Identification of the Main Transition Velocities in a Bubble Column Based on a Modified Shannon Entropy
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Abstract—The gas holdup fluctuations in a bubble column (0.15 m in ID) have been recorded by means of a conductivity wire-mesh sensor in order to extract information about the main transition velocities. These parameters are very important for bubble column design, operation and scale-up. For this purpose, the classical definition of the Shannon entropy was modified and used to identify both the onset (at \( U_0 = 0.034 \) m/s) of the transition flow regime and the beginning (at \( U_0 = 0.089 \) m/s) of the churn-turbulent flow regime. The results were compared with the Kolmogorov entropy (KE) results. A slight discrepancy was found, namely the transition velocities identified by means of the KE were shifted to somewhat higher (0.045 and 0.101 m/s) superficial gas velocities \( U_0 \).

Keywords—Bubble column, gas holdup fluctuations, Modified Shannon entropy, Kolmogorov entropy.

I. INTRODUCTION

The successful identification of the boundaries of the main hydrodynamic regimes is essential for a successful design and scale-up of bubble columns. A good summary of the most famous identification methods has been provided in [1].

The degrees of mixing, mass and heat transfer as well as the reactor volume productivity of bubble columns depend on the prevailing flow regime. Each flow regime transition is usually influenced by many factors (column diameter, sparger design, pressure, temperature, etc.). The differentiation of the flow regime boundaries is important for the improvement of the operation and control of bubble columns.

There are two main transition velocities in bubble column operation. The first transition velocity indicates the transition from bubbly flow regime (or gas maldistribution regime) to transition regime. The second transition velocity distinguishes the onset of the churn-turbulent flow regime. The first transition velocity is more important since it takes part in the calculation of both large bubble diameter and large bubble holdup [2]. In the literature, only empirical correlations [3], [4] for prediction of the first transition velocity exist.

Three major flow regimes are commonly encountered in bubble columns: homogeneous (bubbly flow), transition and heterogeneous (churn-turbulent) flow. The homogeneous regime is characterized by a gentle agitation of the gas-liquid dispersion with relatively small and uniform bubbles. The bubble size distribution is very narrow and it is only influenced by the gas sparger. The bubble streams rise rather rectilinearly and bubble coalescence is insignificant. Bubble coalescence is insignificant. A relatively uniform gas holdup profile and a rather flat liquid velocity profile are observed.

The transition flow regime is characterized by large flow macrostructures (large eddies) and widened bubble size distribution due to the onset of bubble coalescence. This regime is formed due to the development of local circulation patterns in the column. The range of the transition regime depends on both the uniformity and the quality of aeration.

The transition from homogeneous to heterogeneous regime is a gradual process. As the superficial gas velocity \( U_0 \) increases, larger bubbles start to form whose wakes cause gross circulation patterns in the bubble bed leading to the formation of the heterogeneous (churn-turbulent flow) regime. The latter is characterized by a wide bubble size distribution and the existence of a radial gas holdup profile which causes liquid circulation. In this flow regime, coalescence and breakup occur. Bubbles coalesce in the vicinity of the gas distributor to form larger, spherical-cap bubbles. The heterogeneous regime is characterized by vigorous mixing. In this flow regime, the gas distributor has a negligible influence on the main parameters.

In the past five years, Nedeltchev and coworkers [1], [5]-[7] developed several new methods for flow regime identification based on information entropy, reconstruction entropy and new statistical parameters related to the number of signal’s visits into a region. The main objective of the current paper is to demonstrate the usefulness of the modified Shannon entropy for flow regime identification in bubble columns. Different methods are described in the literature but most of them do not identify clearly the flow regime boundaries. Therefore, a new method is needed. The power of the modified Shannon entropy will be illustrated on the basis of gas holdup fluctuations measured in the center of a bubble column by means of a conductivity wire-mesh sensor.

II. SHANNON ENTROPY AND ITS MODIFICATION

The Shannon entropy (SE) is a measure of the amount of information in a certain source (for example, time-dependent signal) and the degree of indeterminacy in a certain system. The SE quantifies the degree of uncertainty involved in predicting the output of a probabilistic event. If one predicts
the outcome of an event exactly before it happens, the probability will be a maximum value and as a result the SE will be a minimum value. If one is absolutely able to predict the outcome of an event, the SE will be zero.

According to [8] the SE of any time-dependent signal can be defined as:

\[ SE = \sum_{i=1}^{N} p(x_i) \ln[p(x_i)] \]  

where \( N \) is the length of the time series signal, \( p(x_i) \) is the probability of every component in the signal satisfying the constraint that the sum of all probabilities is equal to unity. 

Nedeltchev et al. [1] defined the probability as:

\[ p(x_i) = \frac{x_i}{\sum x_i} \]  

Since the total sum of all points (60,000 in our case) is not a good choice for the denominator, the summation was done for every 100 points and then the probability for each point from this particular group was calculated. Then, the same procedure was repeated for the next 100 points and so on. This approach gave more realistic probabilities. Based on them the modified SE was calculated. So, the main originality in the modification of the Shannon entropy algorithm is the new definition of the probability.

The time series were divided into 6 segments consisting of 10,000 points. The division of the time series into different segments is needed in order to identify the maximum (SE_{max}) and minimum (SE_{min}) SE values. In this work, it will be demonstrated that the ratio SE_{min}/SE_{max} and the difference between SE_{max} and SE_{min} are very useful parameters for the flow regime identification in a bubble column.

The SE is measured in nats. The larger SE corresponds to more disorder in the system. This implies more complex and chaotic nature resulting in turbulent motion of the gas or liquid, gas-liquid intensive interactions, flow instability, etc.

III. KOLMOGOROV ENTROPY

The modified SE results will be supported by the Kolmogorov entropy (KE) results. KE quantifies the degree of unpredictability of the system. The KE algorithm is considered as an effective diagnostic tool for the characterization of the degree of chaos and for the flow regime identification.

The KE reflects the rate of information loss of the system, and thus, accounts for the accuracy of the initial conditions that is required to predict the evolution of the system over a given time interval [9]. KE=0 is a sufficient condition for chaos and the behavior of the chaotic system is only predictable over a restricted time interval.

The KE algorithm developed by [10] is used in this work. However, a very high number of state vector pairs have been generated and compared. The number of elements in each state vector is equal to the embedding dimension. Following [11], the embedding dimension was set equal to 50 and delay time equal to unity was used. The maximum interpoint distance (so-called cut-off length) was set equal to three times the average absolute deviation (AAD) from the data’s mean. According to the definition of [10], the KE is a function of the sampling frequency \( f_s \) and the parameter \( b_{av} \):

\[ KE = f_s \ln \left( 1 - \frac{1}{b_{av}} \right) \]  

The variable \( b_{av} \) is the mean of all \( b \) values in the time series. Each \( b \) value equals the number of sequential pair of points, in which the interpoint distance is for the first time bigger than the specified maximum interpoint distance.

The time series (gas holdup fluctuations) for the KE calculation consisted of 60,000 points. They were not divided into six segments since it is considered that the KE estimation is reliable only when very long time series are used [9].

IV. EXPERIMENTAL SETUP

The gas holdup time series data were obtained in a small bubble column (0.15 m in ID) operated with an air-deionized water system under ambient conditions. The clear liquid height was kept constant at 2.0 m. The bubble column was equipped with a perforated plate distributor (14 holes with diameter \( 4 \times 10^{-3} \) m resulting in an open area of 1%). The gas holdup was measured in the center of the column by means of a conductivity wire-mesh sensor (see Fig. 1) installed at a height of 1.3 m above the distributor plate.

![Fig. 1 Design of conductivity wire-mesh sensor](image)

The wire-mesh sensor consisted of two electrode planes (24 stainless-steel wires). The wires were \( 0.2 \times 10^{-3} \) m in diameter and the lateral distance between them was \( 6.125 \times 10^{-2} \) m. The distance between the electrode planes was \( 4.0 \times 10^{-3} \) m and the wires from different planes ran at right angles to each other. These arrangements gave 576 crossings points, thereof 452 crossings inside the circular cross-section of the bubble column, respectively.

The time series of the gas holdup consisted of 60,000 points for every superficial gas velocity \( U_G \). They were obtained in the center of the column by averaging of the matrix data over a certain area (2×2 crossing points) of the sensor. As a result, some zero values occurred in the time series. They corresponded to the condition when only liquid appeared in the center of the column cross-section. It is worth noting that
the raw gas holdup values were estimated in percentage [%] and then they were treated by the proposed analysis methods.

The superficial gas velocity \( U_g \) was varied from 0.01 to 0.15 m/s. At each \( U_g \), the local value of the gas holdup was measured with sampling frequency \( f_s \) of 2000 Hz over a measurement period of 30 seconds.

V. RESULTS AND DISCUSSION

Fig. 2 shows that values of the ratio \( SE_{\text{min}}/SE_{\text{max}} \) can be used successfully for flow regime identification. At \( U_g=0.034 \) m/s the first well-pronounced local minimum is observed. Nedeltchev et al. [7] identified the same first transition velocity. The authors provided also photos to support their findings. The first minimum identifies the end of the gas maldistribution regime. At \( U_g=0.089 \) m/s the \( SE_{\text{min}}/SE_{\text{max}} \) ratio identifies successfully the onset of the churn-turbulent regime.

![Fig. 2 Values of the ratio \( SE_{\text{min}}/SE_{\text{max}} \) as a function of \( U_g \)](image1)

The difference between \( SE_{\text{max}} \) and \( SE_{\text{min}} \) can be also used for the sake of flow regime identification. Fig. 3 exhibits that the two well-pronounced maxima occur at the same \( U_g \) values (0.034 and 0.089 m/s) as in Fig. 2. These results imply that at every regime transition velocity the disturbance of the signal is very strong and that is why the difference \( SE_{\text{max}} - SE_{\text{min}} \) exhibits maximum.

![Fig. 3 Values of the difference \( SE_{\text{max}} - SE_{\text{min}} \) as a function of \( U_g \)](image2)

The average absolute deviation (AAD) values, calculated from the local SE values in the six signal segments, are also capable of identifying the two main transition velocities in a bubble column. Fig. 4 shows that the two local AAD maxima occur at exactly the same \( U_g \) values (0.034 and 0.089 m/s) as in Figs. 2 and 3.

![Fig. 4 Values of AAD (from six segments) as a function of \( U_g \)](image3)

In the churn-turbulent flow regime there are alternating minima and maxima, which are due to the enhanced degree of turbulence in this particular flow regime. The Kolmogorov entropy (KE) is also a very powerful tool for the flow regime identification. Fig. 5 shows that three well-pronounced local KE minima occur at \( U_g=0.045, 0.079 \) and 0.101 m/s, respectively. In other words, the KE results show that the onset of both transition and churn-turbulent flow regimes are shifted to somewhat higher \( U_g \) values in comparison with the results illustrated in Figs. 2-4. Nedeltchev et al. [5] identified also three transition velocities based on the information entropy extracted from the number of crossings.

It is noteworthy that the local KE minimum at \( U_g=0.079 \) m/s distinguishes the boundary between the first and second transition sub-regimes. These two transition sub-regimes have been observed experimentally by [12], [13]. This additional transition velocity at \( U_g=0.079 \) m/s is not very important since it does not affect the degrees of mixing, mass and heat transfer. Moreover, only one KE value (at \( U_g=0.089 \) m/s) belongs to the second transition sub-regime which makes its existence not very reliable. It is worth noting that the KE calculations are more complicated and time-consuming than the SE calculations. Since the SE depends only on the definition of the probability, whereas the KE depends on three parameters (cut-off length, number of elements in a state vector and delay time), the results in Figs. 2-4 should be considered as more important.
VI. CONCLUSION

A new identification method has been developed for the prediction of the two main transition velocities in a bubble column (0.15 m in ID) operated with an air-deionized water system. The new approach is based on a modification of the Shannon entropy (SE) and experimental gas holdup fluctuations measured by means of a conductivity wire-mesh sensor. It was found that the ratio of maximum SE to minimum SE as well as the difference between $SE_{\text{max}}$ and $SE_{\text{min}}$ are capable of identifying the two transition velocities at $U_G=0.034$ and 0.089 m/s. The average absolute deviation results confirmed these findings.

At attempt was made to compare the transition velocities identified by the new method with the ones distinguished by means of the Kolmogorov entropy (KE). Since the KE is a parameter from the nonlinear chaos theory, which is a much more powerful identification tool, the KE values were capable of identifying not only the two main transition velocities (at 0.045 and 0.101 m/s) but also the transition (at 0.079 m/s) between the first and second transition sub-regimes. It was found that the two main transition velocities identified by means of the KE are somewhat higher than the ones identified on the basis of the new method.

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NOMENCLATURE

- $AAD$: average absolute deviation, nats or percentage
- $b$: number of sequential pair of points in which the interpoint distance is lower than the cut-off length,–
- $b_{\text{no}}$: average number of $b$ values in the time series,–
- $f_s$: sampling frequency, 1/s
- $\text{KE}$: Kolmogorov entropy, bits/s
- $N$: number of data points,–
- $p(x)$: probability that a certain value will appear in the signal,–
- $SE$: modified Shannon entropy, nats
- $SE_{\text{max}}$: maximum modified Shannon entropy, nats
- $SE_{\text{min}}$: minimum modified Shannon entropy, nats
- $U_G$: superficial gas velocity, m/s
- $x_i$: point $i$ in the time series, %

REFERENCES