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Data-Reusing Adaptive Filtering Algorithms with Adaptive Error Constraint

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Abstract—We present a family of data-reusing and affine projection algorithms. For identification of a noisy linear finite impulse response channel, a partial knowledge of a channel, especially noise, can be used to improve the performance of the adaptive filter. Motivated by this fact, the proposed scheme incorporates an estimate of a knowledge of noise. A constraint, called the *adaptive noise constraint*, estimates an unknown information of noise. By imposing this constraint on a cost function of data-reusing and affine projection algorithms, a cost function based on the adaptive noise constraint and Lagrange multiplier is defined. Minimizing the new cost function leads to the *adaptive noise constrained* (ANC) data-reusing and affine projection algorithms. Experimental results comparing the proposed schemes to standard data-reusing and affine projection algorithms clearly indicate their superior performance.

Keywords—Data-reusing, affine projection algorithm, error constraint, system identification.

I. Introduction

HIGH eigenvalue spread of the input signal correlation matrix tend to deteriorate the convergence performance of the least mean square (LMS)-type adaptive filters [1]. Recently, the data-reusing LMS (DR-LMS), the normalized DR-LMS (NDR-LMS,) and the affine projection (AP) algorithms have spawned great interest among researchers desiring to improve convergence at reduced computational cost, and to trade off convergence rate as a function of the computational complexity [2]–[5]. In contrast to LMS-type adaptive filters, these algorithms use block error and block input vector for updating the filter coefficient.

For an identification of a noisy linear finite impulse response (FIR) channel, a partial knowledge of a channel, especially noise, can be used to improve the performance of the adaptive filter [6][7]. Motivated by this fact, we expect that the performance of DR-LMS, NDR-LMS and AP algorithms can be further improved by incorporating a knowledge of noise. However, a knowledge of noise generally is not available to the filter. To overcome this obstacle, a constraint, called *adaptive noise constraint*, which estimates an unknown information of noise is introduced. By imposing this constraint on a cost function of DR and AP algorithms, we present the *adaptive noise constrained* (ANC) DR-LMS, NDR-LMS and AP algorithms, which are based on the adaptive noise constraint and Lagrange multiplier. Through experiments, we illustrate that the proposed algorithms possess better performance than standard DR-LMS, NDR-LMS, AP algorithms in terms of the convergence rate and the misadjustment.

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II. Adaptive Noise Constrained DR and AP Algorithms

Consider data $d(i)$ that arise from the system identification model

$$d(i) = \mathbf{u}_i \mathbf{w}^\circ + v(i), \tag{1}$$

where $\mathbf{w}^\circ$ is a column vector for the impulse response of an unknown system that we wish to estimate, $v(i)$ accounts for measurement noise and $\mathbf{u}_i$ denotes the $1 \times M$ row input vector,

$$\mathbf{u}_i = [u(i) \;\; u(i-1) \;\; \cdots u(i-M+1)], \tag{2}$$

and $\mathbf{u}_i$ and $v(i)$ are uncorrelated.

A. Conventional DR-LMS and NDR-LMS and AP Algorithms

Let $\mathbf{w}_i$ be an estimate for $\mathbf{w}^\circ$ at iteration i. The DR-LMS, NDR-LMS, and APA take the forms [2]

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu \mathbf{U}_i^* \mathbf{e}_i \tag{3}$$

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu \mathbf{U}_i^* \mathbf{D}_i \mathbf{e}_i \tag{4}$$

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu \mathbf{U}_i^* (\mathbf{U}_i \mathbf{U}_i^* + \delta \mathbf{I})^{-1} \mathbf{e}_i, \tag{5}$$

respectively, where

$$\mathbf{U}_i = \begin{bmatrix} \mathbf{u}_i \\ \mathbf{u}_{i-1} \\ \vdots \\ \mathbf{u}_{i-K+1} \end{bmatrix}, \quad \mathbf{d}_i = \begin{bmatrix} d(i) \\ d(i-1) \\ \vdots \\ d(i-K+1) \end{bmatrix},$$

$\mathbf{e}_i = \mathbf{d}_i - \mathbf{U}_i \mathbf{w}_{i-1}$, $\mathbf{D}_i = \text{diag}[1/\|\mathbf{u}_i\|^2, \ldots, 1/\|\mathbf{u}_{i-K+1}\|^2]$, μ is the step-size, and * denotes the Hermitian transpose. It is known [2] that these algorithms are obtained to minimize the following cost function

$$J(i) = E[\mathbf{e}_i^* \Pi \mathbf{e}_i], \tag{6}$$

where Π is a positive-definite matrix. The gradient vector of $J(i)$ with respect to $\mathbf{w}_{i-1}$ is given by

$$\frac{\partial J(i)}{\partial \mathbf{w}_{i-1}} = -E[\mathbf{e}_i^* \Pi \mathbf{U}_i]. \tag{7}$$

Then, we can obtain the stochastic gradient algorithm

$$\mathbf{w}_i = \mathbf{w}_{i-1} + \mu \mathbf{U}_i^* \Pi \mathbf{e}_i \tag{8}$$

Note that the choice of Π determines algorithms of (3)–(5). In other words, if we choose $\Pi = I$, i.e., the identity matrix, the DR-LMS algorithm (3) is obtained. And with the choice of data-normalized identity matrix, $\mathbf{D}_i$, we get the NDR-LMS (4). Also the choice $\Pi = (\mathbf{U}_i \mathbf{U}_i^*)^{-1}$ results in the APA (5).

The minimization of (6) over $\mathbf{w}_{i-1}$ yields $\mathbf{w}_{i-1} = \mathbf{w}^{\circ}$ and $\mathbf{e}_i = \mathbf{v}_i$ where $\mathbf{v}_i = [v(i)\ v(i-1)\ldots v(i-K+1)]^T$. This optimal $\mathbf{w}_{i-1}$ results in

$$J(i)|_{\mathbf{w}_{i-1}=\mathbf{w}^{\circ}} = E[\mathbf{v}_i^*\Pi\mathbf{v}_i]. \tag{9}$$

B. Adaptive Noise Constrained (ANC) Algorithms

Along this line of thought, we organize a constrained optimization problem incorporating the knowledge of $\mathbf{v}_i$. The optimum solution is obtained by minimizing $J(i)$ subject to $J(i) = E[\mathbf{v}_i^*\Pi\mathbf{v}_i]$. The augmented cost function, using a Lagrange multiplier λ, is given by

$$J_1(i) = J(i) + \gamma\lambda\left(J(i) - E[\mathbf{v}_i^*\Pi\mathbf{v}_i]\right) - \gamma\lambda^2, \tag{10}$$

where $\gamma > 0$. To get a unique critical λ, a term $-\gamma\lambda^2$ is used [6]. However, this cost function is based on the knowledge of $\mathbf{v}_i$. To avoid this unpractical obstacle, $E[\mathbf{v}_i^*\Pi\mathbf{v}_i]$ is replaced by an unknown variable ζ, which is adjusted at each iteration. Then the proposed cost function is given by

$$J_{\text{ANC}}(i) = J(i) + \gamma\lambda\left(J(i) - \zeta\right) - \gamma\lambda^2 + \rho\zeta^2, \tag{11}$$

where $\gamma, \rho > 0$. In (11), we know that the proposed cost function is minimized with respect to the weight and ζ, and maximized with respect to λ. Then the update equations are as follows:

$$\begin{aligned} \mathbf{w}_i &= \mathbf{w}_{i-1} - \mu_{\mathbf{w}}\nabla_{\mathbf{w}}J_{\text{ANC}} & (12)\\ \lambda_{i+1} &= \lambda_i + \mu_\lambda\nabla_\lambda J_{\text{ANC}} & (13)\\ \zeta_{i+1} &= \zeta_i - \mu_\zeta\nabla_\zeta J_{\text{ANC}}, & (14) \end{aligned}$$

where $\mu_{\mathbf{w}}, \mu_\lambda$ and μ_ζ are positive parameter. The gradients in (12)–(14) are simply derived as

$$\begin{aligned} \nabla_{\mathbf{w}}J_{\text{ANC}} &= -E[\mathbf{e}_i^*\Pi\mathbf{U}_i] & (15)\\ \nabla_\lambda J_{\text{ANC}} &= \gamma\left(E[\mathbf{e}_i^*\Pi\mathbf{e}_i] - \zeta\right) - 2\gamma\lambda & (16)\\ \nabla_\zeta J_{\text{ANC}} &= \gamma\lambda + 2\rho\zeta, & (17) \end{aligned}$$

respectively. Replacing the expected values of (15)–(17) by its instantaneous values and substituting for (12)–(14), we have the following stochastic gradient based update algorithm:

$$\begin{aligned} \mathbf{w}_i &= \mathbf{w}_{i-1} - \mu_{\mathbf{w}}(1+\gamma\lambda_i)\mathbf{U}_i^*\Pi\mathbf{e}_i & (18)\\ \lambda_{i+1} &= \lambda_i + \mu_\lambda\gamma\left[\frac{1}{2}(\mathbf{e}_i^*\Pi\mathbf{e}_i - \zeta_i) - \lambda_i\right] & (19)\\ \zeta_{i+1} &= \zeta_i - \mu_\zeta(\gamma\lambda_i - 2\rho\zeta_i). & (20) \end{aligned}$$

As mentioned above, a different form of Π, i.e., $\Pi = \mathbf{I}$, $\text{diag}[1/\|\mathbf{u}_i\|^2, \ldots, 1/\|\mathbf{u}_{i-K+1}\|^2]$, and $(\mathbf{U}_i\mathbf{U}_i^*)^{-1}$, results in the adaptive noise constrained (ANC) DR-LMS, NDR-LMS, and AP algorithms, respectively.

C. Properties of Proposed Algorithms

Let us consider the steady-state mean behavior of λ_i and ζ_i. By taking the expectation of steady-state value of both sides of (20), it leads to

$$E[\zeta_\infty] = E[\zeta_\infty] - \mu_\zeta(\gamma E[\lambda_\infty] - 2\rho E[\zeta_\infty]), \tag{21}$$

$$E[\zeta_\infty] = \frac{\gamma}{2\rho}E[\lambda_\infty], \tag{22}$$

TABLE I
EXPERIMENTAL PARAMETERS OF ADAPTIVE NOISE CONSTRAINED ALGORITHMS

ANC-DR-LMS	ANC-NDR-LMS	ANC-APA
$\mu_{\mathbf{w}} = 0.004$	$\mu_{\mathbf{w}} = 0.04$	$\mu_{\mathbf{w}} = 0.016$
$\mu_\lambda = 10^{-8}$	$\mu_\lambda = 10^{-7}$	$\mu_\lambda = 3\times10^{-6}$
$\mu_\zeta = 10^{-5}$	$\mu_\zeta = 10^{-8}$	$\mu_\zeta = 10^{-7}$
$\gamma = 2000$	$\gamma = 5000$	$\gamma = 2000$
$\rho = 100$	$\rho = 100$	$\rho = 100$

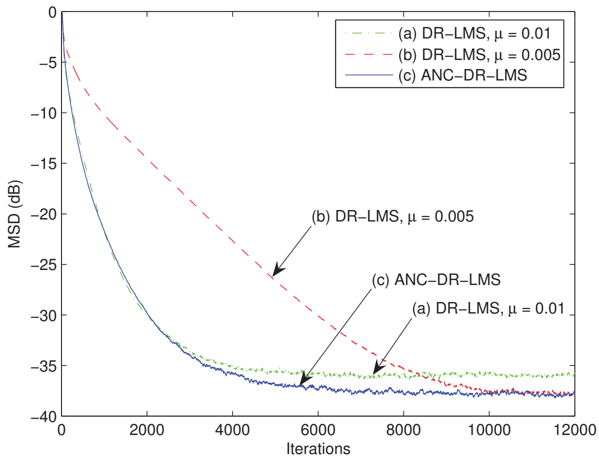

Fig. 1 Plots of MSD for the ANC-DR-LMS and the DR-LMS [K=4, input: ARMA(2,2)]

and

$$E[\lambda_\infty] = \frac{2\rho}{4\rho+\gamma}E[\mathbf{e}_\infty^*\Pi\mathbf{e}_\infty]. \tag{23}$$

In addition, from (20), we find that

$$E[\zeta_\infty] = \frac{\gamma}{2\rho}E[\lambda_\infty]. \tag{24}$$

From (23) and (24), we obtain the following relation as

$$E[\zeta_\infty] = \frac{2\rho}{4\rho+\gamma}E[\mathbf{e}_\infty^*\Pi\mathbf{e}_\infty]. \tag{25}$$

If $\gamma >> 4\rho$, ζ_i converges to $E[\mathbf{e}_\infty^*\Pi\mathbf{e}_\infty]$.

III. EXPERIMENTAL RESULTS

We illustrate the performance of the proposed algorithms by carrying out computer simulations in a channel identification scenario. The unknown channel $H(z)$ has 16 taps and is randomly generated. The adaptive filter and the unknown channel are assumed to have the same number of taps. The input signal is obtained by filtering a white, zero-mean, Gaussian random sequence through a first-order system

$$G(z) = \frac{1+0.5z^{-1}+0.81z^{-2}}{1-0.59z^{-1}+0.4z^{-2}}.$$

This results in a highly correlated Gaussian signal of which the eigenvalue spread is about 105. The signal-to-noise ratio (SNR) is calculated by

$$\text{SNR} = 10\log_{10}\left(E[y^2(i)]/E[v^2(i)]\right), \tag{26}$$

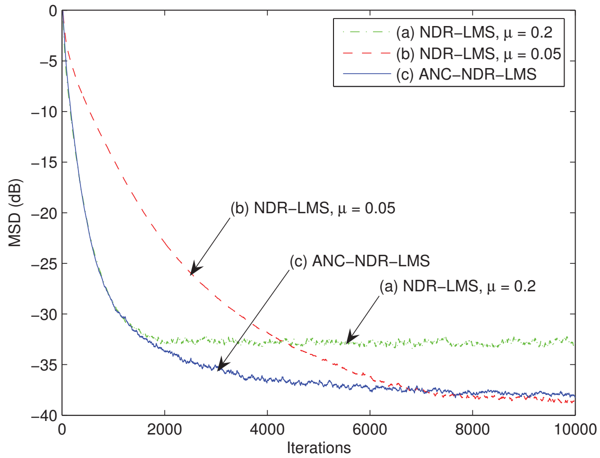

Fig. 2 Plots of MSD for the ANC-NDR-LMS and the NDR-LMS [K=4, input: ARMA(2,2)]

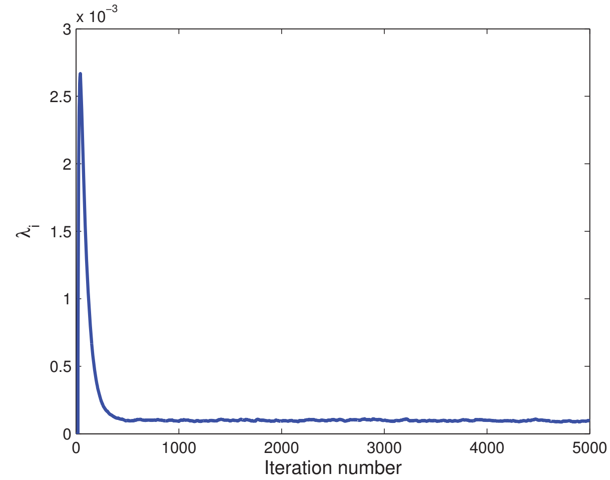

Fig. 4 Time evolution of λ_i of the ANC-APA

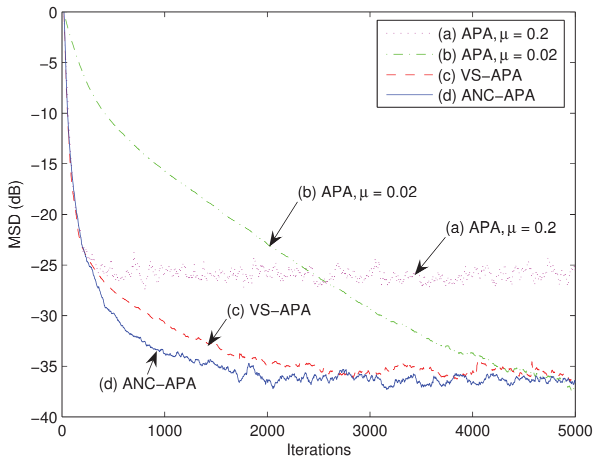

Fig. 3 Plots of MSD for the ANC-APA, the VS-APA [8] and the APA [K=4, input: ARMA(2,2)]

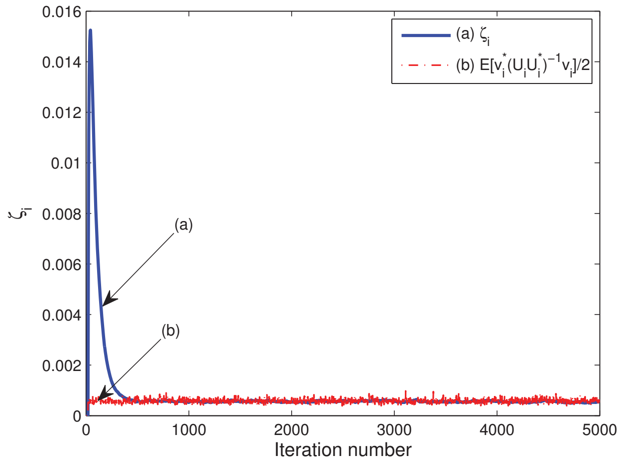

Fig. 5 Time evolution of ζ_i of the ANC-APA

where $y(i) = \mathbf{u}_i\mathbf{w}^\circ$. The measurement noise $v(i)$ is added to $y(i)$ such that SNR $=$ 30dB. The mean square deviation (MSD), $E\|\mathbf{w}^\circ - \mathbf{w}_i\|^2$, is taken and averaged over 100 independent trials. The parameters used for the proposed algorithms are shown in Table. I.

Fig. 1 indicates the MSD curves of the DR-LMS and the ANC-DR-LMS. Dashed lines indicate the results of the DR-LMS with fixed step-sizes when we choose $\mu = 0.01$ and 0.005. As can be seen, the ANC-DR-LMS has the faster convergence and lower misadjustment than the standard DR-LMS. Fig. 2 shows the MSD curves of the NDR-LMS and the ANC-NDR-LMS. We choose the step-sizes of the NDR-LMS as $\mu = 0.2$ and 0.05. A similar result of Fig. 1 is observed in Fig. 2. Fig. 3 indicates the performance of the APA and the ANC-APA. For a comparison purpose, the variable step-size APA (VS-APA) [8] is presented. we use $C = 0.01$ and $\mu_{\max} = 1.0$ for the VS-APA, which are defined in [8]. We know that the ANC-APA outperforms the APA and is comparable to the VS-APA.

In Fig. 4, the transient behaviour of λ_i of the ANC-APA is depicted. It exhibits the time evolution of λ_i which increases rapidly and then converges. Fig. 5 indicates that ζ_i converges to $E[\mathbf{v}_i^*(\mathbf{U}_i\mathbf{U}_i^*)^{-1}\mathbf{v}_i]/2$ which is the noise related constraint.

IV. Conclusion

In this paper, we present adaptive noise constrained (ANC) DR-LMS, NDR-LMS and AP algorithms, which incorporate a knowledge of noise without a prior information of noise. A cost function based on the adaptive noise constrained optimization using Lagrangian multipliers is introduced to estimate a noise information. As a result, the convergence performance of data-reusing and affine projection algorithms is greatly improved.

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