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Optimal Network of Secondary Warehouses for Production-Distribution Inventory Model

G. M. Arun Prasath, N. Arthi

***Abstract*—This work proposed a multi-objective mathematical programming approach to select the appropriate supply network elements. The multi-item multi-objective production-distribution inventory model is formulated with possible constraints under fuzzy environment. The unit cost has taken under fuzzy environment. The inventory model and warehouse location model has combined to formulate the production-distribution inventory model. Warehouse location is important in supply chain network. Particularly, if a company maintains more selling stores it cannot maintain individual secondary warehouse near to each selling store. Hence, maintaining the optimum number of secondary warehouses is important. Hence, the combined mathematical model is formulated to reduce the total expenditure of the organization by arranging the network of minimum number of secondary warehouses. Numerical example has been taken to illustrate the proposed model.

***Keywords*—Fuzzy inventory model, warehouse location model, triangular fuzzy number, secondary warehouse, LINGO software.

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I. Introduction

The need to gain a global competitive advantage on the supply side has forced businesses to search for effective supply network strategies. The effective selection of suppliers [13] is the key ingredient for the success of supply networks. The increasing interest in evaluating the performance of supply network over the last years indicates the need for the development of complex optimization models able to answer unsolved questions in the production distribution network. Implementations of a supply chain system [19] have crucial impacts on organization's financial performance. Overall performance of a supply network is influenced significantly by the decisions taken in its production-distribution plan integrating the decisions in production, transport, and warehousing as well as inventory management [4], [5], [9]. Thus, one key issue in the performance evaluation of supply networks is the modeling and optimization of production-distribution plan considering its actual complexity.

We propose a multi-objective mathematical programming approach to select the most appropriate supply network elements. That is, the real life problem is transformed into its equivalent deterministic form and presented as a multi-objective nonlinear programming. The warehouse space plays an important role in inventory models [7], [8]. The organization can maintain one separate warehouse near to the market place other than the warehouse space available in the selling stores [15]. The separate warehouse is called secondary warehouse [2], [6]. For more than one selling stores [11], maintaining individual secondary warehouses will increase the total expenditure of the organization. By arranging the network of minimum number of secondary warehouses for satisfying the requirements of all selling stores, the total expenditure of the organization will be minimized [1]. To identify the minimum number of secondary warehouses the warehouse location model is taken where the distance parameter is the distance between the secondary warehouses and selling stores. It determines the optimal set of warehouses for a given set of selling stores in the transportation network under defined constraints, which is known as the warehouse location model [18] distributing the items, and resolving the inventory control decision problem simultaneously since the regular warehouse location model does not. In most of the real world situations, the cost parameters are imprecise in nature [14]. Also, if unit cost increases, then the demand will decrease and if the unit cost decreases then the demand will increase. Because of this nature, the demand depends on unit cost. Hence, the unit cost is imposed herein fuzzy environment [16].

Based on the integration of the production plan and distribution plan, a nonlinear mathematical model formulation for a supply network is developed. The model incorporates multi-time periods, multi-products, multi-selling stores, multi-warehouses as well as multi-end users, and consider the real world variables and constraints. A production-distribution inventory model with possible constraints is developed for the optimization of lot size, unit cost, shortage level, percentage of utilization of warehouse space and the number of secondary warehouses [12]. Finally, the developed model can be analyzed in the case of a realistic scenario-based production-distribution problem. Considering detailed production cost elements and a realistic range of variables and constraints in the proposed model indicates the effectiveness of the developed model in the real-world applications.

II. Mathematical Model

A multi-item multi-objective production-distribution inventory model [10] is developed under the following notations and assumptions,

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A. Notations

- n - Number of items
- C_i - Scaling constant for the item i
- S_i - Set up cost of the cycle for the item i
- t - Number of orders per cycle
- H_i - Holding cost per item i
- v_i - Volume of the unit item i
- V - Volume of the warehouse space available in the selling store
- δ_i - Shortage cost of the item i
- k_{u_i} - Lower limit of unit cost of the item i
- k_{m_i} - Middle value of unit cost of the item i
- k_{o_i} - Upper limit of unit cost of the item i
- I - Total investment cost for production
- α - Number of selling stores
- β - Number of proposed secondary warehouses
- m - Number of production plant
- d - Maximum allowed distance between secondary warehouses and selling stores
- t_{ikj} - Unit transportation cost of unit item i from the secondary warehouse k to the selling store j
- $t_{i\gamma k}$ - Unit transportation cost of unit item i from the production plant γ to the secondary warehouse k
- p_i - Unit cost of the item i
- μ_{p_i} - Membership value of the unit cost of the item i
- Q_i - Lot size of the item i
- M_i - Shortage level of the item i
- V_W - Percentage of utilization of volume of the warehouse in the selling store
- S_{c_k} - Set up cost of the secondary warehouse k
- A_{jk} - Distance matrix entries
- x_{ijk} - Equal to 1 if the quantity of product i is transferred from secondary warehouse k to the selling store j, 0 otherwise
- y_k - Equal to 1 if an order is placed on secondary warehouse k, 0 otherwise

B. Assumptions

(i) Production rate is instantaneous

(ii) Shortage is allowed

(iii) Lead time is zero

(iv) The warehouse space is taken in terms of volume

(v) Lot size is considered as a required material for the selling store from the secondary warehouse

(vi) The unit cost is taken in fuzzy environment

(vii) Demand to be dependent on unit cost and it is related to the unit price as: $D_i = \frac{C_i}{p_i^{e_i}}$

C_i (>0) and $e_i\left(0 < e_i < 1\right)$ are the scaling constants which are used to provide the best-fit estimated price function. While $C_i >0$ is an obvious condition, both D_i and p_i must be non-negative [3], [17].

C. Objective Function of the Mathematical Model

Total Expenditure (Z) = [(Number of selling stores × Total Annual cost of the SSs) +Total Annual cost of the SWs]

where,

Total Annual cost of the SSs= [Production cost +Set up cost of the SS + Holding cost + Shortage cost]

Total Annual cost the SWs = [Transportation cost + Set up cost of the SW + Unit transportation cost from production plant to the SWs]

Therefore,

$$Min\ Z = \left\{ \alpha \sum_{i=1}^{n} \left[C_i p_i^{1-e_i} + \frac{C_i}{p_i^{e_i}} \frac{S_i}{Q_i} + \frac{H_i (Q_i - M_i)^2}{2Q_i} + \frac{\delta_i M_i^2}{2Q_i} \right] + \sum_{i=1}^{n}\sum_{j=1}^{\alpha}\sum_{k=1}^{\beta} (A_{jk} \times Q_i \times x_{ijk} \times t_{ikj}) + \sum_{k=1}^{\beta} (S_{c_k} \times y_k) + \sum_{i=1}^{n}\sum_{\gamma=1}^{m}\sum_{k=1}^{\beta} (y_k \times t_{i\gamma k} \times Q_i) \right\}$$

D. Objective Function under Fuzzy Environment

In the objective, the unit cost is taken in fuzzy environment.

$$Min\ Z = \left\{ \alpha \sum_{i=1}^{n} \left[C_i \tilde{p}_i^{1-e_i} + \frac{C_i}{\tilde{p}_i^{e_i}} \frac{S_i}{Q_i} + \frac{H_i (Q_i - M_i)^2}{2Q_i} + \frac{\delta_i M_i^2}{2Q_i} \right] + \sum_{i=1}^{n}\sum_{j=1}^{\alpha}\sum_{k=1}^{\beta} (A_{jk} \times Q_i \times x_{ijk} \times t_{ikj}) + \sum_{k=1}^{\beta} (S_{c_k} \times y_k) + \sum_{i=1}^{n}\sum_{\gamma=1}^{m}\sum_{k=1}^{\beta} (y_k \times t_{i\gamma k} \times Q_i) \right\}$$

E. Constraints under Fuzzy Environment

i. The limitation on the available warehouse space in the store, $\sum_{i=1}^{n} v_i Q_i \le V$

ii. The upper limit of the total amount investment, $\sum_{i=1}^{n} \tilde{p}_i Q_i \le I$

iii. The upper limit on the number of orders can be made in a time cycle on the system, $\sum_{i=1}^{n} \frac{C_i}{\tilde{p}_i^{e_i} Q_i} \le t$

iv. Percentage of utilization of volume of the warehouse, $\frac{V \times V_W}{\left(\sum_{i=1}^{n} v_i Q_i\right) \times 100} = 1$

v. Restricts the total number of sites that are assigned warehouses to a maximum of ε, $\sum_{k=1}^{\beta} y_k \le \beta$

vi. Restricts that each store is served by only one warehouse, $\sum_{i=1}^{n}\sum_{k=1}^{\beta} x_{ijk} = 1,\ j = 1,2,3,....,\alpha$

vii. Makes sure that each site which is assigned with a warehouse serves at least one store, $\sum_{i=1}^{n}\sum_{j=1}^{\alpha} x_{ijk} - y_k \ge 0,\ k = 1,2,3,....\beta$

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viii. Makes sure that each site which is not assigned with a warehouse does not serve any of the stores, $\sum_{i=1}^{n}\sum_{j=1}^{\alpha} x_{ijk} - \alpha y_k \le 0,\ k = 1,2,3,....\beta$

ix. Restricts the maximum allowed distance between the secondary warehouse and selling store, $\sum_{i=1}^{n}\sum_{j=1}^{\alpha} A_{jk} x_{ijk} \le d,\ k = 1,2,3,....\beta$

Here,

$$x_{ijk} = \begin{cases} 1, & \text{if a SW } k \text{ is assigned for the SS } j \text{ for unit item } i \\ 0, & \text{otherwise} \end{cases}$$

$$y_k = \begin{cases} 1, & \text{if a SW } k \text{ is assigned} \\ 0, & \text{otherwise} \end{cases}$$

and $\tilde{p}_i, Q_i > 0\ (i = 1,2,......, n), 0 \le V_W \le 100$

In the above constraints, constraint (vii) will be inactive when (viii) is active, similarly, constraint (viii) will be inactive when (vii) is active.

F. Membership Function

The membership function [20] for the triangular fuzzy number $\tilde{p}_i = (k_{u_i}, k_{m_i}, k_{o_i})$, $i = 1,2,.... n$ is

$$\mu_{p_i}(x) = \begin{cases} \dfrac{p_i - k_{u_i}}{k_{m_i} - k_{u_i}}, & k_{u_i} \le p_i \le k_{m_i} \\ \dfrac{k_{o_i} - p_i}{k_{o_i} - k_{m_i}}, & k_{m_i} \le p_i \le k_{o_i} \\ 0, & \text{otherwise} \end{cases}$$

where k_{u_i} is the lower limit, k_{m_i} is the middle value and k_{o_i} is the upper limit of the unit cost. The pictorial representation of the proposed configuration of supply network is presented in Fig. 1.

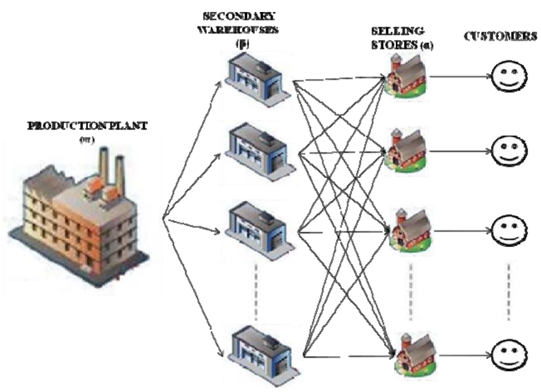

Fig. 1 The proposed configuration of supply network

III. Numerical Example

A manufacturing company has single production plant (m = 1) produces single item (i = 1). The company has five selling stores (α = 5) and five proposed secondary warehouses (β = 5). Company decided to select the network of optimum number of secondary warehouses to satisfy the requirements of the selling stores which are available in various main market places. Volume of the unit item and warehouse space in the selling store is given by $v_1 = 8m^3$ and $V = 3600\ m^3$ respectively. The holding cost, set up cost, and shortage cost of the item are given by \$1, \$100 and \$3 respectively. Total investment cost for production is $I = \$1400$ and number of orders per cycle is 4. The unit cost of the item is $\tilde{p}_1 = \$(10,15,20)$. Maximum allowed distance between the secondary warehouses and selling stores is $d = 12\ kms$. The unit transportation cost of unit item 1 from the production plant γ to the secondary warehouse k is given by $t_{1\gamma k} = \$2$ and the unit transportation cost of unit item 1 from the secondary warehouse k to the selling store j is given by $t_{1kj} = \$2$. The scaling constant value for the unit item 1 is $C_1 = 113$. The set up cost of the secondary warehouses and the distance between the selling stores and the secondary warehouses are given in Table I.

TABLE I
Distance Matrix [A_{JK}] in KMs

		Proposed secondary warehouse (*k*)				
		1	**2**	**3**	**4**	**5**
Store (*j*)	**1**	5	3	7	6	8
	2	3	8	10	9	6
	3	10	9	6	3	9
	4	4	7	3	8	10
	5	9	8	10	9	3
Set up cost (S_{c_k})		100	120	110	140	150

TABLE II
Result of the Example

e_1	p_1	μ_{p_1}	Q_1	V_W	M_1	Z
0.646	12.86	0.572	14.19	3.15	3.55	3164.66
0.650	13.30	0.660	13.97	3.07	3.49	3142.50
0.660	14.54	0.908	13.24	3.05	3.30	3078.53
0.670	15.81	0.838	12.85	2.85	3.21	3032.97
0.675	16.53	0.694	12.57	2.79	3.14	2999.89
0.680	17.30	0.540	12.29	2.73	3.07	2970.54
0.690	18.97	0.206	11.73	2.61	2.93	2911.05
0.693	19.52	0.096	11.57	2.57	2.89	2893.00

The example is solved by using LINGO 8.0 software, and the result of the example is given in Table II. The values of e_1 are lies between 0 and 1. The most suitable values of e_1 for the given example are obtained by trial and error method. The above suitable values of e_1 are said to be selected values of e_1 . Only the selected values of e_1 are substituted in the problem for obtaining the optimum result. Hence, the optimum values for the decision variables and the respective Total Expenditures is presented in Table II.

From Table II, the following result has been observed $x_{111}=1$, $x_{121}=1$, $x_{134}=1$, $x_{141}=1$, $x_{154}=1$, and all remaining $x_{1jk}=0$ and y_1 = 1, y_2 = 0, y_3 = 0, y_4 = 1, y_5 = 0.

From the above result, it is observed that out of 5 secondary warehouses first secondary warehouse and the fourth secondary warehouse serves all the selling stores. That is, for the first selling store the first secondary warehouse is assigned for distributing items, since $x_{111}=1$. For the second selling store the first secondary warehouse is assigned for distributing items, since $x_{121}=1$. For the third selling store the fourth secondary warehouse is assigned for distributing items, since $x_{134}=1$. For the fourth selling store the first secondary warehouse is assigned for distributing items, since $x_{141}=1$. For the fifth selling store the fourth secondary warehouse is assigned for distributing items, since $x_{154}=1$.

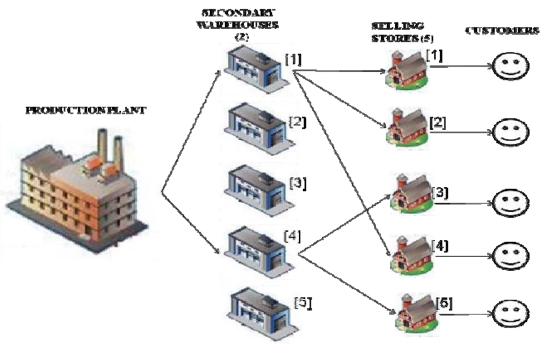

Fig. 2 Network of secondary warehouses

The pictorial representation of the network of secondary warehouses for the optimum result is given in Fig. 2, which shows that two secondary warehouses are assigned to satisfy the requirements of the selling stores which in turn reduces the total expenditure.

IV. Conclusion

A concept of the secondary warehouse selection of the production-distribution inventory model is proposed in this research. Here, a combined mathematical model is developed to optimize the total expenditure of the organization. The constraint goals are restricted up to certain values and the warehouse space is considered in terms of volume. The unit cost of the item is taken under fuzzy environment. The result reveals that network of secondary warehouses can be formed by using first and fourth secondary warehouses to satisfy the requirements of all the five selling stores. In real life inventory control system, the cost parameters such as holding cost, ordering cost, production cost etc., are imprecise in nature. Similarly, in practical situations, resources like warehouse space, number of orders, etc. are also imprecise in nature. Hence, in future, any of the above cost parameters or resources can be taken in fuzzy environment. The proposed model is more general and can be applied to the real inventory problems faced by the practitioners in the industry and other areas.

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