Dynamic Response of Nano Spherical Shell Subjected to Termo-Mechanical Shock Using Nonlocal Elasticity Theory

J. Ranjbarn, A. Alibeigloo

Abstract**—**In this paper, we present an analytical method for analysis of nano-scale spherical shell subjected to thermo-mechanical shocks based on nonlocal elasticity theory. Thermo-mechanical properties of nano shpere is assumed to be temperature dependent. Governing partial differential equation of motion is solved analytically by using Laplace transform for time domain and power series for spacial domain. The results in Laplace domain is transferred to time domain by employing the fast inverse Laplace transform (FLIT) method. Accuracy of present approach is assessed by comparing the the numerical results with the results of published work in literature. Furtheremore, the effects of non-local parameter and wall thickness on the dynamic characteristics of the nano-sphere are studied.

Keywords**—**Nano-scale spherical shell, nonlocal elasticity theory, thermomechanical shock.

I. Introduction

THE discovery of carbon nano tubes (CNTs) and fullerene (as a nano scale spherical shell)created a wonderful evolution in technology. The unique thermao-mechanical and electromechanical properties of carbon nano scale structures (CNSSs) as high elasticity modulus, tensile strength, low density, caused wide applications such as nano wires, nano probes, nano actuators, nano bearings, nano springs and nano sensors. Because of these exceptional properties, researchers aroused to do extensive studies on CNSSs as nano spherical shells. Generally, two kinds of studies are performed on nano structures: experimental studies, theoretical and computational studies. Recently, many experiments were applied to study properties and behavior of nanostructures. Since the experiments in nano and micro scale models are very expensive so the theoretical and computational studies are more important.

Basically, there are two major categories in theoretical studies: atomistic modeling and continuum mechanics modeling. Because of time-consuming, the atomistic modeling is more complex than continuum mechanics modeling. Two approches are applied in continuum mechanics. The first approch is classical continuum mechanics theory and the secend is nonlocal continuum mechanics theory. Because of considering size effect in nano systems, the nonlocal continuum theory is more acceptable than classical theory. Fu et al. [1] investigated nonlinear free vibration of embedded multiwall carbon nanotubes considering intertube radial displacement and the related internal degrees of freedom based on the continuum mechanics model as well as a multiple-elastic beam model. Sun and Liu [2] discussed vibration of multi wall carbon nanotubes (MWCNTs) based on Donnell equations by considering the effect of the van der Waals forces. Aydogdu [3] computed frequencies of MWCNTs by using generalized shear deformation theory. He showed that the effect of van der Waals forces should be considered for small inner radius. Ke et al. [4] incorporated nonlocal continuum theory and Timoshenko beam theory to study free vibration of embedded double-walled carbon nanotubes (DWCNTs) by using differential quadrature method (DQM). The axial vibration of nanorods, based on local and nonlocal rod theories utilizing elastic beam models are developed by Aydogdu [5]. He expressed, that the size effect in nonlocal theory causes lower frequencies than local (classical) theory. Yang et al. [6] investigated nonlinear free vibration of single walled carbon nanotubes (SWCNTs) by using von Ka´rma´n geometric nonlinearity, Timoshenko beam theory and Eringen's nonlocal elasticity theory. Their numerical results show that an increase in nonlocal parameter causes to decrease linear and nonlinear frequencies. Maachou et al. [7] presented a nonlocal Levinson beam model for free vibration analysis of zigzag SWCNTs including thermal effects. In their study, thermo-mechanical properties of SWCNTs are considered independent of temperature. Zidour et al. [8] investigated thermal effect on vibration of zigzag SWCNT using nonlocal Timoshenko beam theory. In their study, thermoelastic properties of SWCNT are considered as constant value and independent of temperature. Yan et al. [9] computated natural frequencies of free vibration of SWCNTs by using a higher-order gradient theory by employing a numerical solution, they found increase of length results in an increase in the critical diameter, approximately a linear function relationship. Hoseinzadeh and Khadem [10] used a nonlocal shell theory to evaluate thermoelastic damping of vibrating DWCNT. Kiani et al. [11] examined pressure-driven water flow through an arrangement of CNTs inside a silicon matrix using non-equilibrium molecular dynamics (MD) simulations. They investigated effects of different parameters such as CNT type, CNT diameter and pressure gradient on water transport through CNTs.

To our knowledge, dynamic behavior of nano spherical shell under thermomechanical shock has not yet been

J. Ranjbarn and A. Alibeigloo are with the Department of Mechanical Engineering, Tarbiat modares University, Tehran, Iran (e-mail: abeigloo@ modares.ac.ir).

investigated. Therefore, this first known solution provides an important benchmark for future assessing the validity of conventional two-dimensional theory. In this work, the governing equations are derived by employing nonlocal elasticity theory with assumption of temperature dependence of termomechanical properties of nano sphere.

II. Governing Equations

A. A Review on Nonlocal Elasticity Theory

According to nonlocal elasticity theory the stress tensor at arbitrary point "q" of a nano material body not only depends on strain tensor at "q" but also depends on all points of body. The nonlocal elasticity basic equation for isotropic, elastic and homogenous materials in the absence of body force is expressed as follow:

$$t_{ij}(q)=\int\varphi(|q-q'|,\eta)\sigma_{ij}(q)dV \tag{1}$$

where t_{ij} is component of nonlocal stress tensor and σ_{ij} is component of classical local stress $\boldsymbol{\sigma}$ at point q. $\varphi(|q-q'|,\eta)$ is nonlocal modulus function which contains the small scale effect. This function depends on $|q-q'|$, μ that $|q-q'|$ is the distance between two points q, q' and μ is the material constant that is defined as:

$$\eta=\frac{e_0 a}{L} \tag{2}$$

where "a" is internal characteristic length as length of C-C bound and "L" is external characteristic length as crack length and e_0 is constant exclusivity of each material.

By employing Eringens' nonlocal formulation, the nonlocal stress tensor "t" can be expressed as:

$$(1-\mathrm{K}^2\nabla^2)\mathbf{t}=\boldsymbol{\sigma}\,,\ \mathrm{K}=e_0 a \tag{3}$$

where $\boldsymbol{\sigma}$ is local stress tensor, ∇^2 is Laplacian operator and "K" is nonlocal parameter.

By balancing the linear momentum, the equation of motion in invariant form can be said as follow:

$$\nabla.\mathbf{t}+\mathbf{f}=\rho\ddot{\mathbf{u}} \tag{4}$$

where $\mathbf{f}$ is body force, $\mathbf{u}$ is displacement vector, ∇ and "$\cdot\cdot$" denotegradient operator and derivation relative to time respectively and ρ is density of material. After using (3) and (4) , the invariant form of nonlocal equation of motion can be derived as follow:

$$\nabla.\boldsymbol{\sigma}+\mathbf{f}=(1-\mathrm{K}^2\nabla^2)\rho\ddot{\mathbf{u}} \tag{5}$$

B. Problem Formulations

In the absence of body force and using (5) in spherical coordinate system, the nonlocal equation of motion for a spherically symmetric model of spherical shell can be written as:

$$\frac{\partial\sigma_r(r,t)}{\partial r}+\frac{2}{r}(\sigma_r(r,t)-\sigma_\theta(r,t)) =(1-\mathrm{K}^2\nabla^2)[\rho\frac{\partial^2 u_r(r,t)}{\partial t^2}] \tag{6}$$

where σ_r, σ_θ, u_r and t are radial stress, hoop stress, radial displacement and time, respectively and ∇^2 is Laplacian operator for an spherically symmetric model of a spherical shell that defined as follow:

$$\nabla^2=\frac{\partial^2}{\partial r^2}+\frac{2}{r}\frac{\partial}{\partial r} \tag{7}$$

Also, strain- displacement relations are:

$$\varepsilon_r(r,t)=\frac{\partial u_r(r,t)}{\partial r}\quad,\quad \varepsilon_\theta(r,t)=\frac{u_r(r,t)}{r} \tag{8}$$

where, ε_r and ε_θ are radial and hoop strains respectively.

Local thermoelastic stress tensor $\boldsymbol{\sigma}$ is related to the strain by generalized Hook's law:

$$\begin{Bmatrix}\sigma_r(r,t)\\ \sigma_\theta(r,t)\end{Bmatrix}= \begin{bmatrix}c_{11} & c_{12}\\ c_{12} & c_{22}\end{bmatrix}\left(\begin{Bmatrix}\varepsilon_r(r,t)\\ \varepsilon_\theta(r,t)\end{Bmatrix}-\begin{Bmatrix}(1+v)\alpha(r,t)\\ (1+v)\alpha(r,t)\end{Bmatrix}T(r,t)\right) \tag{9}$$

where

$$c_{11}=\frac{E(1-v)}{(1+v)(1-2v)}\,,\qquad c_{12}=\frac{Ev}{(1+v)(1-2v)} \tag{10}$$

and $E,\alpha(r,t),v$ are elasticity modulus, thermal expansion coefficient and Poisson's ratio, respectively. In this paper, we assumed that thermal expansion coefficient is dependent to temperature. By substituting (8), and (10), into (9) stress-displacement relations can be derived as:

$$\sigma_r(r,t)=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{\partial u_r(r,t)}{\partial r}+\nu\frac{u_r(r,t)}{r}]-\frac{E\alpha(r,t)}{1-2\nu}T(r,t) \quad (11)$$

$$\sigma_\theta(r,t)=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{u_r(r,t)}{r}\nu\frac{\partial u_r(r,t)}{\partial r}]-\frac{E\alpha(r,t)}{1-2\nu}T(r,t)$$

Substitution of (11), into (6), nonlocal Navier's equation of motion can be obtained as follow:

$$\frac{\partial^2 u_r(r,t)}{\partial r^2}+\frac{2}{r}\frac{\partial u_r(r,t)}{\partial r}-\frac{2}{r^2}u_r(r,t)-\frac{1+\nu}{1-\nu}\frac{\partial\left(\alpha(r,t)\times T(r,t)\right)}{\partial r}=\frac{(1+\nu)(1-2\nu)}{E(r)(1-\nu)}(1-\mathrm{K}^2\nabla^2)\left(\rho\frac{\partial^2 u_r(r,t)}{\partial t^2}\right) \quad (12)$$

Initial and boundary conditions related to (12) are assumed to be:

$$u_r(r,t)=0 \quad , \quad \frac{\partial u_r(r,t)}{\partial t}=0 \quad , \quad t=0 \quad (13)$$

$$\sigma_r(r_1,t)=P_{in}(t),\sigma_r(r_2,t)=P_{out}(t) \quad (14)$$

where r_1 and r_2 are inner and outer radius of sphere, respectively. Also, $P_{in}(t)$ and $P_{out}(t)$ are uniform pressure at the inner and outer surfaces, respectively.

By using (11), boundary conditions, (14), can be written as follow:

$$P_{in}(t)=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{\partial u_r(r_1,t)}{\partial r}+\nu\frac{u_r(r_1,t)}{r_1}]-\frac{E\alpha(r_1,t)}{1-2\nu}T(r_1,t) \quad (15)$$

$$P_{out}(t)=\frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{\partial u_r(r_2,t)}{\partial r}+\nu\frac{u_r(r_2,t)}{r_2}]-\frac{E\alpha(r_2,t)}{1-2\nu}T(r_2,t)$$

For convenience, following non-dimensional parameters are employed:

$$R=\frac{r_1+r_2}{2},\ \xi=\frac{r}{R},\ \bar{E}=\frac{E}{E_0},\ \bar{\rho}=\frac{\rho}{\rho_0},$$
$$\bar{\alpha}(\xi,\bar{t})=\frac{\alpha(r,t)}{\alpha_0},\ c_v=\sqrt{E_0/\rho_0},\ \kappa=\frac{\mathrm{K}}{R},\ \bar{t}=\frac{c_v}{R}t,$$
$$\bar{u}_r(\xi,\bar{t})=\frac{u_r(r,t)}{T_0R\alpha_0},\ \bar{T}(\xi,\bar{t})=\frac{T(r,t)}{T_0},\quad (16)$$
$$\bar{\sigma}_r(\xi,\bar{t})=\frac{\sigma_r(r,t)}{T_0E_0\alpha_0},\ \bar{\sigma}_\theta(\xi,\bar{t})=\frac{\sigma_\theta(r,t)}{T_0E_0\alpha_0},$$
$$\xi_1=\frac{r_1}{R},\ \xi_2=\frac{r_2}{R}$$

where T_0 is reference temperature and E_0, α_0, ρ_0 and c_v are, reference value of elasticity modulus, thermal expansion coefficient, density and stress wave propagation, respectively at reference value respectively.

By employing (16)", dimensionless form of (12) and (15) can be written as:

$$\frac{\partial^2\bar{u}_r(\xi,\bar{t})}{\partial\xi^2}+\frac{2}{\xi}\frac{\partial\bar{u}_r(\xi,\bar{t})}{\partial\xi}-\frac{2}{\xi^2}\bar{u}_r(\xi,\bar{t})-\frac{1+\nu}{1-\nu}\frac{\partial\left(\bar{\alpha}(\xi,\bar{t})\times\bar{T}(\xi,\bar{t})\right)}{\partial\xi}=\frac{(1+\nu)(1-2\nu)}{\bar{E}(r,t)(1-\nu)}(1-\kappa^2\bar{\nabla}^2)\left(\bar{\rho}\frac{\partial^2\bar{u}_r(\xi,\bar{t})}{\partial\bar{t}^2}\right) \quad (17)$$

$$\bar{P}_{in}(\bar{t})=\frac{\bar{E}}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{\partial\bar{u}_r(\xi_1,\bar{t})}{\partial\xi}+\nu\frac{\bar{u}_r(\xi_1,\bar{t})}{\xi_1}]-\frac{\bar{E}\bar{\alpha}(\xi_1,\bar{t})}{1-2\nu}\bar{T}(\xi_1,\bar{t}) \quad (18)$$

$$\bar{P}_{out}(\bar{t})=\frac{\bar{E}}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{\partial\bar{u}_r(\xi_2,\bar{t})}{\partial\xi}+\nu\frac{\bar{u}_r(\xi_2,\bar{t})}{\xi_2}]-\frac{\bar{E}\bar{\alpha}(\xi_2,\bar{t})}{1-2\nu}\bar{T}(\xi_2,\bar{t})$$

where:

$$\bar{\nabla}^2=\frac{\partial^2}{\partial\xi^2}+\frac{2}{\xi}\frac{\partial}{\partial\xi},$$
$$\bar{P}_{in}(\bar{t})=\frac{P_{in}(t)}{T_0E_0\alpha_0},\quad (19)$$
$$\bar{P}_{out}(\bar{t})=\frac{P_{out}(t)}{T_0E_0\alpha_0}$$

III. Solution Procedure

By employing Laplace transform to (17), and (18), and considering initial conditions, (13), following equations will be derived:

$$\frac{\partial^2 U(\xi,s)}{\partial \xi^2}+\frac{2}{\xi}\frac{\partial U(\xi,s)}{\partial \xi}$$
$$-\frac{2}{\xi^2}U(\xi,s)-\frac{1+\nu}{1-\nu}\frac{\partial\left(\bar{\alpha}(\xi,s)\times\theta(\xi,s)\right)}{\partial \xi} \quad (20)$$
$$=\frac{(1+\nu)(1-2\nu)}{\bar{E}(1-\nu)}(1-\kappa^2\bar{\nabla}^2)\left(\bar{\rho}s^2U(\xi,\bar{t})\right)$$

$$G_{in}(s)=$$
$$\frac{\bar{E}}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{\partial U(\xi_1,\bar{t})}{\partial\xi}+\nu\frac{U(\xi_1,s)}{\xi_1}] \quad (21)$$
$$-\frac{\bar{E}\bar{\alpha}(\xi,s)}{1-2\nu}\theta(\xi_1,s)$$
$$G_{out}(s)=$$
$$\frac{\bar{E}}{(1+\nu)(1-2\nu)}[(1-\nu)\frac{\partial U(\xi_2,s)}{\partial\xi}+\nu\frac{U(\xi_2,s)}{\xi_2}]$$
$$-\frac{\bar{E}\bar{\alpha}(\xi,s)}{1-2\nu}\theta(\xi_2,s)$$

where:

$$\begin{aligned} U(\xi,s)&=L(\bar{u}_r(\xi,\bar{t}))\\ G_{in}(s)&=L(\bar{P}_{in}(\bar{t}))\\ G_{out}(s)&=L(\bar{P}_{out}(\bar{t}))\\ \theta(\xi,s)&=L(\bar{T}(\xi,\bar{t}))\\ \bar{\alpha}(\xi,s)&=L(\bar{\alpha}(\xi,\bar{t}))\end{aligned} \quad (22)$$

where L denotes Laplace transform operator. By applying power series method for solution of differential equations, (20), can be solve the Teylor's series at $\xi=1$ as follow:

$$\bar{U}(\xi,s)=\sum_{n=0}\phi_n(s)(\xi-1)^n \quad (23)$$

Substitution of (23), into (20), results in the following recurrence relation will be obtained:

$$\begin{aligned}&(n+1)(n+2)(1+\kappa^2\psi)\phi_{n+2}(s)=\\ &\quad\psi\phi_{n-2}(s)+2\psi\phi_{n-1}(s)\\ &\quad-\left(n(n+1)(1+\kappa^2\psi)-2-\psi\right)\phi_n(s)\\ &\quad-2(n+1)^2(1+\kappa^2\psi)\phi_{n+1}(s)\\ &\quad-(\frac{1+\nu}{1-\nu})\times\sum_{i=0}^{n}[(i-1)\Psi_{i-1}(s)+2i\Psi_i(s)\\ &\quad+(i+1)\Psi_{i+1}(s)]\mathrm{A}_{n-i}(s)\\ &\quad-(\frac{1+\nu}{1-\nu})\times\sum_{i=0}^{n}[\Psi_{i-2}(s)+2\Psi_{i-1}(s)\\ &\quad+\Psi_i(s)]\mathrm{B}_{n-i}(s)\ ,\ n\geq 0\end{aligned} \quad (24\text{-a})$$

where:

$$\phi_{-1}(s)=\phi_{-2}(s)=0 \quad (24\text{-b})$$
$$\psi=\frac{(1+\nu)(1-2\nu)}{(1-\nu)}\frac{\bar{\rho}}{\bar{E}}s^2 \quad (24\text{-c})$$
$$\theta(\xi,s)=\sum_{n=0}\Psi_n(s)(\xi-1)^n \quad (24\text{-d})$$
$$\bar{\alpha}(\xi,s)=\Lambda_1(\xi,s)=\sum_{n=0}a_n(s)(\xi-1)^n \quad (24\text{-e})$$
$$\frac{\partial\bar{\alpha}(\xi,s)}{\partial\xi}=\Lambda_2(\xi,s)=\sum_{n=0}b_n(s)(\xi-1)^n \quad (24\text{-f})$$

where:

$$\mathrm{A}_n(s)=\frac{1}{n!}\Lambda_1^{(n)}(s,\xi=1)$$

and:

$$\mathrm{B}_n(s)=\frac{1}{n!}\Lambda_2^{(n)}(s,\xi=1)$$

From (24-a) and some manipulation it can be concluded that $\phi_n(s)$ is a linear combination of ϕ_0 and ϕ_1 which can be expressed as:

$$\phi_n(s)=\mathcal{X}_n(s)\phi_0+\mathcal{Y}_n(s)\phi_1+\mathcal{Z}_n(s) \quad (25)$$

In (25) ϕ_0 and ϕ_1 are unknown parameters which can be obtained by using surface boundary conditions, (21) , whereas $\mathcal{X}_n$, $\mathcal{Y}_n$ and $\mathcal{Z}_n(s)$ can be derived from recurrence relation (24-a). The concise form of radial non dimensional displacement is obtained by substituting (25) into (23) as follow:

$$\bar{U}(\xi,s)=\sum_{n=0}^{\infty}\left[\mathcal{X}_n(s)\phi_0+\mathcal{Y}_n(s)\phi_1+\mathcal{Z}_n(s)\right](\xi-1)^n \quad (26)$$

Applying FLIT method [12], that is combination of sine and cosine Fourier transform to (26), radial displacement can be derived in term of time domain. Finally, substitution of obtained displacement into (11) leads to the stress fields.

IV. Numerical Results and Discussions

To illustrate numerical results for dynamic behavior of nano scale spherical shell, under thermal and/or mechanical shock, the following specification for sphere is considered: $E=1.06$TPa , $r_1=2.3$ nm , $\rho=2.3$ gr/cm^3 and $\nu=0.35$ at reference temperature T_0 . For an example, we discuss dynamic behavior of nano sphere subjected to impulse temperature load. It is assumed that the temperature change of shell is as the follow:

$$T(r,t) = T_0 H(t) = \begin{cases} 0 & t < 0 \\ T_0 & t \geq 0 \end{cases} \qquad (27)$$

where $H(t)$ Heaviside is step function and T_0 is constant. From (16) , dimensionless form of (27) is as:

$$\bar{T}(\xi, \bar{t}) = H(\bar{t}) = \begin{cases} 0 & \bar{t} < 0 \\ 1 & \bar{t} \geq 0 \end{cases} \qquad (28)$$

It is noted that in thermal loading case, spherical shell has traction free inner and outer surfaces. To validate the accuracy of the present approach, we compute numerical results of local dimensionless hoop stress for an isotropic spherical shell ($\kappa = 0$) subjected to impulse load at inner surface that is considered by Wang and Ding [13]. According to the fig. 1, increases the number of series term cause to converge the results to the results of Wang and Ding [13]. From the figure exactness as well as rapid convergence can be observed. Discrepancy between the numerical results is due to method of solution adopted by Wang and Ding [13].

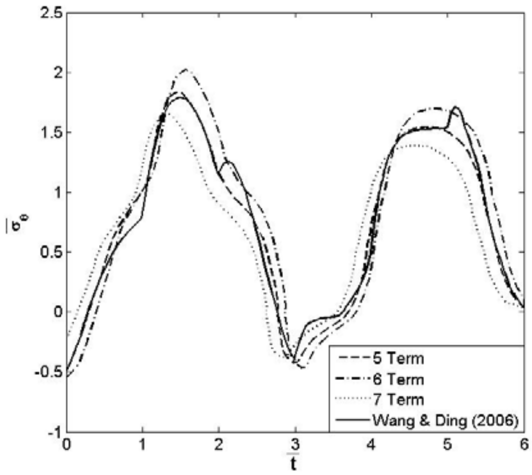

Fig. 1Time histories of hoop stress at inner surface for an elastic hollow sphere based on Wang and Ding [13] and presented methods

Fig. 2 shows time histories of radial displacement in mid radius of shell with $r_1 = 2.3$ nm and thickness $h = 0.02 r_1$ for different nonlocal parameter. From figure it is seen that increase nonlocal parameter results in to decrease the amplitude of radial dimensionless displacement.

The effect of nonlocal parameter on radial stress at mid-radius pf shell with $r_1 = 2.3$ nm and thickness $h = 0.02 r_1$ is showed in Fig.3. As the figure depicts, velocity of wave propagation decreases by increasing nonlocal parameter. Variations of non-dimensional hoop stress at mid-radius of shellwith $r_1 = 2.3$ nm and thickness $h = 0.02 r_1$ for various nonlocal parameters are shown in Fig. (4). As the figures show, increasing the nonlocal parameter causes to decrease of peak value of hoop stress. Also, it can be seen that several point of shell has similar hoop stress. Through the thickness distribution of radial displacement for various nonlocal parameters are depicted in Fig.5. According to the figure this distribution is linear and by increasing the nonlocal parameter the slope of this distribution decreases.

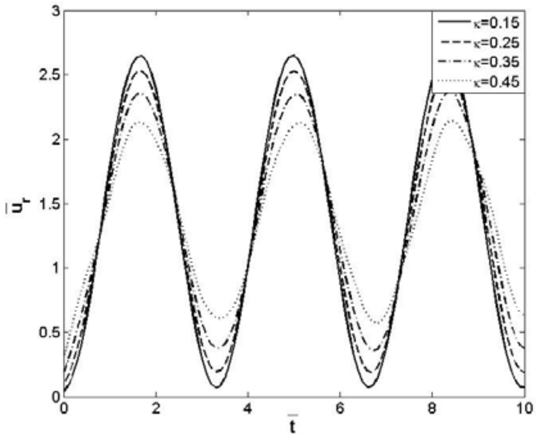

Fig.2 Effect of nonlocal parameter on time histories of non dimensional radial displacement at middle point of thickness

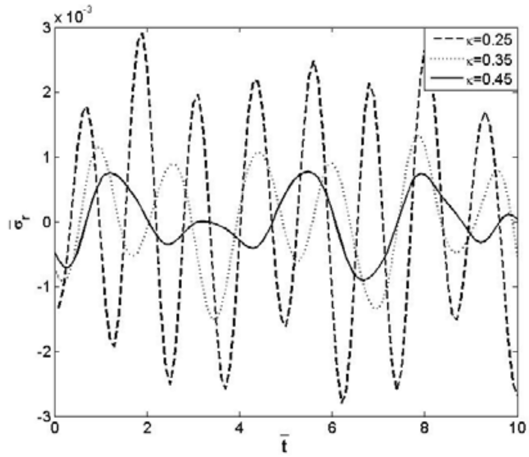

Fig. 3 Effect of nonlocal parameter on time histories of non dimensional radial stress at mid-radius of spherical shell

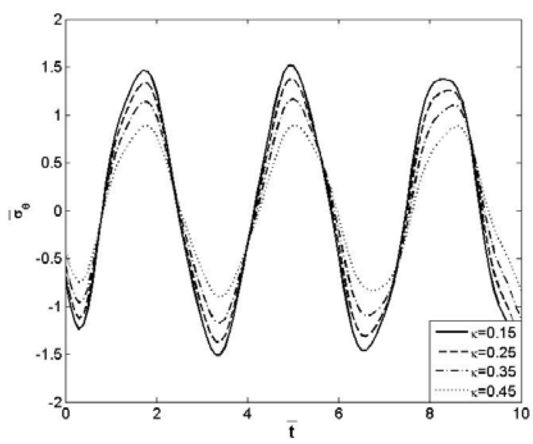

Fig. 4 Effect of nonlocal parameter on time histories of non-dimensional hoop stress at mid-radius of spherical shell

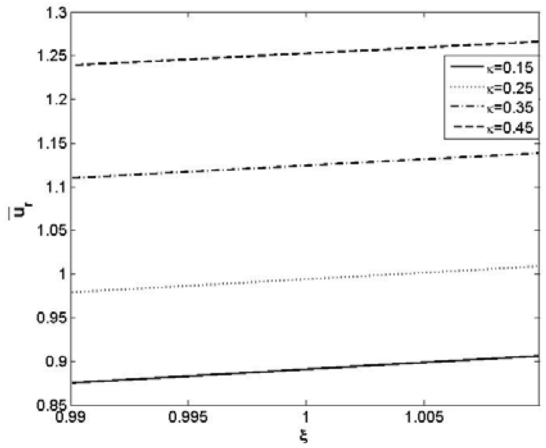

Fig. 5 Effect of nonlocal parameter on nondimentional displacement distribution for spherical shell with, $h = 0.02r_1$ at $\bar{t} = 6$

V. Conclusion

In this paper, we presented thermoelastodynamic solutions of nano spherical shell subjected to thermal and or pressure shock based on analytical method. The nonlocal governing equations of problem were transferred to Laplace domain by employing Laplace transform and then were solved analytically by using Taylor's expansion series. Displacement and stress field were derived in time domain by using FLIT method. Time histories and distribution of displacement, radial stress and hoop stress were obtained and plotted for various nonlocal parameter as well as wall thickness. From parametric study some conclusion can be made:

- The presented method is an effective solution to study time histories of stress field under any time dependent thermo-mechanical loads.
- Nonlocal parameter has significant effect on nanoscale structures. Increasing the nonlocal parameter causes to increase the elastic property and consequently decreasing value and wave propagation of stresses and displacement.
- Rate of variation of radial stress as well as circumferential stress decrease by increasing the nonlocal parameter.
- Increase nonlocal parameter causes to decrease velocity of stresses and displacement wave propagation.
- Radial stress varies along the thickness direction nonlinearly whereas the circumferential stress nearly is independent of radial coordinate.
- As the results show, the longitude of hoop stress is more than radial stress. An other words, hoop stress are more effective than radial stress in study of dynamic behavior of nano spherical shell under thermal shocks.

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