World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:8, No:8, 2014

# Adaptive Kaman Filter for Fault Diagnosis of Linear Parameter-Varying Systems

Rajamani Doraiswami, Lahouari Cheded

***Abstract*****—**Fault diagnosis of Linear Parameter-Varying (LPV) system using an adaptive Kalman filter is proposed. The LPV model is comprised of scheduling parameters, and the emulator parameters. The scheduling parameters are chosen such that they are capable of tracking variations in the system model as a result of changes in the operating regimes. The emulator parameters, on the other hand, simulate variations in the subsystems during the identification phase and have negligible effect during the operational phase. The nominal model and the influence vectors, which are the gradient of the feature vector respect to the emulator parameters, are identified off-line from a number of emulator parameter perturbed experiments. A Kalman filter is designed using the identified nominal model. As the system varies, the Kalman filter model is adapted using the scheduling variables. The residual is employed for fault diagnosis. The proposed scheme is successfully evaluated on simulated system as well as on a physical process control system.

***Keywords*****—**Identification, linear parameter-varying systems, least-squares estimation, fault diagnosis, Kalman filter, emulators.



## I. Introduction

In recent years, many complex, high order, and nonlinear physical systems have been successfully modeled as (LPV) systems with a view to designing gain-scheduling controllers, fault diagnosis schemes, and real-time simulation. Fault Detection and Isolation (FDI) schemes based on assuming linear time-invariant systems are not reliable if the parameter perturbations are large around an operating point. FDI schemes using LPV models have been proposed for approximating a class of nonlinear systems by LPV systems [1]. Model of a physical system is complex and nonlinear. Linearized model is successfully employed in many applications. A model-based fault diagnosis for nonlinear systems is still a challenging problem [2]. In recent years, however, detection of a fault in nonlinear systems has been proposed using robust fault detection filters [3].

In this paper, a piecewise liner model based on the linear parameter varying model approach to approximate a class of nonlinear system is employed. Modeling, identification and fault diagnosis of interconnected system composed of subsystems are developed by extending the results for linear system proposed in [4], [5]. The operating points are tracked by *scheduling variables*, which are measured in real time. They include exogenous signals such as the set point, internal variables such as the velocity and power, environment variables such as the altitude, temperature, pressure and air speed. A reliable identification is proposed by identifying the system at a given operating point by indirectly perturbing the subsystem parameters. Except in the case of subsystems such as a controller, the parameters which characterize the subsystems may not be accessible. To meet the requirement of the accessibility of the subsystem parameters, an emulator is connected at the measured inputs and the outputs. Each emulator is associated with a subsystem, and is connected in cascade with it. Variations of the emulator parameters mimic the macroscopic behavior of the subsystem, namely the variations in the phase and magnitude of its transfer function. The emulator transfer function is a first-order all-pass filter, a finite impulse response (FIR) filter, an infinite impulse response (IIR) filter, a pure delay, a static gain or other.

An adaptive Kalman filter is designed using the identified nominal model. The key property of the Kaman filter established in [4], [5] is exploited in developing the fault diagnosis scheme, namely the residual of the Kalman filter is zero mean white noise process if and only if there is no deviation between the nominal and actual model, otherwise, the residual will contain an additive term representing the deviation in their feature vectors. The Kalman filter model is adapted using the scheduling variable if the operating regime varies.

The proposed scheme is evaluated on a physical process control system. The objective of identification is to help develop a fault diagnosis scheme to detect and isolate sensor and actuator faults

## II. Linear Parameter Varying Model

Physical systems are generally complex and nonlinear. Modeling of a class of nonlinear physical system for intended application to fault diagnosis is presented. To illustrate the proposed scheme, an interconnected system $G$ formed of a number of subsystems $\{G_i : i = 1,2,3,...,m\}$ is considered. Each subsystem may represent a physical entity such as a sensor, actuator, controller or any other system component that is subject to variations, and may be affected by the noise $v$ and the disturbance inputs $\{w_i\}$ as shown in Fig. 1.

The scheduling variable $\xi$ is a function of the state of the system $x(k)$. A finite number of scheduling variables, $\xi^{\ell}$, are selected to cover the relevant operating regimes. The scheduling variable is a $px1$ discrete-time vector:

Rajamani Doraiswami is with the Department of Electrical and Computer Engineering, University of New Brunswick, Fredericton, New Brunswick, Canada (e-mail: dorai@unb.ca).

Lahouari Cheded is with Systems Eng. Department, King Fahd University of Petroleum & Minerals, Dhahran 31261, Saudi Arabia (e-mail: cheded@kfupm.edu.sa).

$$\boldsymbol{\xi}^{\ell} = \begin{bmatrix} \xi_1^{\ell} & \xi_2^{\ell} & \xi_3^{\ell} & . & \xi_p^{\ell} \end{bmatrix}^T, \ \ell = 0,1,2,... \quad (1)$$

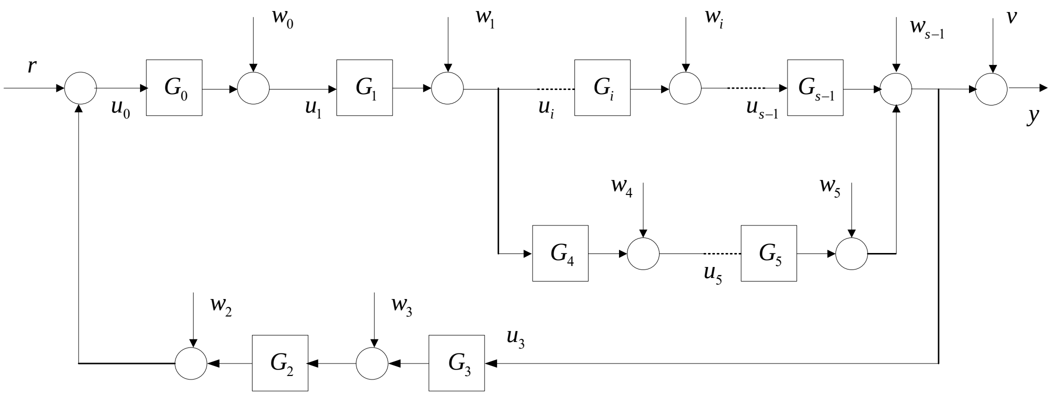


Fig. 1 Interconnected system

The index $\ell$ indicates an operating point, and $\boldsymbol{\xi}^0$ is the scheduling variable indicating the nominal fault-free operating regime. The nonlinear model of the system is linearized at each discrete variable $\boldsymbol{\xi}^{\ell}$ to obtain a set of piecewise linear approximate models $\{G(\boldsymbol{\xi}^{\ell}, z)\}$. The overall system relating the input $r(k)$, and the output $y(k)$ may be expressed using a linear regression model as:

$$y(k) = \boldsymbol{\psi}^T(k)\boldsymbol{\theta}(\boldsymbol{\xi}^{\ell}) + v(k) \quad (2)$$

where $\boldsymbol{\psi}(k)$ is an $M$x1 data vector, with $M = n_a + n_b$, that is given by:

$$\boldsymbol{\psi}^T(k) = [-y(k-1) \ ... - y(k-n_a) \ r(k-1)...r(k-n_b)] \quad (3)$$

$\boldsymbol{\theta}(\xi^{\ell})$ is an $M$x1 feature vector, which is a function of the scheduling variable $\boldsymbol{\xi}^{\ell}$:

$$\boldsymbol{\theta}(\boldsymbol{\xi}^{\ell}) = \left[a_1(\boldsymbol{\xi}^{\ell}) \ a_2(\boldsymbol{\xi}^{\ell})....a_{n_a}(\boldsymbol{\xi}^{\ell}) \ b_1(\boldsymbol{\xi}^{\ell}) \ ...b_{n_b}(\boldsymbol{\xi}^{\ell})\right]^T \quad (4)$$

where $\{a_i(\boldsymbol{\xi}^{\ell})\}$ and $b_i(\boldsymbol{\xi}^{\ell})$ are the denominator and the numerator coefficients of the overall system transfer function. $G(\boldsymbol{\xi}^{\ell}, z)$.

*A Emulator Model*

The structure and the parameters of a physical system may vary due to changes in the operating regime. The difference between the actual system and its model, termed *model uncertainty*, is considered in identification. A model, termed the numerator-denominator perturbation model, is employed herein, where the perturbations in the numerator and denominator polynomials are treated separately instead of being clubbed together as a single perturbation term in the overall transfer function. The numerator-denominator perturbation model takes the following form:

$$G(z) = E(z)G_0(z) \quad (5)$$

where $G_0(z) = \frac{N_0(z)}{D_0(z)}$ ,is the nominal transfer function, $N_0(z)$ is the nominal numerator polynomial, $D_0(z)$ the nominal denominator (scalar) polynomial, and $E(z)$ the multiplicative perturbation, termed here as the *emulator:*

$$E(z) = \frac{1+\Delta_N(z)}{1+\Delta_D(z)} \quad (6)$$

$\Delta_N(z) \in RH_\infty$ and $\Delta_D(z) \in RH_\infty$ represent, respectively, the pertutbations in the numerator and denominator polynomials of the nominal model $G_0(z)$. In many practical problems, for computational simplicity, the perturbation model is chosen to mimic the macroscopic behavior of the system characterized by gain and phase changes in the system transfer function. The emulator $E(z)$ is chosen to be a constant gain ($\gamma_i$), a gain and a pure delay of $d$ time instants ($\gamma_i z^{-d}$), an all-pass first-order filter $\left(\gamma_j \frac{\gamma_i + z^{-1}}{1+\gamma_i z^{-1}}\right)$ or where $\{\gamma_i\}$ are termed herein as the *emulator* parameters. Fig. 2 below shows a position control system formed of subsystems a) a PID controller with gains $k_p$, $k_I$ and $k_d$, b) an actuator which is an amplifier of gain $k_A$, c) position sensor of gain $k_\theta$, d) velocity sensor of gain $k_\omega$, and a plant, which is DC servo-motor with time constant $\alpha$. The system is subject to load disturbance $w(k)$ and the measurement noise $v(k)$. The actuator $k_A$, the velocity sensor $k_\omega$, and the position sensor $k_\theta$ were modeled as constant gain transfer function. Since the subsystem of the plant $G_{p1}(z) = \frac{k_1 z^{-1}}{1-\alpha z^{-1}}$ is subject to a fault as result of variation of the parameter $\alpha$, and since the plant parameter $\alpha$ is not accessible, an emulator, denoted $E_1(z)$, is connected at the output of the PID controller to mimic variations in the plant parameter $\alpha$. The emulator may be chosen to be a first order all pass filter $E_1(z)=\dfrac{\gamma_1+z^{-1}}{1+\gamma_1 z^{-1}}$ .The emulators $E_2$, $E_3$ and $E_4$ to induce faults respectively in the actuator $k_A$, the velocity sensor $k_\omega$, and the position sensor $k_\theta$ are chosen to be static constant gain transfer functions given by:

$$E_2=\gamma_2\,; E_3=\gamma_3;\; E_4=\gamma_3 \tag{7}$$

All the emulators are connected in cascade with respective devices. The nominal diagnostic parameters are chosen such that during the operating regime they have negligible effect on the static and the dynamic behaviour. In the case of the plant emulator $E_1(z)$, the nominal emulator parameters is chosen so that $E_1(z)$ is approximately unity. The emulator and the scheduling parameters play an important role in the identification and fault diagnosis of a LPV system. The emulator parameters mimic variation in the subsystems during the identification, and the associated influence vector is employed in isolating a fault. The scheduling parameters on the other hand track the variation in the operating regime of the system. Both parameters are accessible. Consider an interconnected system shown in Fig. 1.

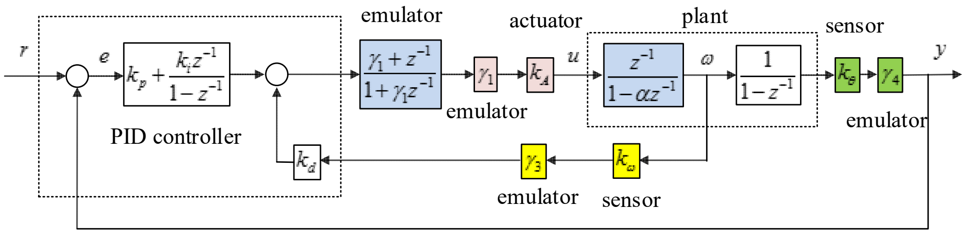


Fig. 2 Position control system

A subsystem $G_i(z)$ may be a process (or a plant), a controller, an actuator, or another device, and is associated with an emulator $E_i(z)$. The parameter $\gamma^i$ of $E_i(z)$ is selected so that it is capable of monitoring solely the health of the subsystem $G_i(z)$. The *emulator parameter vector* $\gamma$ for the entire interconnected system is a $(q\times 1)$ vector that augments $\{\gamma^i, i=1,2,...,s\}$ for all subsystems, $G_i(z)$, $i=1,2,...,s$, that are subject to failure. The overall $qx1$ diagnostic parameter vector $\gamma$ formed of all the subsystem diagnostic parameters $\{\gamma^i\}$ is given by:

$$\gamma=\begin{bmatrix}\gamma^1 & \gamma^2 & \gamma^3 & ... & \gamma^s\end{bmatrix}^T=\begin{bmatrix}\gamma_1 & \gamma_2 & ... & \gamma_q\end{bmatrix}^T \tag{8}$$

Let $\gamma^0$ be nominal value and its variation $\Delta\gamma=\gamma-\gamma^0$

*B. The Feature Vector and the Emulator Parameters*

It can be shown that linear regression model of the system during the identification phase takes the following form:

$$\begin{aligned}y(k)=\psi^T(k)\Big(&\theta^0(\xi^\ell)+\sum_i\Omega_i(\xi^\ell)\Delta\gamma_i+\sum_{i,j}\Omega_{ij}(\xi^\ell)\Delta\gamma_i\Delta\gamma_j\\&+...+\sum_{1,2,3,...,q}\Omega_{123..q}(\xi^\ell)\Delta\gamma_1\Delta\gamma_2\Delta\gamma_3...\Delta\gamma_q\Big)+v(k)\end{aligned} \tag{9}$$

where $\theta^0(\xi^\ell)$ is the nominal fault-free feature vector corresponding to the emulator parameter values $\gamma^0$; $y^0(k)=\psi^T(k)\theta^0(\xi^\ell)$ is the nominal output ; $\Omega_i(\xi^\ell)$, $\Omega_{ij}(\xi^\ell)$, $\Omega_{ijk}(\xi^\ell)$, … $\Omega_{123..q}(\xi^\ell)$ are $Mx1$ vectors, termed *influence vectors*, which denote the first, second, third and up to $q^{th}$ partial derivatives of $\theta$ with respect to $\gamma$ as given by:

$$\Omega_i(\xi^\ell)=\frac{\partial\theta}{\partial\gamma_i},\ \Omega_{ij}(\xi^\ell)=\frac{\partial^2\theta}{\partial\gamma_i\partial\gamma_j},\ ...,\ \Omega_{123..a}(\xi^\ell)=\frac{\partial^q\theta}{\partial\gamma_1\partial\gamma_2\partial\gamma_3\ ...\ \partial\gamma_q}$$

The nominal feature vector and all the partial derivatives are computed at the scheduling variable $\xi^\ell$. The nominal feature vector $\theta^0(\xi^\ell,\gamma^0)$ and influence matrix $\Omega$ formed of influence vectors, $\Omega=\left[\Omega_i(\xi^\ell),\Omega_{ij}(\xi^\ell),\Omega_{ijk}(\xi^\ell),...\Omega_{123...q}(\xi^\ell)\right]$, completely describe the describe the system during the identification .

## III. Identification of the System

The system model is identified by performing a number of parameter-perturbed experiments. Each experiment consists of perturbing one or more emulator parameters. The input $r(k)$ is chosen to be persistently exciting to allow the model to capture as much as possible of the system dynamics.

Consider the $j^{th}$ experiment of perturbing the one or more elements of emulator parameter $\gamma$. Let $\{y^j(k)\}$ be the set of all outputs from the parameter perturbed experiments for the input $r(k)$.The objective here is to identify a) an *optimal nominal model* and b) the influence vectors from the input-output data collected from all perturbed parameter experiments from:

$$\left[\hat{\theta}^0(\xi^\ell),\hat{\Omega}(\xi^\ell)\right]=\arg\left\{\min_{\Omega,\theta^0} y(k)=\sum_f\left\|y^0(k)-y^j(k)\ \right\|^2\right\} \tag{10}$$

The optimal nominal model in the state space form, denoted $\left(\boldsymbol{A}_0\left(\boldsymbol{\xi}^\ell\right), \boldsymbol{B}_0\left(\boldsymbol{\xi}^\ell\right), \boldsymbol{C}_0\left(\boldsymbol{\xi}^\ell\right)\right)$, is derived from the estimate of the nominal feature vector $\hat{\boldsymbol{\theta}}^0(\boldsymbol{\xi}^\ell)$ .

The proposed scheme based on performing a number of experiments and the conventional scheme base on single experiment at the nominal operating points under noise and disturbance corrupting the data and variations the system model is significantly superior based on both simulated examples and the actual physical system as shown in Fig. 3 where the mean squared errors between the identified and the actual model as under model variations are compared. The mean squared error for the proposed is significantly smaller.

## IV. Kalman Filter

The Kalman filter (KF) is essentially a closed-loop filter formed of an exact copy of identified state-space model of the fault-free system $\left(\boldsymbol{A}_0\left(\boldsymbol{\xi}^\ell\right), \boldsymbol{B}_0\left(\boldsymbol{\xi}^\ell\right), \boldsymbol{C}_0\left(\boldsymbol{\xi}^\ell\right)\right)$ and driven by the residual [4]. The structure of the KF is given by:

$$\begin{aligned} \hat{x}(k+1) &= \boldsymbol{A}_0\left(\boldsymbol{\xi}^\ell\right)\hat{x}(k) + \boldsymbol{B}_0\left(\boldsymbol{\xi}^\ell\right)r(k) + \boldsymbol{K}_0\left(\boldsymbol{\xi}^\ell\right)e(k) \\ e(k) &= y(k) - \boldsymbol{C}_0\left(\boldsymbol{\xi}^\ell\right)\hat{x}(k) \end{aligned} \tag{11}$$

where $e(k)$ is the residual, and $\boldsymbol{K}_0\left(\boldsymbol{\xi}^\ell\right)$ is the Kalman gain computed from the statistics of the disturbance and measurement noise . The KF model is adapted along the trajectories of the scheduling variable $\xi^\ell$ in step with the variations in the system model varies. It is shown in [5] that the residual takes the form of:

$$F_0(z,\boldsymbol{\xi}^\ell)e(z) = \boldsymbol{\psi}^T(z)\Delta\boldsymbol{\theta}\left(\boldsymbol{\xi}^\ell\right) + e_0(z) \tag{12}$$

where $F_0(z,\boldsymbol{\xi}^\ell) = \left|zI - \boldsymbol{A}_0\left(\boldsymbol{\xi}^\ell\right) + \boldsymbol{K}_0\left(\boldsymbol{\xi}^\ell\right)\boldsymbol{C}_0(\boldsymbol{\xi}^\ell)\right|$, $\Delta\theta = \theta(\xi,\gamma) - \theta^0(\xi)$, $e_0(z)$ is a zero-mean white noise process

## V. Fault Diagnosis

A unified approach to both detection and isolation of a fault is presented based on the Kalman filter residual. The residual of the Kalman filter is a zero mean white noise process if and only if there is no fault. If there is a fault, the residual will have an additive fault indicating term. The fault indicating term is function of the variation in the feature vector [5].

The fault detection problem is posed as binary hypothesis testing problem and the threshold value is chosen as an acceptable trade-off between the correct detection and false alarm probabilities.

The fault isolation problem is similarly posed as a multiple hypothesis testing problem. The hypotheses include a single fault in a subsystem, simultaneous faults in two subsystems, and so on till simultaneous faults in all subsystems .The unified decision strategy becomes:

$$t_s(\boldsymbol{e}) \begin{cases} \le \eta_{th} & \textit{no fault} \\ > \eta_{th} & \textit{fault} \end{cases} \tag{13}$$

Test statistics for a constant reference input $r(k)$ is:

$$t_s(\boldsymbol{e}) = \left|\frac{1}{N}\sum_{i=0}^{N-1} e(k-i)\right| \tag{14}$$

It is based subjecting the residual to a detailed analysis by hypothesizing whether there is a variation in a single or simultaneous variations in two, three or all emulator parameters. For the single variation, the maximum likelihood estimate $\Delta\hat{\boldsymbol{\gamma}}$ of $\Delta\boldsymbol{\gamma}$ is obtained from:

$$\Delta\hat{\gamma}_j = \arg\left\{\min_{\{\Delta\gamma_i\}}\left\| e(k) - \boldsymbol{\psi}^T(k)\sum_i \boldsymbol{\Omega}_i(\boldsymbol{\xi}^\ell)\Delta\gamma_i\right\|^2\right\} \tag{15}$$

The subsystem $G_j(z)$ is asserted to be faulty if $\left|\Delta\hat{\gamma}_j\right|$ exceeds some threshold value.

## VI. Evaluation on a Physical System

The objective is to detect and isolate leakage, actuator fault, liquid-level sensor fault. In order to simulate faults in the physical system, static fault emulators are connected in cascade with the height sensor, the flow rate sensor, the actuator and the leakage drain pipe as shown in Fig. 4, which is the block diagram of the process control system. The emulator parameters $\gamma_1, \gamma_2$, and $\gamma_3$ are connected in cascade with the leakage drain pipe, the actuator and the height sensor during the off-line identification stage.

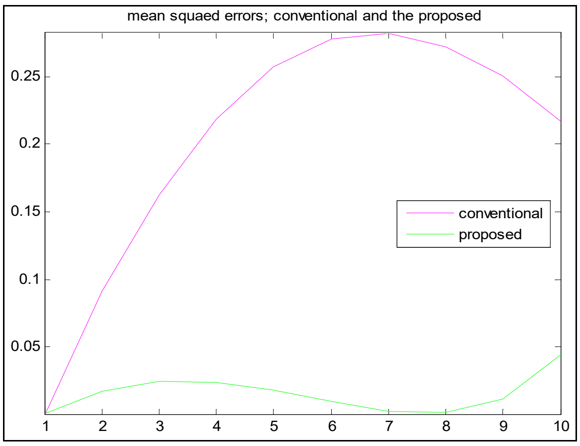


Fig. 3 Mean-squared errors: the proposed (in green) and conventional (in blue) schemes subject to model perturbations

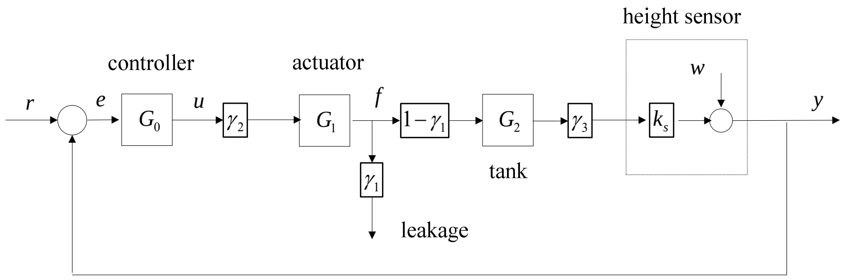


Fig. 4 Process control system

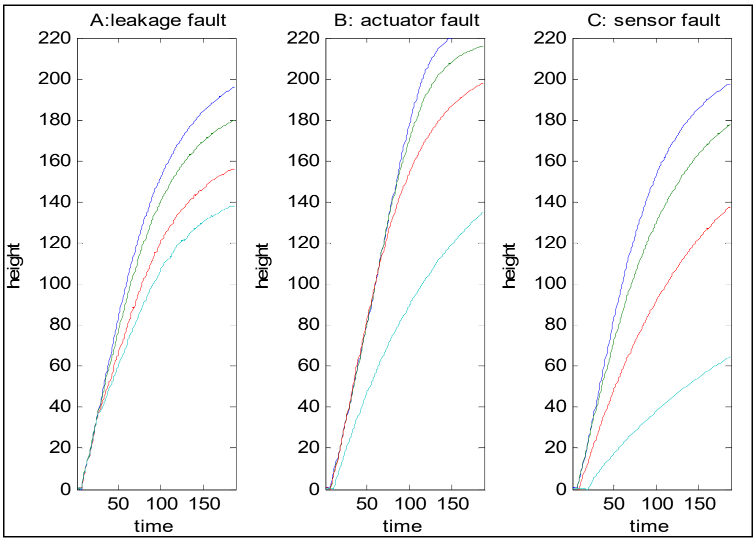


Fig. 5 Identification experiments

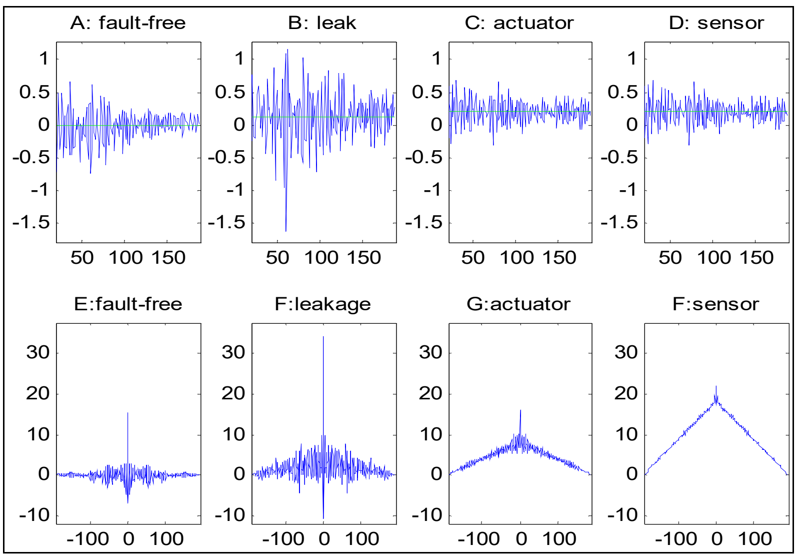


Fig. 6 The residuals and their autocorrelations

Fig. 5 shows the parameter perturbed experiments where the normal, leakage, actuator and sensor faults are induced by varying the emulator parameters. Fig. 6 shows the Kalman filter residual and its auto-correlation for the following cases: (a) nominal (or fault-free), (b) leakage fault, (c) actuator fault and (d) sensor fault. The test statistic value is the lowest and the auto-correlation is that of a zero-mean white noise for the nominal (fault-free) case. The subfigures in Fig. 6 A, B, C, D show the residuals and their test statistics shown as straight lines, whereas the sub-figures E, F, G and H show the corresponding auto-correlations. The proposed scheme was successful in fault.

## VII. Conclusions

The proposed adaptive Kalman filter based scheme is effective for fault diagnosis of LPV system based on simulated as well as physical systems. The residual is a reliable indicator of a fault as well for the adaptation of the Kalman filter.

## Acknowledgement

The authors acknowledge the support of The University of New Brunswick, The National Science and Engineering Research Council (NSERC) of Canada, and the University of KFUPM, Saudi Arabia and Dr. Haris Khaled for help and experimental work.

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