SCATTERING ON ^{11}B

E_d(MeV)	V_o(MeV)	a_v(fm)	W_D(MeV)	r_D(fm)	a_D(fm)	χ^2/N
5.5	86.76	0.793	7.75	1.65	0.54	1.156
11.8	85.794	0.717	3.68	1.592	0.997	5.258
27.7	80.005	0.734	10.27	1.433	0.652	5.24

The real potential depth of deuterons elastically scattered on ^{11}B nuclei at different deuterons energies can be represented by the following equation according to parameters listed in Table III as:

$$V_o = 88.925 - 0.316\, E_d$$

C. Phenomenological and Semi-Microscopic Analysis of the Elastic Scattering ^{11}B (^3He,^3He)^{11}B

Theoretical analysis of the measured angular distributions of the elastically scattered ^3He on ^{11}B nuclei at ^3He energies (8, 10, 12 MeV) [16] and (17.5, 40 MeV) [17] shown in Fig. 4 was performed in order to obtain the global optical potential parameters, which could fairly reproduce these experimental measurements. For data analysis, both phenomenological approach and semi-microscopic approach using code FRESCO [18] were used. The optical model analysis of the experimental data was performed by using Woods-Saxon (WS) form factor for both real and imaginary parts of the potential where the radii r_C=1.3fm, r_V=1.05fm and r_D=0.98 fm were fixed taken from [19], and only the four remaining parameters (V, W, a_V, a_W) were varied. The semi-microscopic analysis was performed by obtaining the real part from the folding procedure and using it with a Woods-Saxon term for the imaginary potential. The real part of optical potential is calculated from a more fundamental basis by the folding model in which effective nucleon-nucleon NN interaction was folded into the densities of both ^3He and ^{11}B nuclei using code FRESCO with normalization factors listed in Table *IV*, R= $r_oA^{1/3}$; where A mass number of the target.

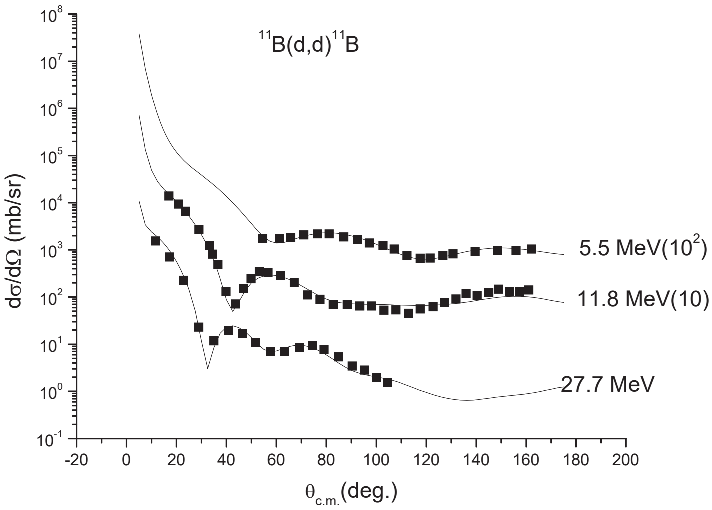

Fig. 3 Optical model fitting of the experimental angular distributions for deuterons elastically scattered from ^{11}B at different energies

TABLE IV
OPTICAL AND DOUBLE FOLDING POTENTIAL PARAMETERS FOR ^{3}HE ELASTICALLY SCATTERED ON ^{11}B USING FRESCO CODE

E_α (MeV)	Model	V (MeV)	a_v (fm)	Nr	W_i (MeV)	r_i (fm)	a_i (fm)	J_R MeV.fm^3	J_W MeV.fm^3
8	OM	99.636	0.658		6.075	0.98	0.735	287.867	44.021
	DF			0.7047	17.786	1.631	0.471		
10	OM	99.077	0.677		12.754	0.98	0.577	293.729	65.354
	DF			1.175	13.461		0.714		
12	OM	145.621	0.747		10.246	0.98	0.864	475.109	95.584
	DF			1.18	17.306	1.63	0.376		
17.5	OM	152.10	0.699		10.558	0.98	0.987	464.651	122.960
	DF			1.236	25.256	1.562	0.391		
40	OM	127.394	0.866		14.696	0.98	0.98	489.626	169.079
	DF			1.18	17.746	1.394	0.576		

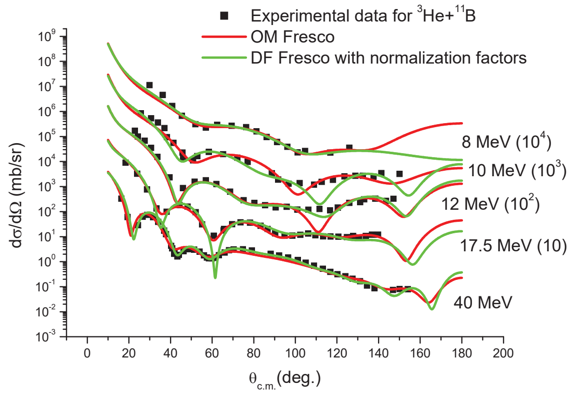

Fig. 4 Angular distributions for ^{3}He+^{11}B elastic scattering at ^{3}He energies (8, 10, 12, 17.5, 40 MeV) where (red line) represents the optical model calculations, (green line) represents the double folding calculations and (dots) represents the experimental data.

Search of optimal optical potential parameters (OP) for protons and deuterons analysis were carried out with the use of ECIS88 Code by means of the minimization of the χ2/N value:

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\left[\frac{(\sigma_i)_T-(\sigma_i)_E}{(\Delta\sigma_i)_E}\right]^2 \tag{12}$$

where $(\sigma_i)_T$ and $(\sigma_i)_E$ calculated and experimental values of differential cross sections for the given angle (Θ_i), respectively; $(\Delta\sigma_i)_E$ the experimental error; N= (P-F), P is the number of measured points and F is the number of varying parameters during the fitting process which equal 4 in this paper.

V. CONCLUSION

An analysis of protons, deuterons and helium-3 elastically scattered from ^{11}B nuclei in a wide energy range has been performed within the framework of the standard optical model as well as double folding model for the last reaction. Analysis of protons and deuterons performed using code (ECIS88), while the analysis of ^{3}He+^{11}B performed using code (FRESCO). Optical potential parameters were achieved on the base of best agreement between theoretical and experimental angular distribution with physical meaning. The automatic search of optimal parameters was carried out by the minimization of χ^2/N value. In DF calculations, the extracted normalization factor N_r was in the range 0.7–1.236. The comparison between the experimental data and the theoretical predictions is good in the whole angular range.

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