Semi Empirical Equations for Peak Shear Strength of Rectangular Reinforced Concrete Walls

Ali Kezmane, Said Boukais, Mohand Hamizi

***Abstract*—**This paper presents an analytical study on the behavior of reinforced concrete walls with rectangular cross section. Several experiments on such walls have been selected to be studied. Database from various experiments were collected and nominal shear wall strengths have been calculated using formulas, such as those of the ACI (American), NZS (New Zealand), Mexican (NTCC), and Wood and Barda equations. Subsequently, nominal shear wall strengths from the formulas were compared with the ultimate shear wall strengths from the database. These formulas vary substantially in functional form and do not account for all variables that affect the response of walls. There is substantial scatter in the predicted values of ultimate shear strength. Two new semi empirical equations are developed using data from tests of 57 walls for transitions walls and 27 for slender walls with the objective of improving the prediction of peak strength of walls with the most possible accurate.

***Keywords*—**Shear strength, reinforced concrete walls, rectangular walls, shear walls, models.

I. Introduction

THE two main structural design options for resisting high lateral loads from wind and earthquakes in the construction of buildings are reinforced concrete frames and shear walls. Reinforced concrete shear walls are constructed using a combination of steel and concrete and their construction is popular because of the abundance of tensile strength in steel and the cost efficiency of concrete. The low cost and high quality resistance to lateral loads makes shear walls a common design option for structures located in areas of high seismic risk. The transition walls with aspect ratio between one and two are widely used in low rise building, in other hand, the slender walls with an aspect ratio higher than two are typically used in slender construction.

The ultimate shear strength of reinforced concrete shear walls and the design criteria to adequately resist shear has been the focus of many experimental and analytical studies [1]. One popular approach to predicting the ultimate shear strength of reinforced concrete walls used by researches is the derivation of empirical expressions based on test results (for example Barda [2] et al. and Wood [3]). Most of the seismic design provisions found in modern building codes, such as American Code provisions (ACI 318, 2008) [4], Mexican code (NTCC 2004) [5], New Zeeland code (NZS 2006) [6] use empirical or semi-empirical equations to estimate the ultimate shear strength of reinforced concrete walls. These procedures use parameters such as aspect ratio, horizontal and vertical reinforcement ratio, and axial load to estimate the ultimate shear strength. A data base of 57 rectangular transition walls with aspect ratio between one and tow, and 27 slender walls with aspect ratio higher than two are used to evaluate the accuracy of the five cited equations. The experimentally measured ultimate shear strengths of the 57 transition walls and 27 slender walls are compared with shear strengths predicted by five pervious cited equations. This comparison has indicated that the scatter in the shear strength predicted by these equations are substantial, which is problematic because shear strength is the key variable for force-based design and performance assessment. The topic of strength degradation in structural walls is not widely reported in the literature. Hence, an evaluation of the parameters those contribute to the shear restating in the transition walls and slender walls is made using the database, one the evaluation was done a tow new semi-empirical equations were proposed for transition and slender walls with the objective of improving the prediction of ultimate strength of transition and slender walls with the most possible accurate.

Ali. Kezmane is with Civil Engineering Faculty of Riga Technical Univerrsity (Latvia) and Civil Engineering Department of Tizi-Ouzou University (Algeria), Laimdotas ilea 2A Riga(phone: +371 26211488, e-mail: ali.kezmane@hotmail.fr).

Said. Boukais, is with Civil Engineering Department of Tizi-Ouzou Universit (Algeria), Rue de Hasnaoua Université de Tizi-Ouzou 15000, Algeria (e-mail: sbouka58@yahoo.fr).

Mohand. Hamizi is with Civil Engineering Department of Tizi-Ouzou Universit (Algeria), Rue de Hasnaoua Université de Tizi-Ouzou 15000, Algeria (e-mail: chamizi@yahoo.fr).

II. Database

The test specimens in the database are selected using the following criteria: 1) a minimum web thickness of 5 cm; 2) symmetric reinforcement layout; 3) no diagonal reinforcement or additional wall-to-foundation reinforcement to control sliding shear; and 4) cross section is rectangular. Walls that do not comply with these criteria are not included in the -wall database and information on these walls is not presented.

A. Transition-Wall Database

The transition-wall database included experiments of 57 specimens at various scales. The data for the 57 transition wall tests were obtained from Hirosava [7], Maier [8], Lefas [9], Rothe [10], Pilakoutas [11], Salonikios [12], Zhang [13], Kuang [14] Tran [15]. Fig. 3 presents summary information on the 57 transition walls included in this database.

B. Slender-Wall Database

The slender-wall database included experiments of 27 specimens at various scales. The data for the 27 slender wall tests were obtained from Wang [16], Dario [17], Thomsen [18], Jiang [19] Ji [20], Caravajal [21], Zahou [22], Tasnimi [23], Aaleti [24]. Fig. 4 presents summary information on the

27 slender walls included in this database.

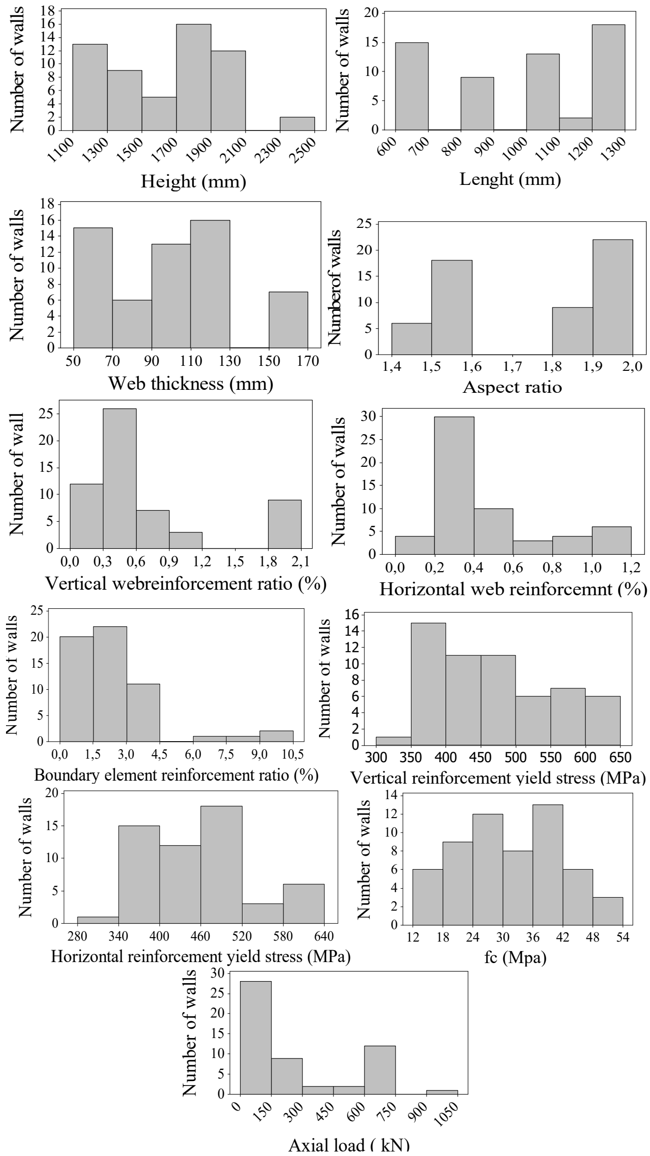
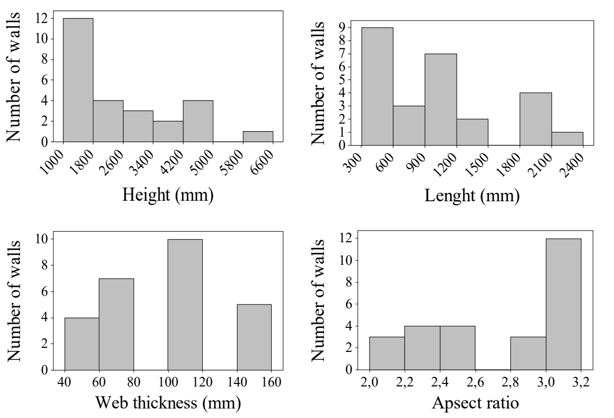


Fig. 1 Histograms of geometric and material properties of the 57 Transition walls



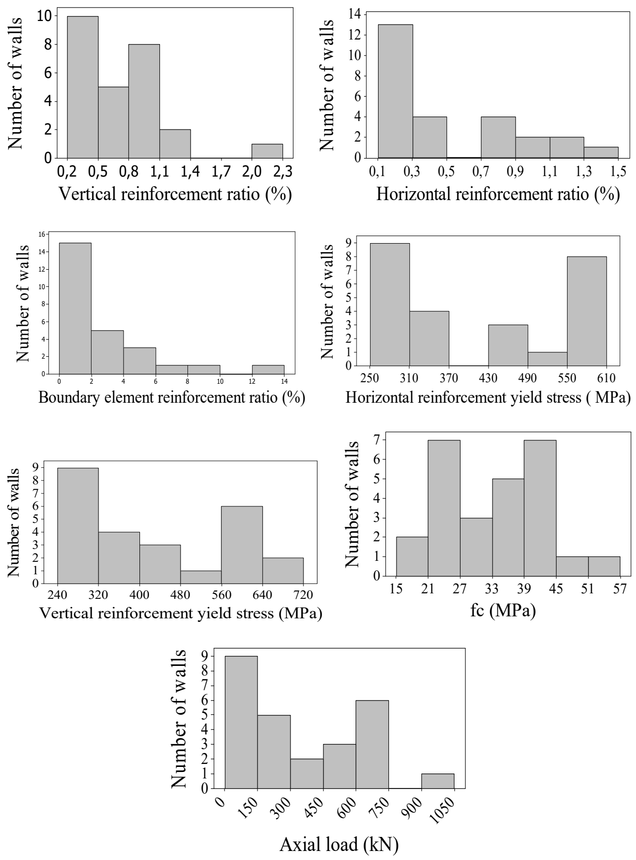


Fig. 2 Histograms of geometric and material properties of the 27 slender walls

III. Models for Calculation the Shear Strength of Walls

Most models available for calculating shear strength of rectangular reinforced concrete walls consider nominal strength to be the addition of the contributions of concrete V_c and steel reinforcement within the web V_s. In general, V_c has been derived by adjusting trends from experimental results. In turn, V_s is based on a truss analogy that assumes both 45-degree cracks and that all horizontal reinforcement crossed by inclined cracking attains yielding at strength. Model equations for calculating shear strength of rectangular reinforced concrete walls studied in this research are presented in Table I. For Code Equations (1)–(3), such as those from the American Concrete Institute ACI 318-08 code, the New Zealand code NZS 2006 and the Mexico Code NTCC2004, nominal shear equations are included in Table I, that's mean, strength reduction factors are not included in Table I.

In ACI 318-08 and NTCC 2004 codes, V_c depends on the aspect ratio; the larger the H/L ratio, the lower the concrete contribution. In both codes, in the equation for calculating the nominal shear strength provided by steel reinforcement, it is assumed that all horizontal reinforcement yields at strength. In the NZS code, V_c depends on the aspect ratio and the axial load and V_s depends on the horizontal reinforcement only as ACI and NTCC codes. Two others models were also assessed in Table I, especially its adequacy to predict shear strength. Wood (1990) developed a lower-bound strength equation for walls reinforced with the minimum steel percentage specified in ACI 318-08 (0.25%). In this model, the steel contribution is not explicitly accounted for. In the model by Barda [2], V_c is a function of aspect ratio (H/L) and axial load. For large H/L ratios, V_c decreases. For small H/L ratios V_c increase. In this model the reinforcement contribution is a function of vertical web reinforcement only.

TABLE I
MODEL EQUATIONS FOR CALCULATING THE SHEAR STRENGTH OF RC WALLS

Model	Concrete contribution (V_c)		Steel contribution (V_s)	Note
ACI 318 [4] 2008	$(\alpha\sqrt{f_c})A$ If $H/L \le 1.5, \alpha = 0.25$ If $H/L \ge 2, \alpha = 0.17$ If $1.5 < H/L < 2$, α : is given by interpolation	(1)	$\rho_h f_{yh} A$	$V_C + V_S \le 0.83\sqrt{f_c}A$
NZS 2006	$Min\begin{cases}(0.27\sqrt{f_c} + \frac{N_u}{4A})t_w d \\ [0.05\sqrt{f_c} + \frac{l(0.1\sqrt{f_c} + \frac{0.2N_u}{A})}{\frac{M_u}{V_u} - \frac{l}{2}}]t_w d\end{cases}$	(2)	$(A_v f_{yh} d/S)A$	$V_C + V_S \le 0.2\sqrt{f_c}t_w d_1$
NTCC [5] 2004	$If\ \frac{H}{L} \le 1.5, V_c = 0.27\sqrt{f_c}Lt_w$ $If\ \frac{H}{L} \ge 2\ \&\ \rho_{vt} < 0.015, V_c = 0.3t_w d(0.2 + 20\rho_v)\sqrt{f_c}$ $If\ \frac{H}{L} \ge 2\ \&\ \rho_{vt} \ge 0.015, V_c = 0.16t_w d\sqrt{f_c}$	(3)	$\rho_h f_{yh} A$	
Wood [3] (1990)	$0.5\sqrt{f_c}A$	(4)	$\rho_v f_{yv} A$	
Barda [2] (1977)	$(8\sqrt{f_c} - 2.5\sqrt{f_c}H/L + N_u/4Lt_w)A$	(5)		

f_c (MPa):concrete compressive strength, A (mm2): area of the wall bounded by web thickness and wall length. t_w (mm), H(mm): wall height: web thickness, L(mm): wall length, $d=0.80L$, M_u (N.m): moment at the section, V_u (N):shear force at the section, N_u(N) : axial load, ρ_h: horizontal web reinforcement ratio; f_{yh} (MPa): yield stress of horizontal web reinforcement, ρ_{vt}: ratio of wall vertical reinforcement in tension, A_v (m^2): area of horizontal reinforcement within a distance S (m), f_{yv} (MPa): yield stress of vertical web reinforcement, ρ_h: vertical web reinforcement ratio

IV. COMPARISON OF PREDICTED AND MEASURED SHEAR STRENGTHS

The experimental shear strength data of the 57 transition and 27 slender rectangular walls documented in section tow are used herein to investigate the accuracy of the calculation procedures presented in Table I to predict the ultimate shear strength of transition and slender rectangular walls. General statistical parameters related to the ratio of the predicted to measured ultimate shear strength of the walls are presented.

A. Transition Walls

A statistical presentation of the ratios of the predicted to measured ultimate shear strength for the 57 transition walls are presented in the rows of Table II for the five equations listed in Table I. Values in columns two (arithmetic mean) or three (median or 50th percentile) in Table II greater than 1.0 indicate that the corresponding strength equation is unconservative in a mean or median sense, respectively, namely, the equation overestimates the measured ultimate shear strength. The standard deviation (column six) and coefficient of variation (COV) (column seven) are also reported to provide supplemental information on the dispersion in the ratios.

The mean and median values of the shear strength ratios presented in Table II for (3), which represent Mexican code (V_{NTC}), indicate that this equation is the most accurate of the five because the mean and median ratio for this equation is 1.20 and 1.14, respectively, and the standard deviation and the coefficient of variation are relatively small compared than others. In other hand, the equation of Wood (V_{WO}) and of the NZS code (V_{NZS}) gives similar results. ACI code presented by (1) (V_{ACI}) give an unsatisfactory results for transition wall, where the mean and the median of the predicted to measured ultimate shear strength is 1.45 and 1.36, respectively, and its utilization must be prudent for transition walls. The Barad (V_{BR}) equation over-predicts strongly the estimation of the shear strength of the 57 transition walls and it cannot be used to predict the shear strength of transition walls.

TABLE II
STATISTICS OF THE RATIO OF ULTIMATE SHEAR STRENGTH PREDICTED USING (1) THROUGH 5 TO MEASURED PEAK SHEAR STRENGTH TRANSITION WALLS

	Mean	Median	Value Max	Value Min	St. Dev	COV
V_{ACI}/V_{EXP}	1,45	1,36	2,98	0.39	0,50	0,34
V_{NZS}/V_{EXP}	1,27	1,20	2,47	0,44	0,44	0,34
V_{NTC}/V_{EXP}	1,20	1,14	2,39	0,33	0,40	0,33
V_{WO}/V_{EXP}	1.31	1.28	2,55	0,27	0,35	0,27
V_{BR}/V_{EXP}	2,01	1,87	3,92	0,86	0,83	0,41

B. Slender Walls

As a transition walls, a statistical presentation of the ratios of the predicted to measured ultimate shear strength for the 27 slender walls are presented in the rows of Table III for the five equations listed in Table I.

The mean and median values of the shear strength ratios presented in Table III indicate that New Zealand code, represents by (2)(V_{NZS}) produce the most accurate predictions because the mean of ratio is 1.70 with the smallest coefficient of variation (0.37). The use of ACI (V_{ACI}) and Wood (V_{WO}) equations produces the lowest estimates of ultimate shear strength under predicting the ultimate shear strength of all the 27 specimens. The Mexican code (4) and Barda (5) give almost a similar results between them.

TABLE III
STATISTICS OF THE RATIO OF ULTIMATE SHEAR STRENGTH PREDICTED USING (1) THROUGH 5 TO MEASURED PEAK SHEAR STRENGTH SLENDER WALLS

	Mean	Median	Value Max	Value Min	St. Dev	COV
V_{ACI}/V_{EXP}	2,53	2,08	4,25	1,21	0,89	0,35
V_{NZS}/V_{EXP}	1,70	1,64	2,92	0,78	0,64	0,37
V_{NTC}/V_{EXP}	2,12	1,97	3,90	1.17	0,81	0,38
V_{WO}/V_{EXP}	2,51	2,51	4,78	0,80	1,17	0,43
V_{BR}/V_{EXP}	2,08	1,94	4,19	0,71	0,88	0,44

In all cases (for transition and slender walls) the coefficients of variation were larger than 20%. Although mean values might be considered as acceptable, especially for Mexican code (V_{NTC}) and New Zealand code (V_{NZS}) made for transition walls, because they approximate to 1.0. Based on the observations made of the results of the prediction of equations (1) through (5) of shear strength of transition and slender walls, it was necessary to develop a models that would better capture the mechanisms involved in rectangular transition and slender reinforced concrete walls resisting shear demands.

V. Effect Of Design Variables On Peak Shear Strength-Evaluation Of Experimental Data

The results presented in Section IV showed that all five equations are inaccurate in the sense that a) the utility of the equations vary significantly with the type of wall (transition and slender), and b) the coefficients of variation associated with the distributions of the ratio of predicted to experimental ultimate shear strength are generally large. An ideal equation would provide a mean ratio of predicted to measured peak shear strengths of 1.0 and a small dispersion as measured by a coefficient of variation. An investigation of the effect of a single design variable on wall behavior is made by performing one-factor at-a-time experiments. The databases of Section II include companion walls for design variables aspect ratio, axial load, horizontal web reinforcement ratio, vertical web reinforcement ratio, and boundary element vertical reinforcement ratio.

A. Aspect Ratio

Table IV presents data on the groups of walls that focused on the effect of aspect ratio on wall behavior. Fig. 1 shows the variation in peak shear strength normalized by wall web area (*A*) with aspect ratio (H/*L)* for the companion walls of Table IV. The data presented in Fig. 1 indicates that the ultimate shear strength of transition walls increase with decreasing aspect ratio. No groups of walls found for slender walls to investigate the aspect ratio effect.

TABLE IV
INFORMATION ON THE TEST PROGRAMS FOCUSED ON THE INFLUENCE OF ASPECT RATIO ON WALL BEHAVIOR

Types of Wall	Group	Authors	ID	(H/L)	f_c (MPa)	V_{MAX} (Kn)
Transition Wall	1	Zhang 2007 [13]	SW2-3	2	37,7	225
		Zhang 2007 [13]	SW2-2	1,5	37,7	275

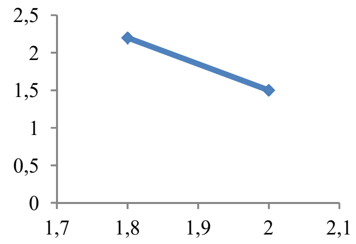


Fig. 1 Variation of peak shear strength with aspect ratio

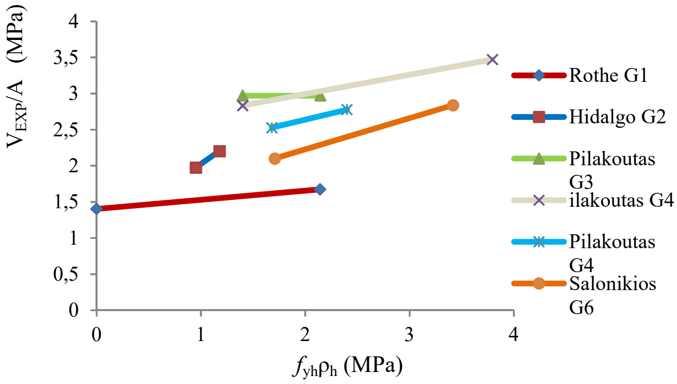


Fig. 2 Variation of ultimate shear strength with the product of horizontal reinforcement ration and horizontal yield stress reinforcement (Rothe [10], Hidalgo [1], Pilakoutas [11], Salonikios [12])

TABLE V
INFORMATION ON THE TEST PROGRAMS FOCUSED ON THE INFLUENCE OF HORIZONTAL REINFORCEMENT RATIO ON WALL BEHAVIOR

Wall types	Group	Authors	ID	ρ_h (%)	f_{yh} (MPa)	f_c (MPa)	V_{MAX} (Kn)
Transition Wall	1	Rothe [10] 1980	T01	0,51	420	24,3	107,2
			T02	0	0	28,8	89,854
	2	Hidalgo [1]	Sp8	0,25	471	15,7	344
			Sp8	0,26	366	17,6	257
	3	Pilakoutas [11] 1995	SW4	0,39	550	36,9	107
			SW6	0,35	400	38,6	107
	4	Pilakoutas [11] 1995	SW5	0,35	400	31,8	102
			SW7	0,69	550	32	125
	5	Pilakoutas [11] 995	SW8	0,42	400	45,8	91
			SW9	0,6	400	38,9	100
	6	Salonikios [12] 1999	MSW5	0,28	610	22	252,21
			MSW6	0,56	610	27,5	340,73

B. Horizontal Web Reinforcement Ratio

Table V presents data on the groups of walls that focused on the effect of horizontal web reinforcement ratio on transition wall behavior. Fig. 2 shows the variation in ultimate shear strength with the horizontal web reinforcement ratio for transition wall. No group is selected for slender wall, because there are no walls with a different horizontal web reinforcement ratio for identical material properties.

Fig. 2 shows that the shear strength of transition walls groups 1, 2, 4,5 and 6 increases with increasing of horizontal web reinforcement, however for group 3 the shear strength remains observed with increasing horizontal web reinforcement ratio. (This steady state of Group 3 may be due to the spacing of steel that is different between the webs of group 3).

C. Vertical Web Reinforcement Ratio

Table VI presents data on the groups of walls that focused on the effect of vertical web reinforcement ratio on transition wall behavior. No group is selected for slender wall.

TABLE VI
INFORMATION ON THE TEST PROGRAMS FOCUSED ON THE INFLUENCE OF VERTICAL WEB REINFORCEMENT RATIO ON WALL BEHAVIOR

Wall types	Group	Authors	ID	ρ_v (%)	f_{yv} (MPa)	f_c (MPa)	V_{MAX} (Kn)
Transition wall	1	Hidalgo 2008	Sp 7	0,13	471	18,1	373
			Sp 8	0,26	471	15,7	344

Fig. 3 shows that the shear strength of the selected group of the transition wall decreases with increasing the vertical steel, but in this case the strength of the concrete is different in the group. So we cannot make any conclusion about the influence of the vertical web reinforcement ratio. More experimental work is needed to identify the effect of vertical web reinforcement ratio transition and slender walls behavior.

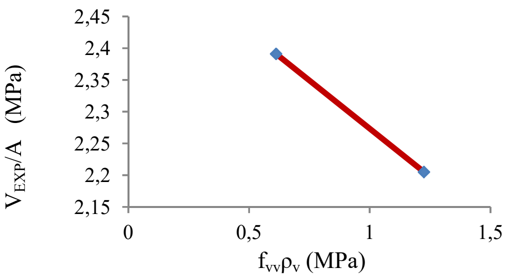


Fig. 3 Variation of ultimate shear strength with the product of horizontal reinforcement ration and horizontal yield stress reinforcement

D. Compressive Axial Load

Table VII presents selected groups of wall for the two types of walls to investigate the influence of the axial load on the behavior of the walls.

Figs. 4 (a) and (b) shows the evolution of shear stress as a function of the axial load for transition and slender walls. This figure shows that the shear strength of the two types of walls increases with increasing axial load. This means, the axial load is parameter influencing the shear strength of rectangular transition and slender reinforced concrete walls.

E. Concrete Compressive Strength

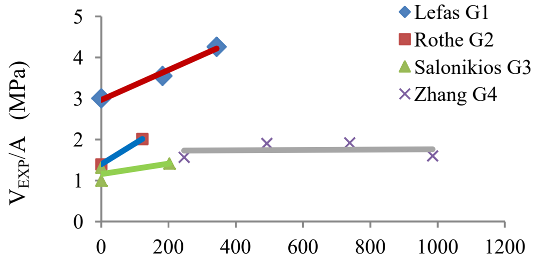
A concrete contribution term, as a function of $\sqrt{f_c}$, is included ACI 318-08, NZS 2006, and NTTC 2004 Wood and Barda ultimate shear strength procedures. Table VIII presents data on the groups of companion walls that focused on the effect of f_c on transition and slender wall behavior. Fig. 5 shows the variation in ultimate shear strength with f_c. As seen in Fig. 5 (a), the effect of f_c on ultimate shear strength varies across the different groups of companion walls. For group 4, ultimate shear strength increases with increasing f_c. For group 3 the effect of f_c on ultimate shear strength is insignificant. For group 1 and 2 ultimate shear strength decreases with increasing f_c. As showed in Fig. 5 (b), ultimate shear strength increases with increasing f_c for slender walls.

TABLE VII
INFORMATION ON THE TEST PROGRAMS FOCUSED ON THE INFLUENCE OF THE AXIAL LOAD ON WALL BEHAVIOR

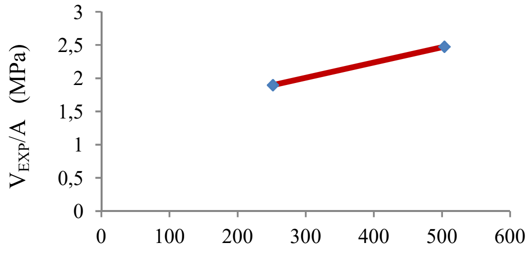
Wall types	Group	Authors	ID	Axial load (Kn)	f_c (MPa)	V_{MAX} (Kn)
Transition Wall	1	Lefas 1990	SW21	0	39,61	127
			SW22	182	47,51	150
			SW23	343	44,67	180
	2	Rothe 1992	T10	0	33,57	89,41
			T11	122,05	26,86	129
	3	Salonikios [12] 1999	MSW2	0	26,2	120
			MSW3	202,44	24,1	170
			MSW4	0	24,6	158
	4	Zhang [13] 2007	SW1-1	246,25	19,7	196
			SW1-2	492,5	19,7	238
			SW1-3	738,5	19,7	240
			SW1-4	985	19,7	200
Slender Wall	1	Zahou 2004	SW-3	251,775	37,3	128
			SW-4	503,55	37,3	167

TABLE VIII
INFORMATION ON THE TEST PROGRAMS FOCUSED ON THE INFLUENCE OF THE COMPRESSIVE CONCRETE STRENGTH ON WALL BEHAVIOR

Wall types	Group	Authors	ID	fc (MPa)	VMAX (Kn)
Transition Wall	1	Lefas [9] 1990	SW21	39,61	127
			SW24	45,18	120
	2	Lefas [9] 1990	SW31	31,92	117,877
			SW32	50,55	115,653
			SW33	46,09	111,65
	3	Salonikios [12] 1999	MSW2	26,2	120
			MSW4	24,6	158
			MSW5	22	187
			MSW6	27,5	200
	4	Zhang [16] 2007	SW1-3	19,7	240
			SW2-2	37,7	275
Slender wall	1	Tasnimi [23] 2000	SHW1	26,2	15,43
			SHW2	24,6	19,57
			SHW3	22	17,52
			SHW4	27,5	19,77

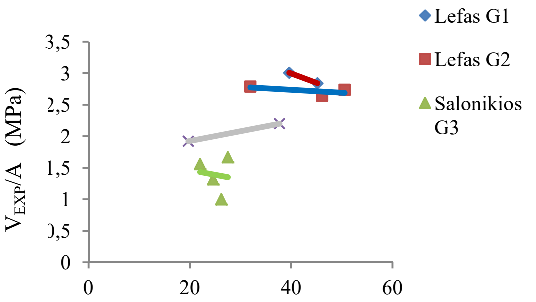


(a) Transition wall

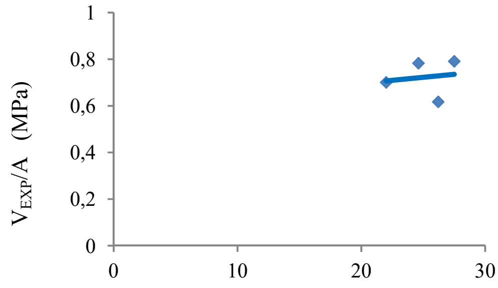


(b) Slender wall

Fig. 4 Variation of ultimate shear strength with axial load (Kn)



(a) Transition Wall



(b)Slender Wall

Fig. 5 Variation of ultimate shear strength with the compressive concrete strength (MPa)

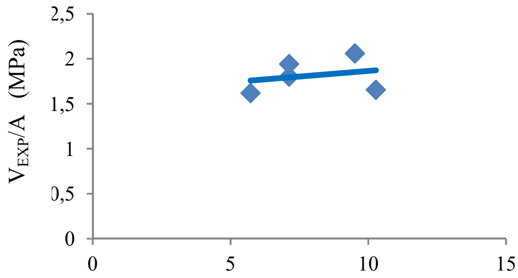
F. Boundary Element Reinforcement

Vertical boundary element reinforcement ratio is not considered in the ACI 318-08, NZS 2006, and NTCC 2004 codes, and in Barda ultimate shear strength calculations. The Wood (1990) equation, which is based on shear-friction, considers indirectly all vertical reinforcement in the horizontal cross-section of a wall to calculate its ultimate shear strength. Table IX presents data on the groups of companion walls that focused on the effect of vertical boundary-element reinforcement ratio on transition and slender wall behavior.

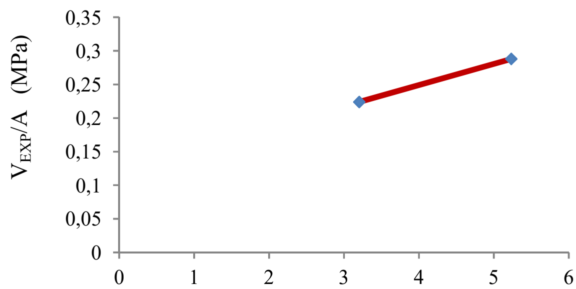
Fig. 6 shows the variation in ultimate shear strength with the vertical boundary-element reinforcement ratio (ρ_{be}). As seen in the figure, the ultimate shear strength of transition and slender walls increased consistently with increasing boundary element reinforcement ratio for all companion wall groups considered.

TABLE IX
INFORMATION ON THE TEST PROGRAMS FOCUSED ON THE INFLUENCE OF BOUNDARY ELEMENT REINFORCEMENT RATIO ON WALL BEHAVIOR

Walls types	Group	Auteur	ID	ρ_{be} (%)	f_{ybe} (MPa)	f_c (MPa)	V_{MAX} (Kn)
Transition Wall	1	Zhang 2007	SW4-4	2,71	379	37,7	225
			SW5-1	1,51	379	37,7	220
			SW5-3	2,51	379	37,7	280
			SW6-1	1,88	379	37,7	245
			SW6-3	1,88	379	37,7	264
Slender Wall	1	caravajal 1983	M-3	1,27	412	28,2	36
			M-5	0,71	451	28,7	28



(a) Transition wall



(b) Slender wall

Fig. 6 Variation of ultimate shear strength with the product of boundary element reinforcement ration and boundary element yield stress reinforcement

VI. PROPOSED MODELS FOR ULTIMATE SHEAR STRENGTH

The important task in creating a new model for the ultimate shear strength is to determine the model parameters and their functional relationship. As seen in Section IV, several models with substantially different functional forms are used for the ultimate shear calculations and all of the models investigated to date provide less-than-satisfactory estimates of the ultimate shear demands. Experimental data analysis results presented in

Section V show that a robust model to predict the ultimate shear strength of rectangular transition and slender reinforced concrete walls should consider the following design variables: 1) aspect ratio, 2) vertical web reinforcement ratio, 3) axial force, 4) boundary element reinforcement ratio, and 5) concrete compressive strength. The data presented in Section V shows that the effect of aspect ratio, horizontal and vertical web reinforcement ratio on ultimate shear strength is not reported for slender walls for missing tests. However, aspect ratio, horizontal and vertical web reinforcement ratio will also be included in the model for slender wall for completeness.

To determine the general form of the regression model, a simple free body diagram that is based on the occurrence of inclined (shear) cracks in a reinforced concrete wall is presented in Fig. 7. The forces along a crack that crosses through the upper corner of the wall web are used to form the free body diagram.

In the Fig. 7, F_H and F_V is lateral and vertical load respectively, and represent external forces. F_{ae}, F_{ah} and F_{av} represent total force carried by the boundary element reinforcement, horizontal and vertical web reinforcement, respectively, F_{bv} and F_{bh} are the vertical and the horizontal components of the compression strut force. F_{ae}, F_{ah}, F_{av}, F_{bv} and F_{bh} represent the internal forces. h is the wall height, *l* is the wall length, and α is the angle of inclination for the crack. x_1 to x_3, y_1 and y_2 are the horizontal and vertical distances used to identify the moment.

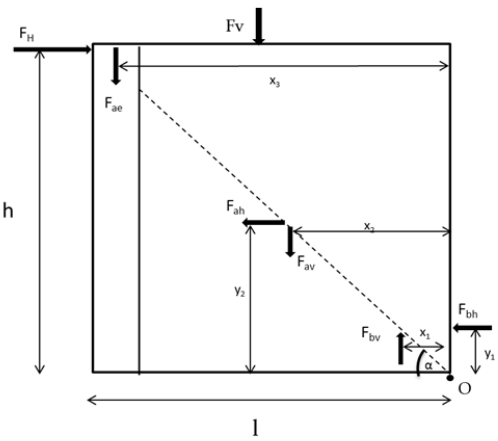


Fig. 7 Free body diagram

Based on the elementary calculation of strength of materials, a relationship is established between the external and internal forces, as shown in (6) and (7).

$$\sum M_O = F_H h - F_{AH} y_2 - F_{AV} x_2 - F_V L/2 - F_{BH} y_1 + F_{BV} x_1 - F_{AE} \times x_3 = 0 \quad (6)$$

$$F_H = (F_{AH} y_2 + F_{AV} x_2 + F_V L/2 + F_{BH} y_1 - F_{BV} x_1 + F_{AE} x_3) h^{-1} \quad (7)$$

Equation (7) gives the ultimate shear of the wall (free body diagram) as a function of all forces contributes to the shear strength except the aspect ratio. To introduce the parameter, a simplified form of (7) is given as follows:

$$F_H = \left[((\alpha_1 \rho_{AH} \sigma_{AH} + \alpha_2 \rho_{AV} \sigma_{AV} + \alpha_3 \rho_{AE} \sigma_{AE} + \alpha_4 f_c^{\alpha 5}) \times (A)) + (\alpha_6 F_V)\right](h/l)^{\alpha 7} \quad (8)$$

If we divide (8) by the cross section (A) of the wall, we get the ultimate shear stress represent by the (9):

$$\sigma_H = \left[((\alpha_1 \rho_{AH} \sigma_{AH} + \alpha_2 \rho_{AV} \sigma_{AV} + \alpha_3 \rho_{AE} \sigma_{AE} + \alpha_4 f_c^{\alpha 5})) + (\alpha_6 F_V / A)\right](h/l)^{\alpha 7} \quad (9)$$

In (9), ρ_{AH}, ρ_{AV} and ρ_{AE}, represent the horizontal, vertical and boundary element reinforcement ratio, respectively, and σ_{AH}, σ_{AV}, σ_{AE} represent its reinforcement yield stress, respectively. f_c represents the compressive concrete strength, F_v represents the axial load, *h* and *l* is the height and the length of the wall, respectively.

The coefficients (α_1 to α_3, and α_6) associated with reinforcement and axial load are defined using linear functions and the coefficients addressing the concrete contribution and wall aspect ratio (α_4, α_5, and α_7) are defined using more general functional forms to improve the model accuracy.

The values for the unknown coefficients (α_1 to α_7) of the models are calculated using the nonlinear regression based on the nonlinear least square method.

The method of least squares assumes that the best-fit curve of a given type is the curve that has the minimal sum of the deviations squared (*least square error*) from a given set of data.

The calibration of (9) is to find the optimal values of unknowns coefficients (α1 to α7), which is characterized by the set of variables (ρ_{AH} σ_{AH}, ρ_{AV} σ_{AV}, ρ_{AE} σ_{AE}, f_c, F_v and h/l) from the set of experimental measurements (σ_{Hexp}) reported in the database of Section II by minimizing the mean squared error.

$$\varepsilon = \sum_{i=1}^{N} \left[(\sigma_{H\exp})_i - (\sigma_H)_i\right]^2 \quad (10)$$

σ_{Hexp} represents the experimental ultimate shear stress reported in the database.

Then the nonlinear least square problem can be defined as the following minimization problem:

$$\alpha_{opt} = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, \alpha_7) = \underset{\alpha_1,\alpha_1,\alpha_3,\alpha_4,\alpha_5,\alpha_6,\alpha_7}{\arg\min} \varepsilon^T \varepsilon = \sum_{i=1}^{N} \left[(\sigma_{H\exp})_i - (\sigma_H)_i\right]^2 \quad (11)$$

To solve (11) a Gauss–Newton method or Levenberg–Marquardt method can be used. In this study, the STATISTICA software is used for data fitting.

A. Model for transition wall

Based on (9) tow model are developed using the transition wall database. In the first model α_5 is taken equal to 0.5. The concrete contribution term in widely used ultimate shear strength equations is generally a function of $\sqrt{f_c}$, which is related to the tensile strength of concrete. In the second model α_5 is estimated by the nonlinear regression.

$$V_{TM} = \sigma_H A = \left[((\alpha_1 \rho_{AH} \sigma_{AH} + \alpha_2 \rho_{AV} \sigma_{AV} + \alpha_3 \rho_{AE} \sigma_{AE} + \alpha_4 f_c^{0.5})) + (\alpha_6 F_V / A)\right](h/l)^{\alpha 7} A \quad (12)$$

$$V_{TM2} = \sigma_H A = \left[\left(\left(\alpha_1 \rho_{AH} \sigma_{AH} + \alpha_2 \rho_{AV} \sigma_{AV} + \alpha_3 \rho_{AE} \sigma_{AE} + \alpha_4 f_c^{\alpha_5}\right)\right) + \left(\alpha_6 F_V / A\right)\right] (h/l)^{\alpha_7} A \quad (13)$$

The calculated coefficients for the two models are presented in Table X. Table XI presents the statistics for the ratio of the predicted to experimental ultimate shear strength using the five procedures investigated in Section III and two models developed in this study (12) and (13).

TABLE X
COEFFICIENTS CALCULATED FOR THE TWO MODELS DEVELOPED USING (12) AND (13)

Model	α_1	α_2	α_3	α_4	α_5	α_6	α_7
V_{TM1}	0,0315	0,0053	0,0186	0,40203	0,5	-0,0862	-0,0142
V_{TM2}	0,0143	0,0109	0,0152	0,60713	0,356	-0,08	-0,1627

TABLE XI
STATISTICS OF THE RATIO OF ULTIMATE SHEAR STRENGTH PREDICTED USING EQUATIONS (1) THROUGH (5), (12) AND (13) TO MEASURED PEAK SHEAR STRENGTH TRANSITION WALLS

	Mean	Median	Value Max	Value Min	St. Dev	COV	SSE
V_{ACI}/V_{EXP}	1,45	1,36	2,98	0.39	0,5	0,34	7,90E+11
V_{NZS}/V_{EXP}	1,27	1,2	2,47	0,44	0,44	0,34	5,15E+11
V_{NTC}/V_{EXP}	1,20	1,14	2,39	0,33	0,4	0,33	4,41E+11
V_{WO}/V_{EXP}	1.31	1.28	2,55	0,27	0,35	0,27	1,02E+12
V_{BR}/V_{EXP}	2,01	1,87	3,92	0,86	0,83	0,41	2,25E+12
V_{TM2}/V_{EXP}	1,00	0,99	1,39	0,76	0,164	0,163	9.60E+09
V_{TM1}/V_{EXP}	0,99	0,97	1,38	0,73	0,171	0,171	1.38E+10

Table XI shows that the models V_{TM1} and V_{TM2} provide the best estimates of the ultimate shear strength with a median ratio of predicted to experimentally measured shear strength of 1.00 and 0.99, respectively, and a coefficient of variation of 0.163and 0.171, respectively, which is the smallest among the procedures investigated. The error sum of squares statistics associated with each model presented in the last column of Table XI also reveal that models V_{TM1} and V_{TM2} produces the smallest error in calculating the ultimate shear strength. Based on this remarks, it's allow to say that the forms of the five equations presented in Section II do not take into account all factors that affect the shear strength of transition walls. As seen in Table XI, model V_{TM2} gives better estimation than V_{TM1}, that's mean, the function of compressive concrete strength performed better results when it is taken as a function of exponential then square function.

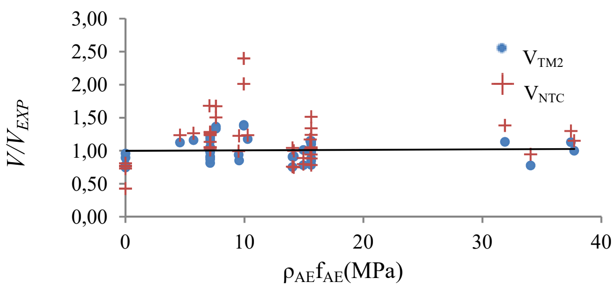


Fig. 8 Variation of the ratio V_{TM2}/V_{EXP} and V_{NTC}/V_{EXP} with the product of boundary element reinforcement ratio and the boundary element reinforcement yield stress

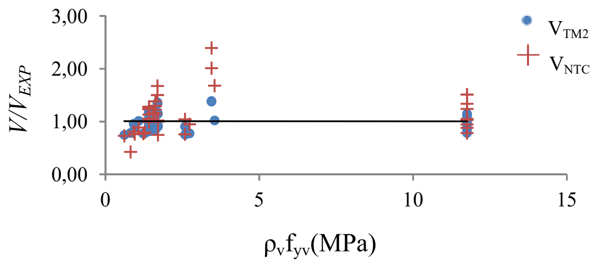


Fig. 9 Variation of the ratio V_{TM2}/V_{EXP} and V_{NTC}/V_{EXP} with the product of vertical reinforcement ratio and the vertical reinforcement yield stress $\rho_v f_{yv}$(MPa)

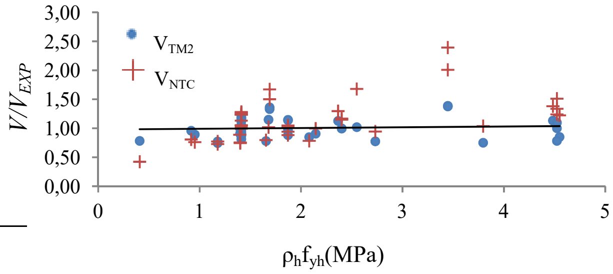


Fig. 10 Variation of the ratio V_{TM2}/V_{EXP} and V_{NTC}/V_{EXP} with the product of vertical reinforcement ratio and the vertical reinforcement yield stress $\rho_h f_{yh}$(MPa)

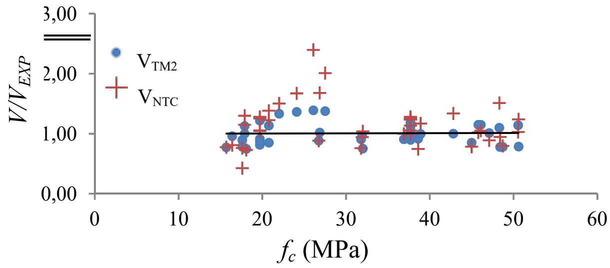


Fig. 11 Variation of the ratio V_{TM2}/V_{EXP} and V_{NTC}/V_{EXP} with the concrete compressive strength (MPa)

Fig. 8 through Fig. 13 present the variation of the ratio of the calculated to experimental shear strengths for models V_{TM2} and V_{NTC} with the design parameters, namely, boundary element reinforcement ratio, vertical web reinforcement ratio, horizontal web reinforcement ratio, vertical web reinforcement ratio, concrete compressive strength, aspect ratio, and axial load. In a well-specified model, the data points in Fig. 8 through Fig. 13 should be scattered without a trend in a shallow band around the value of 1.0 for the ratio of calculated to experimental ultimate shear strength. The figures indicate that model V_{TM2} captures the ultimate shear strength accurately for all design variables over their corresponding ranges. The majority of the ratios associated with model V_{TM2} are between 0.74 and 1.38 whereas the ratios for model V_{NTC} are widely scattered and range between 0.33 and 2.39.

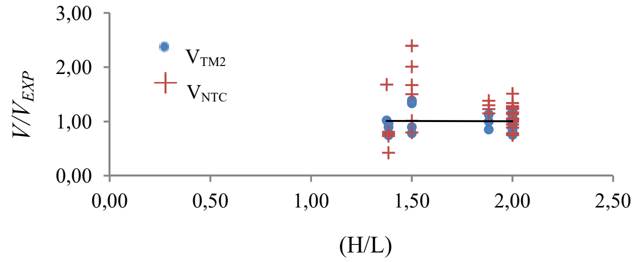


Fig. 12 Variation of the ratio V_{TM2}/V_{EXP} and V_{NTC}/V_{EXP} with the aspect ratio (H/L)

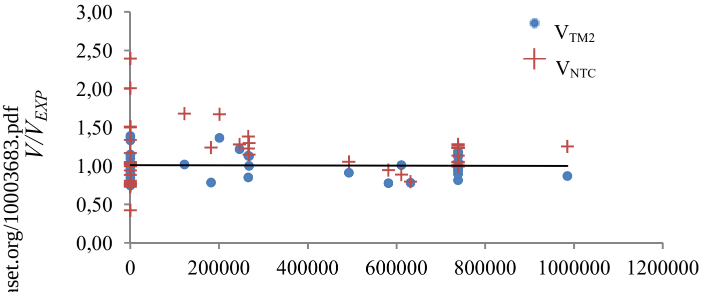


Fig. 13 Variation of the ratio V_{TM2}/V_{EXP} and V_{NTC}/V_{EXP} with the axial load (Kn)

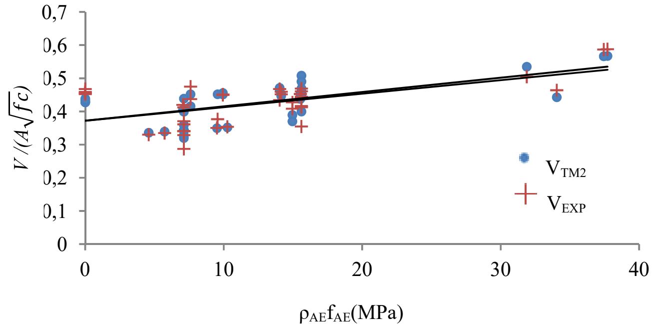


Fig. 14 Variation of the ratio of predicted (V_{MT2}) and experimental shear strengths (normalized using total wall area and $\sqrt{fc}$) with boundary element reinforcement ration

Fig. 14 presents the variation of experimental ultimate shear and ultimate shear calculated using model V_{TM2} normalized with total wall area and $\sqrt{fc}$ with boundary element reinforcement ratio. Fig. 14 shows that normalized experimental ultimate shear strengths for the transition walls in the database are below the shear stress of $0.6\sqrt{fc}$. As seen in Fig. 14, the normalized peak shear strengths calculated using model V_{TM2} do not exceed the shear stress limit of $0.6\sqrt{fc}$ (MPa). The need for an upper shear stress limit is therefore inconclusive at this time since the procedure does not yield unconservative estimations of the peak shear strength for walls that developed relatively high shear stresses.

The final model for the ultimate peak shear strength is given by (14):

The equation *VTM1* is valid for the range of data it was created from, namely, between 1 and 2 for aspect ratio; 0 and 4.5 MPa for $\rho_h f_{yh}$; 0 and 12.5 MPa for $\rho_v f_{yv}$; 0 and 40 MPa for $\rho_h f_{yh}$; 0 and 1000 KN for axial load; and 15 and 54 MPa for f_c.

$$V_{TW} = \left[\left[(0.014\rho_H\sigma_{AH} + 0.011\rho_V\sigma_{AV} + 0.015\rho_{BE}\sigma_{BE} + 0.607 f_c^{0.356} + (-0.078\frac{F_V}{A})\right](h/l)^{-0.163}\right] \times A \leq 0.61\sqrt{f_c}A \qquad (14)$$

B. Model for Slender Wall

As transition walls, tow model are developed for slender walls using the slender wall database based on (9). However, in this case after started the nonlinear regressions, it found that the parameter α7 associated with the aspect variable made a convergence problem of the algorithm solution and gave results without sense, for this, we removed the parameter, and took the aspect ratio variable as logarithm function. The two equations are:

$$V_{SM1} = \sigma_H A = \left[((\alpha_1\rho_{AH}\sigma_{AH} + \alpha_2\rho_{AV}\sigma_{AV} + \alpha_3\rho_{AE}\sigma_{AE} + \alpha_4 f_c^{0.5})) + (\alpha_6 F_V / A)\right]\ln(h/l)A \qquad (15)$$

$$V_{SM2} = \sigma_H A = \left[((\alpha_1\rho_{AH}\sigma_{AH} + \alpha_2\rho_{AV}\sigma_{AV} + \alpha_3\rho_{AE}\sigma_{AE} + \alpha_4 f_c^{\alpha_5})) + (\alpha_6 F_V / A)\right]\ln(h/l)A \qquad (16)$$

Table XII presents calculated coefficients for (15) and (16) using nonlinear regression.

TABLE XII
COEFFICIENTS CALCULATED FOR THE TWO MODELS DEVELOPED USING (15) AND (16)

Model	α_1	α_2	α_3	α_4	α_5	α_6
V_{SM1}	0,25686	0,21699	0,01497	-0.0418	0,5	0,08356
V_{SM2}	0,2615	0,21964	0,01458	-0,0953	0,273	0,0827

Figs. 15 through Fig. 20 present the variation of the ratio of the calculated to experimental shear strengths for models V_{SM2} and V_{NZS} with the design parameters; boundary element reinforcement ratio, vertical web reinforcement ratio, horizontal web reinforcement ratio, vertical web reinforcement ratio, concrete compressive strength, axial load, and aspect ratio. In a well-specified model, the data points in Figs. 15 through Fig. 20 should be scattered without a trend in a shallow band around the value of 1.0 for the ratio of calculated to experimental peak shear strength. The figures indicate that model V_{SM2} captures the ultimate shear strength accurately for all design variables over their corresponding ranges. The majority of the ratios associated with model V_{SM2} are between 0.73 and 1.33 whereas the ratios for model V_{NZS} are widely scattered and range between 0.78 and 2.92.

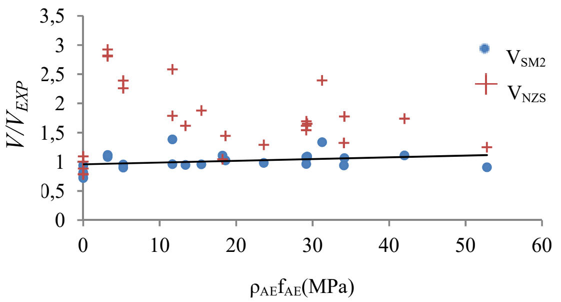


Fig. 15 Variation of the ratio V_{SM2}/V_{EXP} and V_{NZS}/V_{EXP} with the product of boundary element reinforcement ratio and the boundary element reinforcement yield stress

TABLE XIII
STATISTICS OF THE RATIO OF ULTIMATE SHEAR STRENGTH PREDICTED USING EQUATIONS (1) THROUGH (5), (14) AND (15) TO MEASURED PEAK SHEAR STRENGTH SLENDER WALLS

	Mean	Median	Value Max	Value Min	St. Dev	COV	SSE
V_{ACI}/V_{EXP}	2,53	2,08	4,25	1,21	0,89	0,35	8,1E+11
V_{NZS}/V_{EXP}	1,70	1,64	2,92	0,78	0,64	0,37	2,2E+11
V_{NTC}/V_{EXP}	2,12	1,97	3,9	1.17	0,81	0,38	5,2E+11
V_{WO}/V_{EXP}	2,51	2,51	4,78	0,8	1,17	0,43	1,6E+12
V_{BR}/V_{EXP}	2,08	1,94	4,19	0,71	0,88	0,44	1,6E+12
V_{SM1}/V_{EXP}	1,01	0,97	1,38	0,72	0,149	0,148	7,68E+09
V_{SM2}/V_{EXP}	1,00	0,98	1,38	0,73	0,148	0,148	7,59E+09

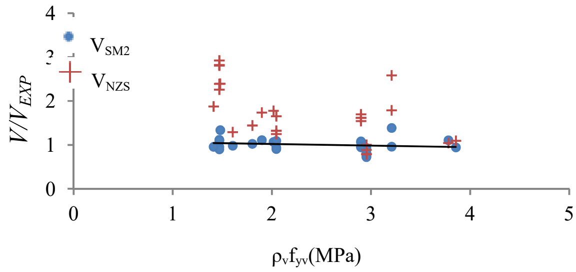


Fig. 16 Variation of the ratio V_{SM2}/V_{EXP} and V_{NZS}/V_{EXP} with the product of vertical reinforcement ratio and the vertical reinforcement yield stress $\rho_v f_{yv}$(MPa)

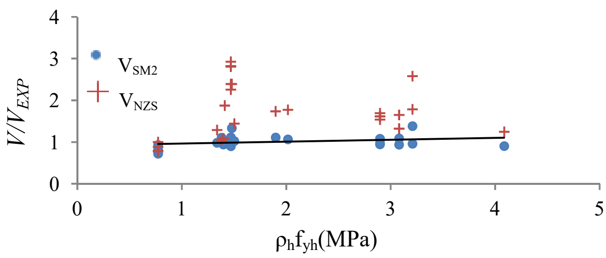


Fig. 17 Variation of the ratio V_{SM2}/V_{EXP} and V_{NZS}/V_{EXP} with the product of vertical reinforcement ratio and the vertical reinforcement yield stress $\rho_h f_{yh}$(MPa)

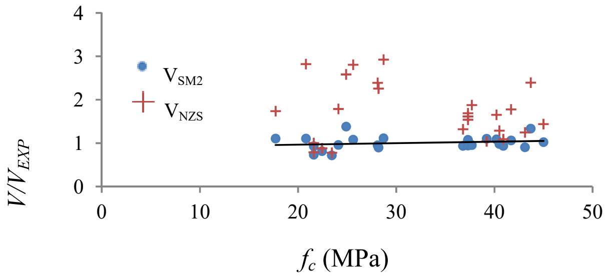


Fig. 18 Variation of the ratio V_{SM2}/V_{EXP} and V_{NZS}/V_{EXP} with the concrete compressive strength (MPa)

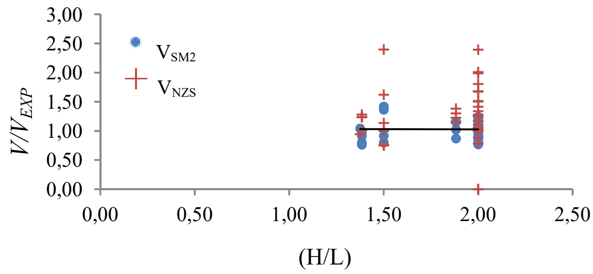


Fig. 19 Variation of the ratio V_{SM2}/V_{EXP} and V_{NZS}/V_{EXP} with the aspect ratio (H/L)

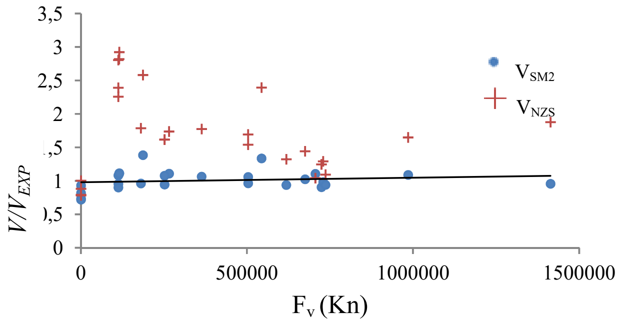


Fig. 20 Variation of the ratio V_{SM2}/V_{EXP} and V_{NZS}/V_{EXP} with the axial load (Kn)

Fig. 21 presents the variation of experimental peak shear and peak shear calculated using model V_{SM2} (normalized with total wall area and $\sqrt{fc}$) with aspect ratio. Fig. 21 shows that normalized experimental ultimate shear strengths for the slender walls in the database are below the shear stress of 0.39 $\sqrt{fc}$ except three values, and these three values we can consider it's as outlines values.

As seen in Fig. 21, the normalized peak shear strengths calculated using model V_{SM2} exceed the shear stress limit of $0.39\sqrt{fc}$ (MPa) . The need for an upper shear stress limit is therefore conclusive at this time since the procedure yields unconservative estimations of the peak shear strength for walls.

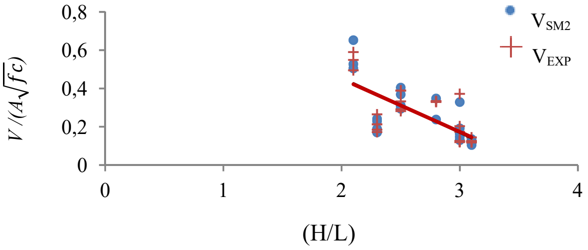


Fig. 21 Variation of the ratio of predicted (V_{SM2}) and experimental shear strengths (normalized using total wall area and $\sqrt{fc}$) with aspect ratio

The final equation for slender rectangular reinforced concrete wall is given in (17).

The equation V_{SW} is valid for the range of data it was created from, namely, between 2.1 and 3.1 for aspect ratio; 0.75 and 4.2 Mpa for $\rho_h f_{yh}$; 1.5 and 4 MPa for $\rho_v f_{yv}$; 0 and 55 MPa for $\rho_b f_{yb}$; 0 and 1400 KN for axial load, and 17 and 46 MPa for f_c.

$$V_{SW} = \left[\left[(0.257\rho_{AH}\sigma_{AH} + 0.217\rho_{AV}\sigma_{AV} + 0.015\rho_{AE}\sigma_{AE} - 0.042 f_c^{0.5} + (0.084\frac{F_V}{A})\right] Ln(h/l)\right] \times A \le 0.38\sqrt{fcA} \quad (17)$$

VII. Optimization of Mexican New Zealand Equation According to the Data of Our Database

As already found in the Section III, the Mexican model NTCC2004 provides the best estimation for transition walls and the code of New Zealand NZS2006 offers the best estimation for the slender walls. So to conclude the performance and the validity of the proposed models (equations), an optimization of the two models discussed in Section III is made by the same method used for the proposed regression models. This optimization will allow us to compare the two proposed models with the Mexican and new Zealand equations optimized using the same adjustment data.

A. Transition Wall

(18) and (17) represent the optimized equations of the Mexican code according transition walls data, in these equations we removed the coefficient of 0.5 and 1.0 of the original (3), which affects the contribution of concrete and steel, respectively, and replaced by β_1 and β_2 coefficients. β_1 and β_2 were determined using the least square method and the transition walls data (as the proposed model)

$$V_{NTCopt} = V_{Copt} + V_{Sopt} \quad (18)$$

$$V_{Copt} = \beta_1 t_w d_2 \sqrt{f_c} \quad (19)$$

$$V_{Sopt} = \beta_2 \rho_h f_{yh} A \quad (20)$$

Table XIV presents the optimized coefficients for Mexican (18).

Table XV shows that the Mexican optimized equation provides a better estimation of the ultimate shear strength of transition wall compared to the original equation. This new equation gives a mean and median of 1.05 and 1.20 versus to 1.03 and 1.14 for the original equation, respectively. Also, it is noted that the coefficient of variation and the square sum errors are smaller in the optimized, but are poorer than those of proposed models (V_{TM1} and V_{TM2}). This observation suggests that the functional forms of Mexican code (NTCC2004) is not successful in accounting for the factors that affect the peak shear strength of rectangular transition reinforced concrete walls.

TABLE XIV
Coefficients Calculated for the Mexican Equation (V_{NTC}) (19) and (20)

Model	β_1	β_2
$V_{NTC\ (original)}$	0.50	1,00
V_{NTCopt}	1,43	0,145

TABLE XV
Statistics of the Ratio of Ultimate Shear Strength Predicted Using Equations (3), (17) and (14) to Measured Peak Shear Strength Transition Walls

	Mean	Median	Value Max	Value Min	St. Dev	COV	SSE
V_{NTC}/V_{EXP}	1,20	1,14	2,39	0,33	0,4	0,33	4,41E+11
V_{NOP}/V_{EXP}	1,05	1,03	2,08	0,61	0,29	0,28	3,61E+12
V_{TM2}/V_{EXP}	1,00	0,99	1,39	0,76	0,164	0,163	9.60E+09

B. Slender Wall

As transition walls, ultimate shear model based on the procedures of New Zealand (6), (7) is optimized similarly and the performance of this models is compared with the currently used procedure and proposed model for slender wall.

$$V_{NZSopt} = V_{Copt} + V_{Sopt} \quad (21)$$

$$V_{Copt} = \left(\lambda_1 \sqrt{f_c'} + \lambda_2 \frac{N_u}{A_w}\right) t_w d_1 \quad (22)$$

$$V_{Sopt} = \lambda_3 \frac{A_v f_{yh} d_1}{S} \quad (23)$$

Table XVI represents the optimized coefficients of (22) and (23) of the New Zealand code according slender walls data.

TABLE XVI
Coefficients Calculated for the Mexican Equations (V_{NTC}) (19) and (20)

Model	λ_1	λ_2	Λ_3
V_{NZS} (original)	0,27	0,25	1
V_{NZSopt}	0,103	110,33	0,522

TABLE XVII

STATISTICS OF THE RATIO OF ULTIMATE SHEAR STRENGTH PREDICTED USING EQUATIONS (2), (21) AND (17) TO MEASURED PEAK SHEAR STRENGTH TRANSITION WALLS

	Mean	Medain	Value Max	Value Min	St, D	COV	SSE
V_{NZS}/V_{EXP}	1.70	1.64	2.92	0,78	0,64	0,37	2,2E+11
V_{NZSopt}/V_{EXP}	1,26	1,1	2,24	0,73	0,44	0,34	8,4E+10
V_{SW}/V_{EXP}	1,00	0,98	1,38	0,72	0,148	0,148	7,6E+09

Table XVII show that, the optimized New Zealand equation (V_{NZSopt}) performs better than the codified version (V_{NZS}) but is poorer than the proposed model for slender wall. This observation suggests that the functional forms New Zealand equation (V_{NZS}) is not successful in accounting for the factors that affect the ultimate shear strength of slender rectangular reinforced concrete walls.

VIII. CONCLUSION

Based on the study reported herein on the ultimate shear strength for rectangular transition and slender reinforced concrete walls, the following key conclusions can be obtained:

1. Tow databases were assembled; the first database contained all information of 57 rectangular transition walls and the second contained 27 rectangular slender walls.
2. The scatter in the values of ultimate shear strength predicted by the five equations evaluated in this study is substantial. The utility of these equations is affected significantly by the type of wall (transition or slender). For transition walls, average ratios of predicted to measured ultimate shear strength is between 1.20 and 1.45 for ACI, NZS, NTCC and wood equation, however, for slender walls the average of predicted to measured rations is between 1.71 and 2.53. The Barad equation gives a similar results for slender and transition walls, this equation overestimate strongly the prediction of the both transition and slender walls.
3. The best predictions of ultimate strength (mean, median ratio of predicted to measured ultimate shear strength close to 1.0 and a small coefficient of variation) are obtained using the Mexican equations (NTCC2004) for transition walls, and New Zealand equation (NZS2006) for slender wall.
4. The evaluation of experiments data included in the tow databases show that the ultimate shear walls affected by all design variables, namely, boundary element reinforcement ratio, vertical and horizontal reinforcement ratio, boundary element yield reinforcement stress, vertical and horizontal reinforcement yield stress, compressive concrete strength, axial load and aspect ratio.
5. The contribution of compressive concrete strength is more efficient when is taken as an exponential function.
6. Two new improving predictive equations for ultimate shear strength of transition and slender walls are proposed taken in account all design variables and using nonlinear regression optimization.
7. From the comparison of measured strengths in tests and calculated strengths using the tow proposed models, it is clear that the model is reliable. Average measured-to-calculated strengths ratio was 1.00 and a coefficient of variation of 0.163. For slender walls the proposed equation provides average ratios of predicted to measured peak shear strength of 1.0 with a coefficient of variation of 0.148.
8. The optimized equation for Mexican (V_{NTC}) and New Zealand equation codes (V_{NZS}) show that the functional forms the both codes are not successful in accounting for the factors that affect the ultimate shear strength of transition and slender rectangular reinforced concrete walls.
9. The proposed model (V_{TW} and V_{SW}) perform significantly better than the equations currently used for predicting the ultimate shear strength of transition and slender walls and tale in account all design variable those affect the ultimate shear strength.

REFERENCES

[1] Hidalgo P A. Ledezma C A. and Jordan R. M. Seismic behavior of squat reinforced concrete shear walls. Earthquake Spectra, EERI, Vol. 18, No. 2, pp 287-308.
[2] Barda F. Hanson J M. and Corley W G. Shear Strength of Low-Rise Walls with Boundary Elements. ACI Special Publications. Reinforced Concrete in Seismic Zones SP-53-8, 1977, pp.149-202.
[3] Wood S. Shear Strength of Low-Rise Reinforced Concrete Walls. ACI Structural Journal Vol. 87, No. 1, January-February 1990, pp. 99-107.
[4] ACI Committee 318. Building Code Requirements for Reinforced Concrete (ACI 318- 2008). American Concrete Institute.Farmington Hill. Michigan 2008.
[5] Concrete Design Committee P 3101.2006. Concrete Structures Standard, Part 1−The Design of Concrete Structures, Standards New Zealand, New Zealand, Wellington 2006.
[6] G. del Distrito Federal. 2004. Normas Técnicas Complementarias para Diseño y Construcción de Estructuras de Concreto (NTCC). Gaceta Oficial del Departamento del Distrito Federal. Mexico2004.
[7] Hirosawa. M. Past Experimental Results on Reinforced Concrete Shear Walls and Analysis on Them, Kenchiku Kenkyu Shiryo, Building Research Institute, Ministry of Construction, Tokyo, Japan, 1975 No. 6. 277 pp.
[8] Maier J. and Thürlimann B. Bruchversuche an Stahlbetonscheiben Institut für Baustatik und Konstruktion, Eidgenössische Technische Hochschule (ETH) Zürich, Zürich, Switzerland, 1985 130 pp.
[9] Lefas DI. Kotsovos DM. Ambraseys NN. Behavior of reinforced concrete structural walls: deformation characteristics, and failure mechanism. ACI Struct J 1990;87(1):23-31.
[10] Rothe, D. Untersuchungen zum Nichtlinearen Verhalten von Stahlbeton Wandschieben unter Erdbebenbeanspruchung, PhD Dissertation, Fachbereich Konstruktiver Ingenieurbau, der Technischen Hochschule Darmstadt, Darmstadt, Germany,1992 161 pp.
[11] Pilakoutas K, Lopes MS. Shear resistance determination of RC members. In: 5th. National conf. on earthquake eng., vol. 2. 1995. p. 201-10.
[12] Salonikios TN, Kappos AJ, Tegos IA, Penelis GG. Cyclic load behavior of low-slenderness reinforced concrete walls: failure modes, strength and deformation analysis, and design implications. ACI Structure J 2000;97(1):132-41.
[13] Zhang L X B. and Hsu T T C. "Behavior and Analysis of 100 MPa Concrete Membrane Elements. Journal of Structural Engineering, ASCE, Vol. 124, No. 1, pp. 24-34.
[14] Kuang J S. and Ho Y B. Seismic Behavior and Ductility of Squat ReinforcedConcrete Shear Walls with Nonseismic Detailing. ACI Structural Journal, Vol. 105, No. 2,
[15] Tran T A. Lateral Load Behavior and Modeling of Low-Rise RC Walls for Performance Based Design. Ph.D. Seminar, University of California, Los Angeles 2010.

[16] Zhang, Y. Wang Z. Seismic Behavior of Reinforced Concrete Shear Walls Subjected to High Axial Loading. ACI Journal of Structural Engineering, 2000 Vol. 97, No. 5, pp. 739-750.
[17] Dazio A. Beyer K. Bachmann H. Quasi-static cyclic tests and plastic hinge analysis of RC structural walls. Engineering Structures, Volume 31, Issue 7, July 2009, pp. 1556-1571.
[18] Thomsen JH. and Wallace .W. Displacement-Based Design of Reinforced Concrete Structural Walls: An Experimental Investigation of Walls with Rectangular and T-Shaped Cross-Sections. Report No. CU/CEE-95/06, Department of Civil Engineering Clarkson University, Postdam, N.Y.1995 353 pp.
[19] Jiang H. Research on seismic behavior of shear walls dissipating energy along vertical direction with application. Doctoral Dissertation, Tongji University, China 1999.
[20] Ji S. Dynamic Analysis of the Elastic-plastic Response of High-rise Reinforced Concrete Frame-wall Structures Subjected to Ground Motion. Doctoral Dissertation. Tongji University. China 2002.
[21] Carvajal O. and Pollner E. Muros de Concreto Reforzados con Armadura Minima. Boletin Tecnico, Universidad Central de Venezuela, Facultad de Ingenieria, 1983 Ano 21 (72 73), Enero-Diciembre, 5-36
[22] Zhou G. Research on the hysteric behavior of high-rise reinforced concrete shear walls. Master's Thesis, Tongji University, China 2004.
[23] Tasnimi AA. Strength and deformation of mid-rise shear walls under load reversal. Eng Struct 2000;22:311-22.
[24] Aaleti, Sriram, "Behavior of rectangular concrete walls subjected to simulated seismic loading. (2009). Graduate Theses and Dissertations. Paper 11047.