

# Assessing the Effect of Grid Connection of Large-Scale Wind Farms on Power System Small-Signal Angular Stability

Wenjuan Du, Jingtian Bi, Tong Wang, Haifeng Wang

**Abstract**—Grid connection of a large-scale wind farm affects power system small-signal angular stability in two aspects. Firstly, connection of the wind farm brings about the change of load flow and configuration of a power system. Secondly, the dynamic interaction is introduced by the wind farm with the synchronous generators (SGs) in the power system. This paper proposes a method to assess the two aspects of the effect of the wind farm on power system small-signal angular stability. The effect of the change of load flow/system configuration brought about by the wind farm can be examined separately by displacing wind farms with constant power sources, then the effect of the dynamic interaction of the wind farm with the SGs can be also computed individually. Thus, a clearer picture and better understanding on the power system small-signal angular stability as affected by grid connection of the large-scale wind farm are provided. In the paper, an example power system with grid connection of a wind farm is presented to demonstrate the proposed approach.

**Keywords**—Power system small-signal angular stability, power system low-frequency oscillations, electromechanical oscillation modes, wind farms, double fed induction generator (DFIG).

## I. INTRODUCTION

LOW frequency power oscillations threaten the safe operation of power systems. They occur as the result of inter-connection of local power networks and the installation of fast-acting automatic voltage regulators (AVRs). It has been reckoned that the occurrence of the oscillations is normally due to the lack of damping of power system electromechanical oscillation modes. Low-frequency power oscillations are the main concern of power system small-signal angular stability.

Connection of a large-scale wind farm with a great number of variable speed wind generators (VSWGs) into a power system may affect the system small-signal angular stability [1]-[8]. To examine the damping of power system low-frequency oscillations as affected by the large-scale grid-connected wind farm, the work in the first step may be the investigation of the change of the damping of power system electromechanical oscillation modes brought about by the grid

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connection of the wind farm. This seems not difficult as the change can be computed by comparing the damping of the modes before and after the wind farm is connected into the power system. The comparison can be illustrated by a simple example power system shown by Figs. 1 and 2.

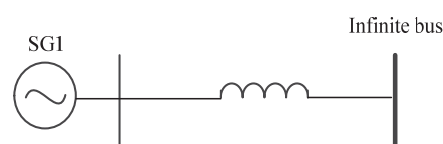


Fig. 1 A single-machine infinite-bus power system

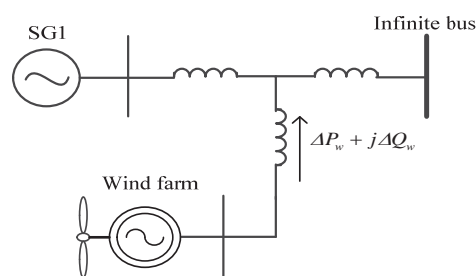


Fig. 2 Addition of a wind farm into the system of Fig. 1

Fig. 1 is a single-machine infinite-bus power system. With a wind farm connected, the system may become that illustrated by Fig. 2. Denote  $\Delta\lambda$  the change of the electromechanical oscillation mode of the power system after the wind farm is connected.  $\Delta\lambda$  can be easily obtained by modal computation of the system of Figs. 1 and 2 to have  $\Delta\lambda = \lambda - \lambda_w$  where  $\lambda$  is the oscillation mode of the system of Fig. 1 and  $\lambda_w$  is that of the system of Fig. 2. However,  $\Delta\lambda$  is caused by two changes with the wind farm being added. The first change is the load flow and system configuration introduced by the wind farm which is obviously different between the system of Figs. 1 and 2. The second is the addition of the dynamic interaction between the wind farm and the synchronous generator. A simple comparison between the oscillation mode of the system of Fig. 1 and that of Fig. 2 cannot distinguish those two factors that the wind farm affects the system small-signal angular stability. Thus exactly how and why the addition of the wind farm into the power system of Fig. 1 causes the change of system oscillation modes remains ambiguous by the comparison.

This paper proposes a method to assess the effect of large-scale wind farms on power system small-signal angular

stability. In establishing the linearized model of a power system connected with a large-scale wind farm, the dynamic power exchange of the wind farm,  $\Delta P_w + j\Delta Q_w$  (Fig. 2) with the synchronous generators (SGs) in the power system is modelled as the input to the system. Thus the established model clearly indicates that the effect of the load flow and system configuration introduced by the wind farm on the system small-signal angular stability can be examined by modelling the wind farm as a constant power source. The proposed method enables the separation of two factors that the wind farm affects the electromechanical oscillation modes in the assessment. Hence by using the proposed method, a clearer picture and deeper insight can be provided about exactly how and why the system small-signal angular stability is influenced by the grid connection of the wind farm.

## II. METHOD TO ASSESS THE EFFECT OF WIND FARMS ON POWER SYSTEM SMALL-SIGNAL ANGULAR STABILITY

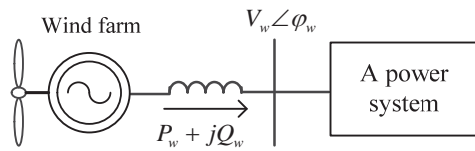


Fig. 3 Addition of a wind farm into a system

Fig. 3 shows a power system where a wind farm is connected at the PCC (point of common connection). Denote the power injection from the wind farm to the power system by  $P_w + jQ_w$ , the magnitude and phase of the voltage at the PCC by  $V_w$  and  $\phi_w$  respectively. Establish the following linearized state-space model of the power system:

$$s\Delta X_G = A_G \Delta X_G + b_{G1} \Delta P_w + b_{G2} \Delta Q_w$$

$$\begin{bmatrix} \Delta V_w \\ \Delta \phi_w \end{bmatrix} = C_G \Delta X_G + d_{G1} \Delta P_w + d_{G2} \Delta Q_w \quad (1)$$

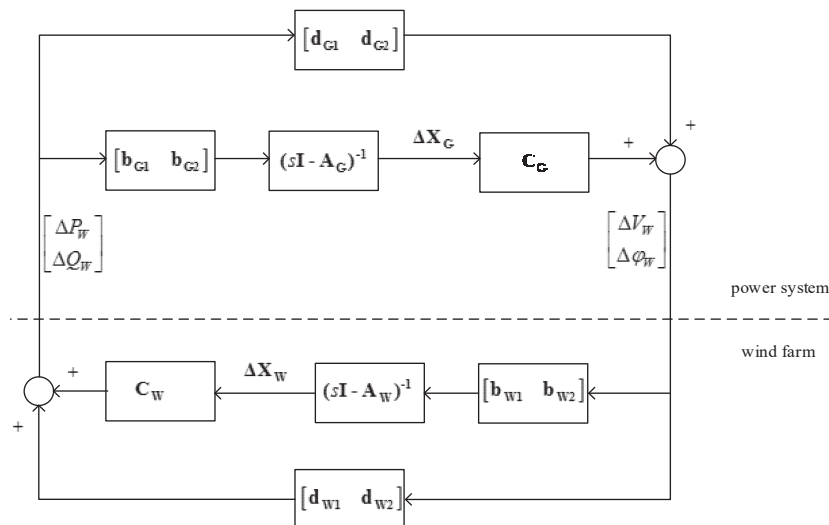


Fig. 4 Linearized model of the power system with the wind farm

$$s\Delta X_w = A_w \Delta X_w + b_{w1} \Delta V_w + b_{w2} \Delta \phi_w$$

$$\begin{bmatrix} \Delta P_w \\ \Delta Q_w \end{bmatrix} = C_w \Delta X_w + d_{w1} \Delta V_w + d_{w2} \Delta \phi_w \quad (2)$$

Equation (1) is the linearized model of the power system where  $\Delta X_G$  is the state variable vector of the synchronous generators (SGs). Equation (2) is the linearized model of the wind farm where  $\Delta X_w$  is the state variable vector of the wind farm. Seeing the SGs as the open-loop plant and the wind farm as the feedback controller, the power system with the wind farm connected forms a closed-loop system as shown by Fig. 4.

Fig. 4 clearly describes the two-ways dynamic interactions between the SGs and the wind farm. One of the ways is the response of the wind farm to  $\Delta V_w \angle \Delta \phi_w$  to inject a variable power  $\Delta P_w + j\Delta Q_w$  into the power system. The other way is the response of the power system to  $\Delta P_w + j\Delta Q_w$  to generate a variation of the voltage at the PCC of the wind farm  $\Delta V_w \angle \Delta \phi_w$  during the electromechanical transient of the power system. If  $\Delta P_w + j\Delta Q_w = 0$ , the wind farm is decoupled dynamically with the power system and there is no dynamic interaction between the wind farm and the power system. This is the case that the **effect of the dynamic interaction** of the wind farm with the power system on system small-signal angular stability is equal to zero. In this assumed case, the influence of the wind farm on the system small-signal angular stability only exists in the way of the load flow and system configuration change it introduces. This influence is represented in the state matrix of the open-loop plant  $A_G$  and can be determined by the modal computation of  $A_G$ . Since  $\Delta P_w + j\Delta Q_w = 0$  means that the wind farm is degraded into a constant power source, electromechanical oscillation modes computed from  $A_G$  must have included the effect of the load flow and change of system configuration brought about by the wind farm on the small-signal angular stability.

From (1) and (2), the state-space model of the closed-loop system of Fig. 5 can be obtained to be:

$$\begin{bmatrix} s\Delta X_G \\ s\Delta X_W \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \Delta X_G \\ \Delta X_W \end{bmatrix} = A_{G+W} \begin{bmatrix} \Delta X_G \\ \Delta X_W \end{bmatrix} \quad (3)$$

where;

$$\begin{aligned} A_{11} &= A_G + [b_{G1} \ b_{G2}](I - [d_{W1} \ d_{W2}][d_{G1} \ d_{G2}]^{-1}[d_{W1} \ d_{W2}])C_G \\ A_{12} &= [b_{G1} \ b_{G2}](I - [d_{W1} \ d_{W2}][d_{G1} \ d_{G2}]^{-1})C_W \\ A_{21} &= [b_{W1} \ b_{W2}]C_G \\ &+ [b_{W1} \ b_{W2}][d_{G1} \ d_{G2}](I - [d_{W1} \ d_{W2}][d_{G1} \ d_{G2}]^{-1}[d_{W1} \ d_{W2}])C_G \\ A_{22} &= A_W + [b_{W1} \ b_{W2}][d_{G1} \ d_{G2}](I - [d_{W1} \ d_{W2}][d_{G1} \ d_{G2}]^{-1})C_W \end{aligned}$$

The electromechanical oscillation modes computed from  $A_{G+W}$  includes the effect of the load flow and change of system configuration introduced by the wind farm and the dynamic interaction between the wind farm and the power system.

Therefore, modal analysis to examine the effect of adding the wind farm into the power system on the system small-signal angular stability can be carried out in two steps as:

Step 1: Model the wind farm as a constant power and establish the linearized model of the power system of Fig. 4 in the form of (1) and (2) and compute the electromechanical oscillation modes of the power system from the open-loop state matrix  $A_G$  as  $\bar{\lambda}_{0i}, i=1,2,\dots$ .

Step 2: Derive the closed-loop state-space model of the power system of (3) and compute the electromechanical oscillation modes of the power system from the closed-loop state matrix  $A_{G+W}$  as  $\bar{\lambda}_i, i=1,2,\dots$ .

Thus  $\bar{\lambda}_{0i}, i=1,2,\dots$  indicate the effect of the load flow and system configuration change introduced by the wind farm on the system small-signal angular stability.  $\Delta\bar{\lambda}_i = \bar{\lambda}_i - \bar{\lambda}_{0i}, i=1,2,\dots$  is the effect of the dynamic interaction between the wind farm and the power system on the system small signal angular stability. Two affecting factors of adding the wind farm into the power system on the system small-signal angular stability are successfully separated.

### III. AN EXAMPLE

Fig. 5 shows the configuration of New England 10-machine 39-bus example power system which has been used for studying power system low-frequency oscillations in many occasions. Parameters of the system and generators provided in [9] are used.

A wind farm is represented by a DFIG wind generator. Its model and parameters are presented in the Appendix. When the wind farm is connected at bus 16 without displacing any synchronous generators, the computational results of electromechanical oscillation modes of the power system are presented in Table I. The 1<sup>st</sup> to the 8<sup>th</sup> oscillation modes are local electromechanical oscillation modes and the 9<sup>th</sup> is the inter-area mode of G10 to G1~G9.

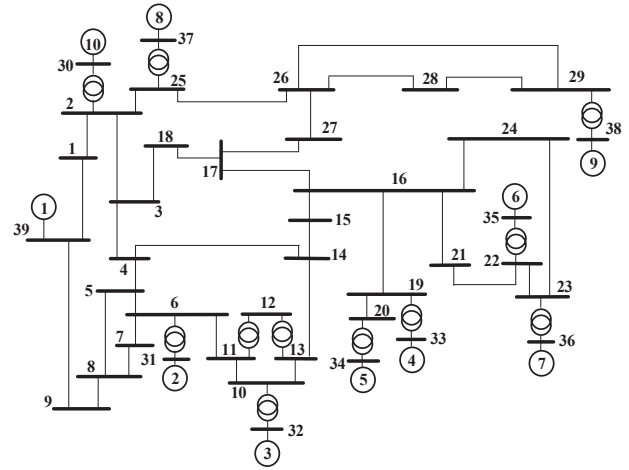


Fig. 5 Configuration of New England power system

- 1) By comparing results in column B (without the wind farm added) and D (with the wind farm added) in Table I it can be seen that the maximum impact of adding the wind farm is on the inter-area oscillation mode. The impact is detrimental to the system small-signal angular stability as the inter-area mode becomes less damped as the results of the addition of the wind farm with total change of the real part of the mode to be 0.0494
- 2) From column E in Table I it can be seen that the impact of dynamic interaction between the wind farm and the system is also biggest on the inter-area oscillation mode. This impact benefits the system small-signal angular stability as the impact moves the inter-area mode further left on the complex plane by -0.0045.
- 3) Hence the impact of load flow and system configuration change introduced by adding the wind farm moves the inter-area oscillation mode on the complex plane by  $0.0494 - 0.0045 = 0.0449$ . It is detrimental to the system small-signal angular stability and much greater than that brought about by the dynamic interaction between the wind farm and the system.

### IV. CONCLUSIONS

This paper proposes a method to assess the effect of grid connection of a large-scale wind farm on power system small-signal angular stability. When the wind farm is connected to a power system, the total effect of the connection includes that from the change of load condition/system configuration added dynamic interactions between wind farm and the synchronous generators in the power system. By using the proposed method, those two aspects of effect can be examined separately. Thus a clearer picture and better understanding is provided on the power system small-signal angular stability as affected by the grid connection of the wind farm. Application of the proposed approach is demonstrated by an example power system in the paper.

TABLE I  
 COMPUTATIONAL RESULTS OF ELECTROMECHANICAL OSCILLATION MODES WHEN THE WIND FARM IS ADDED ON NODE 16 IN THE NEW ENGLAND POWER SYSTEM

A	B	C ( $\bar{\lambda}_{0i}$ )	D ( $\bar{\lambda}_i$ )	E ( $\Delta\bar{\lambda}_i$ )	F
1	-0.4254 + 7.7704j	-0.4397 + 7.8457j	-0.4396 + 7.8457j	0.0001 + 0.0000j	-0.0143 + 0.0753i
2	-0.3796 + 7.7454j	-0.3889 + 7.7762j	-0.3891 + 7.7762j	-0.0002 + 0.0000j	-0.0093 + 0.0308i
3	-0.4034 + 7.6132j	-0.4024 + 7.6670j	-0.4020 + 7.6670j	0.0004 + 0.0000j	0.0010 + 0.0538i
4	-0.1546 + 6.4331j	-0.1589 + 6.4350j	-0.1588 + 6.4351j	0.0001 + 0.0001j	-0.0043 + 0.0019i
5	-0.2784 + 6.5730j	-0.2841 + 6.2278j	-0.2846 + 6.2286j	-0.0005 + 0.0008j	-0.0057 - 0.3452i
6	-0.2858 + 6.1251j	-0.2513 + 5.8822j	-0.2495 + 5.8811j	0.0018 - 0.0011j	0.0345 - 0.2429i
7	-0.2375 + 5.3408j	-0.2273 + 5.3448j	-0.2278 + 5.3450j	-0.0005 + 0.0002j	0.0102 + 0.0040i
8	-0.2184 + 5.6232j	-0.2155 + 4.4747j	-0.2154 + 4.4761j	0.0001 + 0.0014j	0.0029 - 1.1485i
9	-0.0592 + 3.2629j	-0.0098 + 3.1483j	-0.0143 + 3.1567j	<b>-0.0045 + 0.0084j</b>	<b>0.0494 - 0.1146i</b>

A: number of electromechanical oscillation modes; B: oscillation modes without the wind farm added; C ( $\bar{\lambda}_{0i}$ ): oscillation modes with the wind farm modelled as a constant power; D ( $\bar{\lambda}_i$ ): oscillation modes with the full dynamics of the wind farm included; E ( $\Delta\bar{\lambda}_i = \bar{\lambda}_i - \bar{\lambda}_{0i}$ ): effect of dynamic interaction between the wind farm and the SGs on the oscillation modes, F(C-B): effect of system load flow and configuration change on the oscillation modes

V. APPENDIX: MODEL AND PARAMETERS OF THE WIND FARM USED IN THE EXAMPLE POWER SYSTEM

Take the direction of stator voltage  $\bar{v}_s$  as that of q axis of d-q coordinate of the DFIG. Dynamic model of a DFIG can be written as [10], [11]:

$$\begin{aligned}
 pE_{wd}' &= \omega_0 \left( -\frac{R_r}{X_{rr}} E_{wd}' + sE_{wq}' + \frac{R_r X_m^2}{X_{rr}^2} I_{sq} - \frac{X_m}{X_{rr}} V_{rq} \right) \\
 pE_{wq}' &= \omega_0 \left( -sE_{wd}' - \frac{R_r}{X_{rr}} E_{wq}' - \frac{R_r X_m^2}{X_{rr}^2} I_{sd} + \frac{X_m}{X_{rr}} V_{rd} \right) \\
 ps &= \frac{1}{J} (T_e - T_{wm})
 \end{aligned} \quad (4)$$

where  $I_{sd}$  and  $I_{sq}$  are the direct and quadrature component of stator current,  $V_{rd}$  and  $V_{rq}$  the direct and quadrature component of rotor voltage,  $T_e$  and  $T_{wm}$  the electromagnetic and mechanical torque,  $R_r$  and  $X_{rr}$  the resistance and self-inductance of rotor,  $X_m$  the magnetizing inductance,  $J$  the inertia,  $\omega_0$  is the synchronous speed and

$$T_e = E_{wd}' I_{sd} + E_{wq}' I_{sq} \quad (5)$$

Thus voltage equation of stator winding of the DFIG is

$$V_{sd} = E_{wd}' - X' I_{sq} = 0, V_{sq} = V_s = E_{wq}' + X' I_{sd} \quad (6)$$

where  $X'$  is the transient inductance. Relation between stator and rotor current is:

$$I_{rd} = -\frac{X_{ss}}{X_m} I_{sd} - \frac{1}{X_m} V_s, I_{rq} = -\frac{X_{ss}}{X_m} I_{sq} \quad (7)$$

where  $X_{ss}$  is the self-inductance of stator winding.

Variation of DC voltage of the AC/DC converter and its effect on power system small-signal stability is very small. For the simplicity of derivation, it is assumed to be a constant (If considered, similar derivation can be carried out and

conclusions of the paper will not be affected). Hence the dynamics of converter and network-side converter control system are not included in the discussions.

Configuration of rotor-side converter control system [10], [11] is shown by Fig. 6, where superscript ref indicates the reference of associated signal,  $P_s$  and  $Q_s$  is the stator active and reactive power output of the DFIG respectively.

$$P_s = V_{sd} I_{sd} + V_{sq} I_{sq} = V_s I_{sq}, Q_s = V_{sq} I_{sd} - V_{sd} I_{sq} = V_s I_{sd} \quad (8)$$

Denote the transfer function of active and reactive power PI controller by  $K_{Ps}(p)$  and  $K_{Qs}(p)$ , that of direct and quadrature current by  $K_{rd}(p)$  and  $K_{rq}(p)$  respectively. According to Fig.6,

$$\begin{aligned}
 I_{sq}^{ref} &= K_{Ps}(p)(P_s^{ref} - P_s) \\
 I_{sd}^{ref} &= K_{Qs}(p)(Q_s^{ref} - Q_s) \\
 I_{rd}^{ref} &= -\frac{X_{ss}}{X_m} I_{sd}^{ref} - \frac{V_s}{X_m} \\
 I_{rq}^{ref} &= -\frac{X_{ss}}{X_m} I_{sq}^{ref} \\
 V_{rd} &= K_{rd}(p)(I_{rd}^{ref} - I_{rd}) + s(X_{rr} - \frac{X_m^2}{X_{ss}})I_{rq} \\
 V_{rq} &= K_{rq}(p)(I_{rq}^{ref} - I_{rq}) - s(X_{rr} - \frac{X_m^2}{X_{ss}})I_{rd} + s\frac{X_m}{X_{ss}}\Delta V_s
 \end{aligned} \quad (9)$$

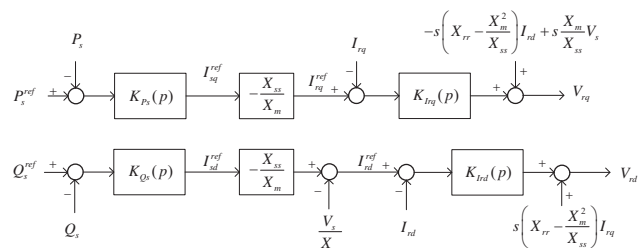


Fig. 6 Configuration of rotor-side converter control system

The rotor active and reactive power output of the DFIG generator are:

$$P_r = V_{rd} I_{rd} + V_{rq} I_{rq}, Q_r = 0 \quad (10)$$

The total output active and reactive power of DFIG generator is:

$$P_w = P_s + P_r, Q_w = Q_s + Q_r = Q_s \quad (11)$$

Parameters of the DFIG used in the example in p.u. are as follows:

$$T_r = 8s, R_s = 0, R_r = 0.0145, X_m = 2.4012, X_s = 0.1784, X_r = 0.1225$$

$$K_{ps}(p) = 0.2 + \frac{12.56}{p}, K_{qs}(p) = 0.2 + \frac{12.56}{p}, K_{rd}(p) = 1 + \frac{62.5}{p}, K_{rq}(p) = 1 + \frac{62.5}{p}$$

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