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An Application of Generalized Fuzzy Soft Sets in a Social Decision Making Problem

Nisha Singhal, Usha Chouhan

***Abstract*—**At present, application of the extension of soft set theory in decision making problems in day to day life is progressing rapidly. The concepts of fuzzy soft set and its properties have been evolved as an area of interest for the researchers. The generalization of the concepts recently got importance and a rapid growth in the research in this area witnessed its vital-ness. In this paper, an application of the concept of generalized fuzzy soft set to make decision in a social problem is presented. Further, this paper also highlights some of the key issues of the related areas.

***Keywords*—** Soft set, Fuzzy Soft set, Generalized Fuzzy Soft set, Membership and Non-Membership Score.

I. Introduction

In real world, we face so many uncertainties in all walks of life fields. However, most of the existing mathematical tools for formal modeling, reasoning and computing are crisp deterministic and precise in character. There are many theories like theory of Probability, Fuzzy Set theory [1], Rough Set theory, Intuitionistic Fuzzy Set theory, Vague Set theory, Interval Mathematics for dealing with uncertainties. These theories have their own limitations as pointed out by [2]. Molodtsov [2] initiated the theory of soft sets as a new mathematical tool to deal with uncertainties while modeling the problems in Engineering, Computer Science, Physics, Social Sciences, and Medical Sciences.

Soft set is a parameterized general mathematical tool which deals with a collection of approximate descriptions of objects. Each approximate description has two parts, a predicate, and an approximate value set. Since initial description of the object has an approximate nature so, we do not need to introduce the notion of exact solution. The absence of any restrictions on the approximate description in soft set theory makes this theory very convenient and easily applicable in practice. Maji et al. [3] presented an application of soft sets in decision making problem and studied basic notions of Soft Set Theory [4]. In recent times, researches have contributed a lot towards fuzzification of Soft Set Theory. Maji et al. [5] introduced the concept of Fuzzy Soft Sets and some properties regarding Fuzzy Soft Sets. Some results were further revised and improved by [6] given by [7] have further generalized the concept of Fuzzy Soft Sets.

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II. Preliminaries

In this section, we first recall the basic definitions related to soft sets and fuzzy soft sets which would be used in the sequel.

Definition 1 - Soft Set **[2]:** A pair (F,E) is called a soft set (over U) if and only if F is a mapping of E into the set of all subsets of the set U. In other words, the soft set is a parameterized family of subsets of the set U. Every set $\mathrm{F}(e), e \in E$, from this family may be considered as the set of e -approximate elements of the soft set.

Example 1: Let U be a set of four factories, say, $U = \{f_1, f_2, f_3, f_4\}$ and

$$E = \begin{Bmatrix} e_1(\cos tly), e_2(excellent\ work\ culture), \\ e_2(assured\ production), \\ e_4(good\ location), e_5(cheap) \end{Bmatrix}$$

(Let say) be a set of parameters. Let $A = \{e_{2,} e_3, e_4\} \subseteq E$ then

$$(F,A) = \begin{Bmatrix} F(e_2) = \{f_1, f_4\}, \\ F(e_3) = \{f_1, f_2, f_4\} \\ F(e_4) = \{f_3\} \end{Bmatrix}$$

is the soft set representing the "good choice of the factory", Mr. X is going to buy? This soft set can be represented in a tabular form as in Table I.

TABLE I
Soft Set

	e2	e2	e4
f1	1	1	0
f2	0	1	0
f2	0	0	1
f4	1	1	0

This style of representation will be useful for storing a soft set in a computer.

Definition 2: Fuzzy Soft Set **[5]:** A pair (F,A) is called a fuzzy soft set over U where $F: A \to \tilde{P}(U)$ is a mapping from A into $\tilde{P}(U)$ where, $A \subseteq E$.

Example 2: If we take Example 1 with

$$F(e_1) = \{f_1/0.9, f_2/0.2, f_3/0.4, f_4/0.8\}$$

$$F(e_2)=\{f_1/0.8, f_2/0.3, f_3/0.2, f_4/0.7\}$$
$$F(e_3)=\{f_1/0.9, f_2/0.4, f_3/0.1, f_4/0.1\}$$
$$F(e_4)=\{f_1/0.3, f_2/0.2, f_3/0.6, f_4/0.1\}$$
$$F(e_5)=\{f_1/0.1, f_2/0.7, f_3/0.5, f_4/0.2\}, then$$
$$(F,A)=\begin{Bmatrix} F(e_2)=\{f_1/0.8, f_2/0.3, f_3/0.2, f_4/0.7\}, \\ F(e_3)=\{f_1/0.9, f_2/0.4, f_3/0.1, f_4/0.8\} \\ F(e_4)=\{f_1/0.3, f_2/0.2, f_3/0.6, f_4/0.1\} \\ \end{Bmatrix}$$

Then it is the fuzzy soft set, representing the 'great choice of the factory' which Mr. X is going to buy. This fuzzy soft set can be represented in Table II.

TABLE II
FUZZY SOFT SET

(F,A)	e2	e2	e4
f1	0.8	0.9	0.3
f2	0.3	0.4	0.2
f2	0.2	0.1	0.6
f4	0.7	0.8	0.1

Definition 3 - Generalized fuzzy soft sets [7]: Let $U=\{x_1, x_2, x_3, ...x_n\}$ be the universal set of elements and $E=\{e_1, e_2, e_3, ...e_n\}$ be the universal set of parameters. The pair (U,E) will be called a soft universe. Let $F:E\to I^U$ and μ be a fuzzy subset of *E*, i.e. $\mu:E\to I=[0,1]$, where I^U is the collection of all fuzzy subsets of *U*.

Let F_μ be the mapping $F_\mu:E\to I^U\times I$ be a function defined as follows: $F_\mu(e)=(F(e),\mu(e))$, where $F(e)\in I^U$, then F_μ is called generalized fuzzy soft sets the soft universe (U,E). For each $e_i, F_\mu(e_i)=(F(e_i),\mu(e_i))$ indicates not only the degree of belongingness of the elements of U in $F(e_i)$ but also the degree of possibility of such belongingness which is represented by $\mu(e_i)$.

Example 3: We take fuzzy soft set (F, A) given in the Example 2 and Let $\mu:E\to I=[0,1]$, be defined as:

$$\mu(e_2)=0.7, \mu(e_3)=0.6, \mu(e_4)=0.8 .$$

We define a function - $F_\mu:E\to I^U\times I$ as:

$$F_\mu(e_2)=\{\{f_1/0.8, f_2/0.3, f_3/0.2, f_4/0.7\}, 0.7\},$$
$$F_\mu(e_3)=\{\{f_1/0.9, f_2/0.4, f_3/0.1, f_4/0.8\}, 0.6\},$$
$$F_\mu(e_4)=\{\{f_1/0.3, f_2/0.2, f_2/0.6, f_2/0.1\}, 0.8\}$$

Then F_μ is a generalized fuzzy soft set over (U,E), which can be expressed in Table III.

TABLE III
GENERALIZED FUZZY SOFT SET F_μ

		e2	e2	e4
$F_\mu=$	f1	0.8	0.9	0.3
	f2	0.3	0.4	0.2
	f2	0.2	0.1	0.6
	f4	0.7	0.8	0.1
	$\mu(e_i)$	0.7	0.6	0.8

$\mu(e_i)$ is the degree of possibility of belongingness and $\mu(e_2)=0.7, \mu(e_3)=0.6, \mu(e_4)=0.8$

Definition 4 - Generalized fuzzy soft subset [7]: Let F_μ and G_δ be two generalized fuzzy soft set over (U,E) then if

i. δ is a fuzzy subset of μ and
ii. $G(e)$ is also a fuzzy subset of $F(e)$, i.e. $G(e)\subseteq F(e), \forall e\in E$.

In this case, we write as $G_\delta \tilde{\subseteq} F_\mu$. In this case, G_δ is said to be a generalized fuzzy soft subset of F_μ.

Example 4: If we take generalized fuzzy soft subset F_μ given in the Example 3 with another generalized fuzzy soft subset G_δ. Over the same universe (U, E) defined by

$$G_\delta(e_2)=\{\{f_1/0.5, f_2/0.2, f_3/0.1, f_4/0.4\}, 0.6\}$$
$$G_\delta(e_3)=\{\{f_1/0.7, f_2/0.2, f_3/0.1, f_4/0.5\}, 0.3\}$$
$$G_\delta(e_4)=\{\{f_1/0.2, f_2/0.1, f_3/0.5, f_2/0.0\}, 0.1\}$$

Then G_δ is a generalized fuzzy soft subset of F_μ over (*U*, *E*). This fuzzy soft set can be represented in Table IV.

TABLE IV
GENERALIZED FUZZY SOFT SET G_δ

		e2	e2	e4
$G_\delta=$	f1	0.5	0.7	0.2
	f2	0.2	0.2	0.1
	f2	0.1	0.1	0.5
	f4	0.4	0.5	0.0
	$\mu(e_i)$	0.6	0.3	0.1

$\mu(e_i)$ is the degree of possibility of belongingness.

Definition 5 - Complement of Generalized fuzzy soft sets [7]: Let F_μ is a generalized fuzzy soft set over (U, E). Then F_μ^c is said to be complement of F_μ and is defined by

$$F_\mu^c(e_i)=(F^c(e_i), \mu^c(e_i)) \text{ for each } e_i .$$

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Example 5 - If we take generalized fuzzy soft set F_μ given in the Example 3 then complement of F_μ is defined as

$$F_\mu^c(e_2)=\{\{f_1/0.2, f_2/0.7, f_3/0.8, f_4/0.3\}, 0.3\},$$
$$F_\mu^c(e_3)=\{\{f_1/0.1, f_2/0.6, f_3/0.9, f_4/0.2\}, 0.4\},$$
$$F_\mu^c(e_4)=\{\{f_1/0.7, f_2/0.8, f_2/0.4, f_2/0.9\}, 0.2\}$$

whose tabular form is given in Table V.

TABLE V
COMPLEMENT OF GENERALIZED FUZZY SOFT SET F_μ^c

		e2	e2	e4
$F_\mu^c =$	f1	0.2	0.1	0.7
	f2	0.7	0.6	0.8
	f2	0.8	0.9	0.4
	f4	0.3	0.2	0.9
	$\mu(e_i)$	0.3	0.4	0.2

$\mu(e_i)$ is the degree of possibility of belongingness.

Now, we illustrate the approach by giving an application of generalized fuzzy soft set in social decision making problem.

III. CASE STUDY

Suppose a working woman Miss G wishes to marry a man on the basis of some criteria*. Our purpose is to find out the most appropriate partner for Miss G. Criteria are selected arbitrarily and can be varies as per the problem i.e. The Alternatives and Attributes. Suppose F_μ is the generalized fuzzy soft set representing eligibility performance of candidates (alternatives).

Assume there are seven* men in universal set U s.t.

$$U\{m_1, m_2, m_3, m_4, m_5, m_6, m_7\}$$

and the parameter* set

$$E=\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$$

Where each $e_i, 1 \le i \le 10$ indicates an attributes of m_j for $1 \le j \le 7$. (*can be varied as per the problem)

Let e_1 stand for "government employee", e_2 stand for "non-government employee", e_3 stand for "businessman", e_4 stand for "education qualification", e_5 stand for "expert in households", e_6 stand for "hard working", e_7 stand for "responsible", e_8 stand for "family status", e_9 stand for "spiritual and ideal", e_{10} stand for "handsome".

Suppose the wishing parameter for Miss G is $A \subseteq E$ where $A=\{e_1, e_4, e_5, e_7, e_8, e_{10}\}$. Let the preference of the criterions for miss G be described by the fuzzy subset $\mu : A \to [0,1]$ of A as

$$\mu(e_1)=0.6, \mu(e_4)=0.7, \mu(e_5)=0.3, \mu(e_7)=0.6$$
$$\mu(e_8)=0.4, \mu(e_{10})=0.3$$

Consider the generalized fuzzy soft set F_μ,

$$F_\mu(e_1)=\left\{\begin{Bmatrix} m_1/0.0, m_2/1.0, m_3/1.0, m_4/0.0, m_5/1.0, \\ m_6/0.0, m_7/1.0 \end{Bmatrix}, 0.6\right\},$$

$$F_\mu(e_4)=\left\{\begin{Bmatrix} m_1/0.1, m_2/0.6, m_3/0.7, m_4/0.2, m_5/0.9, \\ m_6/0.3, m_7/0.8 \end{Bmatrix}, 0.7\right\}$$

$$F_\mu(e_5)=\left\{\begin{Bmatrix} m_1/0.5, m_2/0.3, m_3/0.6, m_4/0.4, m_5/0.3, \\ m_6/0.5, m_7/0.4 \end{Bmatrix}, 0.6\right\}$$

$$F_\mu(e_7)=\left\{\begin{Bmatrix} m_1/0.6, m_2/0.8, m_3/0.5, m_4/0.6, m_5/0.3, \\ m_6/0.5, m_7/0.4 \end{Bmatrix}, 0.6\right\}$$

$$F_\mu(e_8)=\left\{\begin{Bmatrix} m_1/0.5, m_2/0.4, m_3/0.6, m_4/0.5, m_5/0.6, \\ m_6/0.7, m_7/0.2 \end{Bmatrix}, 0.4\right\}$$

$$F_\mu(e_{10})=\left\{\begin{Bmatrix} m_1/0.3, m_2/0.2, m_3/0.4, m_4/0.5, m_5/0.6, \\ m_6/0.3, m_7/0.4 \end{Bmatrix}, 0.3\right\}$$

Comparison table is obtained by multiplying each entry of the table representing the generalized fuzzy soft set by corresponding values of $\mu(e)$.

Finally, we find the *lowest value* from the final score table which would corresponds to the best selection for Miss G.

If there are more than one values are lowest then one of m_i them may be chosen.

The Algorithm

Step 1: Input the generalized fuzzy soft set F_μ.

Step 2: Write F_μ in tabular form.

Step 3: Compute the complement of F_μ i.e. F_μ^c.

Step 4: Write F_μ^c in tabular form.

Step 5: Compute the comparison table for F_μ and F_μ^c both.

Step 6: Compute the membership score "α" for F_μ and "β" for F_μ^c.

Step 7: Compute the final score by "$\alpha+\beta-\alpha.\beta$".

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TABLE VI
F_μ IN TABULAR FORM

(U,E)	e2	e4	e5	e7	e8	e10
m1	0.0	0.1	0.5	0.6	0.5	0.3
m2	1.0	0.6	0.3	0.8	0.4	0.2
m3	1.0	0.7	0.6	0.5	0.6	0.4
m4	0.0	0.2	0.4	0.6	0.5	0.5
m5	1.0	0.9	0.3	0.3	0.6	0.6
m6	0.0	0.3	0.8	0.5	0.7	0.3
m7	1.0	0.8	0.2	0.4	0.2	0.4
	μ(e2) =0.6	μ (e4) =0.7	μ(e5) =0.3	μ(e7) =0.6	μ(e8) =0.4	μ(e10) =0.3

TABLE VII
COMPARISON TABLE FOR F_μ

(U,E)	e2	e4	e5	e7	e8	e10
m1	0.00	0.07	0.15	0.36	0.20	0.09
m2	0.60	0.42	0.09	0.48	0.16	0.06
m3	0.60	0.49	0.18	0.30	0.24	0.12
m4	0.00	0.14	0.12	0.36	0.20	0.15
m5	0.60	0.63	0.09	0.18	0.24	0.18
m6	0.00	0.21	0.24	0.30	0.28	0.09
m7	0.60	0.56	0.06	0.24	0.08	0.12

TABLE VIII
F_μ^c IN TABULAR FORM

(U,E)	e2	e4	e5	e7	e8	e10
m1	1.0	0.9	0.5	0.4	0.5	0.7
m2	0.0	0.4	0.7	0.2	0.6	0.8
m3	0.0	0.3	0.4	0.5	0.4	0.6
m4	1.0	0.8	0.6	0.4	0.5	0.5
m5	0.0	0.1	0.7	0.7	0.4	0.4
m6	1.0	0.7	0.2	0.5	0.3	0.7
m7	0.0	0.2	0.8	0.6	0.8	0.6
	μ(e2) =0.4	μ(e4) =0.3	μ(e5) =0.7	μ(e7) =0.4	μ(e8) =0.6	μ(e10) =0.7

TABLE IX
COMPARISON TABLE FOR F_μ^c

(U,E)	e2	e4	e5	e7	e8	e10
m1	0.40	0.27	0.35	0.16	0.30	0.49
m2	0.00	0.12	0.49	0.08	0.36	0.56
m3	0.00	0.09	0.28	0.20	0.24	0.42
m4	0.40	0.24	0.42	0.16	0.30	0.35
m5	0.00	0.03	0.49	0.28	0.24	0.28
m6	0.40	0.21	0.14	0.20	0.18	0.49
m7	0.00	0.06	0.56	0.24	0.48	0.42

Now we calculate the complement of generalized fuzzy soft set F_μ, which is given by

$$F_\mu^c(e_1)=\left\{\begin{Bmatrix} m_1/1.0, m_2/0.0, m_3/0.0, m_4/1.0, m_5/0.0, \\ m_6/1.0, m_7/0.0 \end{Bmatrix}, 0.4\right\}$$

$$F_\mu^c(e_4)=\left\{\begin{Bmatrix} m_1/0.9, m_2/0.4, m_3/0.3, m_4/0.8, m_5/0.1, \\ m_6/0.7, m_7/0.2 \end{Bmatrix}, 0.3\right\}$$

$$F_\mu(e_5)=\left\{\begin{Bmatrix} m_1/0.5, m_2/0.7, m_3/0.4, m_4/0.6, m_5/0.7, \\ m_6/0.2, m_7/0.8 \end{Bmatrix}, 0.7\right\}$$

$$F_\mu(e_7)=\left\{\begin{Bmatrix} m_1/0.4, m_2/0.2, m_3/0.5, m_4/0.4, m_5/0.7, \\ m_6/0.5, m_7/0.6 \end{Bmatrix}, 0.4\right\}$$

$$F_\mu(e_8)=\left\{\begin{Bmatrix} m_1/0.5, m_2/0.6, m_3/0.4, m_4/0.5, m_5/0.4, \\ m_6/0.3, m_7/0.8 \end{Bmatrix}, 0.6\right\}$$

$$F_\mu(e_{10})=\left\{\begin{Bmatrix} m_1/0.7, m_2/0.8, m_3/0.6, m_4/0.5, m_5/0.4, \\ m_6/0.3, m_7/0.6 \end{Bmatrix}, 0.7\right\}$$

To find the Membership value '**α**', we sum-up the row value of Table VII.

TABLE X
MEMBERSHIP SCORE TABLE

Alternatives	Row sum
m1	0.87
m2	1.81
m3	1.93
m4	0.97
m5	1.92
m6	1.12
m7	1.66

To find then on-Membership value '**β'**, we sum up the row value of Table IX.

TABLE XI
NON-MEMBERSHIP SCORE TABLE

Alternatives	Row sum
m1	1.97
m2	1.61
m3	1.23
m4	1.87
m5	1.32
m6	1.62
m7	1.76

TABLE XII
FINAL SCORE TABLE

Alternatives	Membership Value '**α**'	Non- Membership Value '**β**'	Final Value "**α + β - α .β**"
m1	0.87	1.97	1.0291
m2	1.81	1.61	0.4171
m3	1.93	1.23	0.7585
m4	0.97	1.87	0.9565
m5	1.92	1.32	0.6864
m6	1.12	1.62	0.6864
m7	1.66	1.76	0.4680

In Table XII, the lowest value gives the best option among all the alternatives.

IV. Result

Selection priorities based on the criteria decided by Miss G are $m_2 > m_7 > m_5 > m_3 > m_6 > m_4 > m_1$. It is clear that *Mr.* m2*is the most eligible candidate for Miss* G.

V. Conclusion

In this paper, concept of generalized fuzzy soft set and some of its properties are highlighted and presented. An application of this theory has been applied to solve a social decision making problem. The authors are hopeful that this approach will be useful in dealing with several problems related to uncertainty.

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