

# Advanced Numerical and Analytical Methods for Assessing Concrete Sewers and Their Remaining Service Life

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**Abstract**—Pipelines are extensively used engineering structures which convey fluid from one place to another. Most of the time, pipelines are placed underground and are encumbered by soil weight and traffic loads. Corrosion of pipe material is the most common form of pipeline deterioration and should be considered in both the strength and serviceability analysis of pipes.

The study in this research focuses on concrete pipes in sewage systems (concrete sewers). This research firstly investigates how to involve the effect of corrosion as a time dependent process of deterioration in the structural and failure analysis of this type of pipe. Then three probabilistic time dependent reliability analysis methods including the first passage probability theory, the gamma distributed degradation model and the Monte Carlo simulation technique are discussed and developed. Sensitivity analysis indexes which can be used to identify the most important parameters that affect pipe failure are also discussed.

The reliability analysis methods developed in this paper contribute as rational tools for decision makers with regard to the strengthening and rehabilitation of existing pipelines. The results can be used to obtain a cost-effective strategy for the management of the sewer system.

**Keywords**—Reliability analysis, service life prediction, Monte Carlo simulation method, first passage probability theory, gamma distributed degradation model.

## I. INTRODUCTION

**B**URIED pipes are subject to chemical and mechanical loading in their environment of service and these stresses cause failure that is costly to repair. Methods to predict pipe performance are poorly developed and require improvement, through the introduction of time dependent reliability analytical tools. In this paper the background and significance of reliability analysis and service life prediction for concrete sewers is established.

Pipelines are widely used engineering structures for collecting wastewater in urban and/or rural areas. Most of the time, pipelines are placed underground and are encumbered by soil weight and traffic loads. Evidently, underground pipelines are required to resist the influence of the external loads (soil and traffic) and internal fluid pressure [1]-[3].

In many cases underground pipelines are required to withstand particular environmental hazards. Corrosion of pipe material is the most common form of pipeline deterioration

and should be considered in both strength and serviceability analysis of buried pipes [4], [5].

It is known that sewer collapses are predominantly caused by deterioration of the pipes. For cementitious sewers, sulphide corrosion is the primary cause of these collapses [6] and [7]. In Los Angeles USA, approximately 10% of the sewer pipes are subject to significant sulphide corrosion, and the costs for the rehabilitation of these pipelines are roughly estimated at £325 million [8]. In the UK there is approximately 310,000 km of sewer pipes with an estimated total asset value of £110 billion [9]. The investment for repair and maintenance of this infrastructure is approximately £40 billion for the period 1990 to 2015 [10]. Looking at Europe, in Belgium the cost of sulphide corrosion of sewers is estimated to be £4 million per year, representing about 10% of the total cost for wastewater collection and treatment systems [11]. These statistics indicate that sewer systems are faced with high emergency repair and renewal costs, and frequent charges arising from increasing rates of deterioration. On the other hand, budget limitations are significantly restricting sewer systems and reducing their capabilities in terms of addressing these needs. Therefore to eliminate the high costs associated with sewer failures, sewer system managers need to generate proactive asset management strategies and prioritise inspection, repair and renewal needs of sewer pipes by utilising reliability analysis. The failure assessment and advanced reliability analysis methods can help asset managers to provide an improved level of service and publicity, gain approval and funding for capital improvement projects and manage operations and maintenance practices more efficiently [12], [13].

Large investments are required for building new wastewater collection systems. It is unlikely that the existing pipe networks will be replaced completely over a short period of time. Therefore the solution is to maintain and rehabilitate the existing pipes. Accurate prediction of the service life of pipes is essential to optimize strategies for maintenance and rehabilitation in the management of pipe assets. Service life (of building components or materials) is the period of time after installation during which all the properties exceed the minimum acceptable values when routinely maintained [14].

The basis for making quantitative predictions of the service life of structures is to understand the mechanisms and kinetics of many degradation processes of the material whether it is steel, concrete or other materials. Material corrosion in concrete sewers is a matter of concern for both strength and

serviceability functions. Loss of wall thickness through general corrosion affects the strength of the pipe. To that effect, incorporating the effect of corrosion into the structural analysis of a pipeline is of paramount importance. There are several parameters which may affect corrosion rate and hence the reliability of pipelines. To consider uncertainties and data scarcity associated with these parameters, various researches on the probabilistic assessment of buried pipes have been undertaken [13], [15]-[20].

Since the deterioration of buried pipelines is uncertain over time, it should ideally be represented as a stochastic process. A stochastic process can be defined as a random function of time in which for any given point in time the value of the stochastic process is a random variable depending on some basic random variables. Therefore a robust method for the reliability analysis and service life prediction of corrosion affected pipes should be a time dependent probabilistic (i.e., stochastic) method which considers the randomness of variables to involve uncertainties in a period of time.

In this paper, first the degradation process (i.e., corrosion) in concrete sewers is modelled and then advanced time dependent methods are discussed and developed for the reliability analysis of concrete sewers. To ensure a comprehensive study, procedures for sensitivity analysis are also discussed.

## II. CONCRETE CORROSION MODEL

The most corrosive agent that leads to the rapid deterioration of concrete pipelines in sewers is  $H_2S$ . Approximately 40% of the damage in concrete sewers can be attributed to biogenous sulphuric acid attack. There are many cases in which sewer pipes designed to last 50 to 100 years have failed due to  $H_2S$  corrosion in 10 to 20 years [21].

The rate of corrosion of a concrete sewer can be calculated from the rate of production of sulphuric acid on the pipe wall, which is in turn dependent upon the rate that  $H_2S$  is released from the surface of the sewage stream. The average flux of  $H_2S$  to the exposed pipe wall is equal to the flux from the stream into the air multiplied by the ratio of the surface area of the stream to the area of the exposed pipe wall, which is the same as the ratio of the width of the stream surface ( $b$ ) to the perimeter of the exposed wall ( $\hat{P}$ ). The average flux of  $H_2S$  to the wall is therefore calculated as follows [6]:

$$\Phi = 0.7(su)^{3/8}j[DS](b/\hat{P}) \quad (1)$$

where  $s$  is pipe slope,  $u$  is velocity of stream (m/s),  $j$  is pH-dependent factor for proportion of  $H_2S$  and  $[DS]$  is dissolved sulphide concentration (mg/lit). A concrete pipe is made of cement-bonded material, or acid-susceptible substances, so the acid will penetrate the wall at a rate inversely proportional to the acid-consuming capability ( $A$ ) of the wall material. The acid may partly or entirely react. The proportion of acid that reacts is variable ( $k$ ), ranging from 100% when the acid formation is slow, to perhaps 30% to 40% when it is formed rapidly. Thus, the average rate of corrosion (mm/year) can be calculated as follows:

$$c = \frac{11.5k\Phi}{A} (CCF)(TCF) \quad (2)$$

where  $k$  is the factor representing the proportion of acid reacting, to be given a value selected by the judgment of the engineer and  $A$  is the acid-consuming capability, alkalinity, of the pipe material, expressed as the proportion of equivalent calcium carbonate. Value for granitic aggregate concrete ranges from 0.17 to 0.24 and for aggregate concrete,  $A$  ranges from 0.9 to 1.1 [1]. CCF and TCF are the crown corrosion factor and turbulence corrosion factor, respectively. Substituting (1) into (2):

$$c = 8.05k(su)^{3/8}j[DS] \frac{b}{P'A} (CCF)(TCF) \quad (3)$$

Therefore corrosion depth in elapsed time  $t$ , is:

$$d(t) = \int_0^t c \cdot dt = 8.05k(su)^{3/8}j[DS] \frac{b}{P'A} (CCF)(TCF)t \quad (4)$$

## III. TIME DEPENDENT RELIABILITY ANALYSIS METHODS

Reliability analysis and the prediction of the service life of structures is one of the major challenges for infrastructure managers and structural engineers. Historically, the reliability theory has most often been introduced in the military, aerospace and electronics fields [22]. Over recent years, the significance of the reliability theory has been increasingly realised in the field of civil engineering. Structural reliability began as a subject for academic research about 50 years ago [23]. The topic has grown rapidly during the last three decades and has evolved from being a topic for academic research to a set of well-developed or developing methodologies with a wide range of practical applications.

Structural reliability can be defined as the probability that the structure under consideration has a sufficient performance throughout its service life. Reliability methods are used to estimate the service life of structures.

In addition to the prediction of initial service life, reliability methods are effective tools to evaluate the efficiency of repair and replacement. The impact of any repair and maintenance option upon the future performance of the structure can be evaluated by decision makers using reliability analysis methods.

Furthermore, the reliability analysis of a structure or a system can be used at the conceptual design stage to evaluate various design choices and to determine the impact that their implementation could have upon their service lives.

The uncertain nature of the loading and performance aspects of structures could have led the planners to probabilistic approaches for service life assessment. In probabilistic methods dealing with uncertainties, the safety and service/performance requirements are measured by their reliability. The reliability of a structure or a component is defined as its probability of survival [24]:

$$P_s = 1 - P_f \quad (5)$$

where

$P_s$ : Probability of survival

$P_f$ : Probability of failure

Failure can be defined in relation to different possible failure modes, commonly referred as limit states. Reliability is considered to be the probability that these limits will not be exceeded and is equal to the probability of survival. Each of the limit state function variables is attributed to a probability density function that presents its statistical properties.

To summarise, structural reliability analysis can be generally used for the following purposes:

- Service life prediction of existing structures, for funding allocation to most critical parts of the structure or infrastructure.
- Evaluation of the effect of repair, maintenance and rehabilitation actions on the service life of the structure (ability to examine the consequences of potential action or inaction relative to operational and maintenance procedures).
- To be used at the conceptual design stage to evaluate various design choices and to determine the impact that their implementation would have upon service life.

To predict the service life of existing structures, information is required on the present condition of the structure, rates of degradation and past and future loading, in addition to a definition of the failure of the structure. Based on remaining life predictions, cost-benefit analysis can also be made on whether or not a structure should be repaired, rehabilitated, or replaced.

When a structure is subjected to a time dependent degradation process, probabilistic time dependent methods can be used. In this paper three advanced methods for the time-dependent reliability analysis of corrosion affected pipes are presented. These methods will allow for:

- The application to all types of corrosion-affected structures and/or pipelines,
- The capability to consider multi failure mechanisms/modes,
- Consideration of the scarcity of monitoring data from real world examples.

The first passage probability theory has been introduced and developed for time dependent reliability analysis [24]-[26]. The gamma process concept has also successfully been used as a model for the reliability analysis of structures subject to monotonic degradation processes [20], [27] and [28]. Time-dependent Monte Carlo simulation methods have also been employed to quantify the probability of failure of concrete sewers [18]. These methods are discussed below for the reliability analysis of concrete sewers. By using these techniques a quantitative measure of structural reliability is provided to integrate information on design requirements, material and structural degradation, damage accumulation, environmental factors, and non-destructive evaluation technology.

Structural loads, engineering material properties, and strength-degradation mechanisms are random. The resistance,  $R(t)$ , of a structure and the applied loads,  $S(t)$  are both

stochastic functions of time. At any time,  $t$ , the safety limit state,  $G(R, S, t)$ , is [24]:

$$G(R, S, t) = R(t) - S(t) \quad (6)$$

The probability that failure occurs for any one load application is the probability of limit state violation.

#### IV. FIRST PASSAGE PROBABILITY METHOD

The service life of a pipe or structure in general is a time period at the end of which the pipe stops performing the functions it is designed and built for. With the limit state function of (6), the probability of pipe (structural) failure,  $P_f$ , can be determined by:

$$P_f(t) = P[G(t) \leq 0] = P[S(t) \geq R(t)] \quad (7)$$

At the time that  $P_f(t)$  is greater than a maximum acceptable risk in terms of the probability of pipe failure,  $P_a$ , the pipe becomes unsafe or unserviceable and requires replacement or repair. This can be determined from the following:

$$P_f(T_L) \geq P_a \quad (8)$$

where  $T_L$  is the service life for the pipe for the given assessment criterion and acceptable risk. In principle, the acceptable risk,  $P_a$ , can be determined from a risk-cost optimisation of the pipeline system during its whole service life. This is beyond the scope of this paper and will not be discussed herein but readers can be referred to [29] and [30].

Equation (8) represents a typical up-crossing problem in mathematics and can be dealt with using time-dependent reliability methods. In this method, the structural failure depends on the time that is expected to elapse before the first occurrence of the action process  $S(t)$  up-crossing an acceptable limit (the threshold)  $L(t)$  sometime during the service life of the structure  $[0, T_L]$ . Equally, the probability of the first occurrence of such an excursion is the probability of failure  $P_f(t)$  during that time period. This is known as "first passage probability" and can be determined by [24]:

$$P_f(t) = 1 - [1 - P_f(0)]e^{-\int_0^t v d\tau} \quad (9)$$

where  $v$  is the mean rate for the action process  $S(t)$  to up-cross the threshold  $R(t)$ . In many practical problems, the mean up-crossing rate is very small ( $e^{-\int_0^t v d\tau} \cong 1 - \int_0^t v d\tau$ ), so the above equation can be approximated as follows:

$$P_f(t) = p_f(0) + \int_0^t v d\tau \quad (10)$$

Because it is unlikely that the corrosion depth in a given pipe exceeds the wall thickness at the beginning of structural service, the probability of failure due to corrosion at  $t = 0$  is zero, i.e.,  $P_f(0) = 0$ ; therefore:

$$P_f(t) = \int_0^t v d\tau \quad (11)$$

The up-crossing rate in the above equation can be determined from the Rice formula [24]:

$$v = v_R^+ = \int_R^\infty (\dot{S} - \dot{R}) f_{SS}(R, \dot{S}) d\dot{S} \quad (12)$$

where  $v_R^+$  is the up-crossing rate of the action process  $S(t)$  relative to the threshold  $R$ ,  $\dot{R}$  is the slope of  $R$  with respect to time,  $\dot{S}(t)$  is the time derivative process of  $S(t)$  and  $f_{SS}(\cdot)$  is the joint probability density function for  $S$  and  $\dot{S}$ .

The solution for  $f_{SS}(R, \dot{S})$  for the special case when  $S(t)$  is a stationary normal process given by:

$$f_{SS}(R, \dot{S}) = \frac{1}{2\pi\sigma_S\sigma_{\dot{S}}} \exp\left\{-\frac{1}{2}\left[\left(\frac{R-\mu_S}{\sigma_S}\right)^2 + \frac{\dot{S}^2}{\sigma_{\dot{S}}^2}\right]\right\} \quad (13)$$

in which  $S(t)$  is normal distributed  $N(\mu_S, \sigma_S^2)$  and  $\dot{S}(t)$  is  $N(0, \sigma_{\dot{S}}^2)$ . The mean of  $\dot{S}(t)$  is zero for a stationary process. Noting that:

$$\int_0^\infty \dot{S} \exp\left(-\frac{\dot{S}^2}{2\sigma_{\dot{S}}^2}\right) d\dot{S} = \sigma_{\dot{S}}^2 \quad (14)$$

and substituting (13) into (12) and integrating produces [24]:

$$v_R^+ = \frac{1}{2\pi} \frac{\sigma_{\dot{S}}}{\sigma_S} \exp\left[-\frac{(R-\mu_S)^2}{2\sigma_S^2}\right] \quad (15)$$

#### V. GAMMA PROCESS CONCEPT

In order to model the monotonic progression of a deterioration process, the stochastic gamma process concept can be used for modeling the reduction of pipe wall thickness due to corrosion. The gamma process is a stochastic process with independent, non-negative increments having a gamma distribution with an identical scale parameter and a time-dependent shape parameter.

A stochastic process model, such as the gamma process, incorporates the temporal uncertainty associated with the evolution of deterioration [31]-[33].

The mathematical definition of the gamma process is given in (16). A random quantity  $d$  has a gamma distribution with shape parameter  $\alpha > 0$  and scale parameter  $\lambda > 0$  if its probability density function is given by:

$$Ga(d|\alpha, \lambda) = \frac{\lambda^\alpha}{\Gamma(\alpha)} d^{\alpha-1} e^{-\lambda d} \quad (16)$$

Let  $\alpha(t)$  be a non-decreasing, right continuous, real-valued function for  $t \geq 0$ , with  $\alpha(0) \equiv 0$ .  $\Gamma(\alpha)$  denotes the gamma function of  $\alpha$  with a mathematical definition of  $\Gamma(\alpha) = (\alpha - 1)!$ . The gamma process is a continuous-time stochastic process  $\{d(t), t \geq 0\}$  with the following properties:

1.  $d(0) = 0$  with probability one;
2.  $d(\tau) - d(t) \sim Ga(\alpha(\tau) - \alpha(t), \lambda)$  for all  $\tau > t \geq 0$ ;
3.  $d(t)$  has independent increments.

Let  $d(t)$  denote the deterioration at time  $t$ ,  $t \geq 0$ , and let the probability density function of  $d(t)$ , in accordance with the definition of the gamma process, be given by:

$$f_{d(t)}(d) = Ga(d|\alpha(t), \lambda) \quad (17)$$

with mean and variance as follows:

$$E(d(t)) = \frac{\alpha(t)}{\lambda} \quad (18)$$

$$Var(d(t)) = \frac{\alpha(t)}{\lambda^2} \quad (19)$$

A pipe is said to fail when its corrosion depth, denoted by  $d(t)$ , is more than a specific threshold ( $a_0$ ). Assuming that the threshold  $a_0$  is deterministic and the time at which failure occurs is denoted by the lifetime  $T$ , due to the gamma distributed deterioration, (17), the life time distribution can then be written as:

$$F(t) = Pr(T \leq t) = Pr(d(t) \geq a_0) = \int_0^{a_0} f_{d(t)}(d) dd = \frac{\Gamma(\alpha(t), a_0 \lambda)}{\Gamma(\alpha(t))} \quad (20)$$

where  $\Gamma(v, x) = \int_{t=x}^\infty t^{v-1} e^{-t} dt$  is the incomplete gamma function for  $x \geq 0$  and  $v > 0$ .

To model corrosion in a pipe, in terms of a gamma process, the question that remains to be answered is how its expected deterioration increases over time. The expected corrosion depth at time  $t$  may be modelled empirically by a power law formulation [4]:

$$\alpha(t) = ct^b \quad (21)$$

for some physical constants  $c > 0$  and  $b > 0$ .

Because there is often engineering knowledge available about the shape of the expected deterioration in terms of the exponential parameter  $b$  in (15), this parameter may be assumed constant.

In the event of expected deterioration in terms of a power law (i.e., (21)), the parameters corresponding to shape and scale parameters ( $\alpha$  and  $\lambda$ ) should be estimated. The steps for this purpose are:

Determining the approximate moments (mean and variance); we estimate values for  $\alpha$  and  $\lambda$  by using (18) and (19). Assuming  $X_1, X_2, \dots, X_n$  as basic random variables, moment approximation (i.e., step (a)) can be carried out by expanding the function  $Y = Y(X_1, X_2, \dots, X_n)$  in a Taylor series about the point defined by the vector of the means  $(\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n})$ . By truncating the series, the mean and variance are [34]:

$$E(Y) \approx Y(\mu_{X_1}, \mu_{X_2}, \dots, \mu_{X_n}) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 Y}{\partial X_i \partial X_j} cov(X_i, X_j) \quad (22)$$

$$var(Y) \approx \sum_i^n \sum_j^n c_i c_j cov(X_i, X_j) \quad (23)$$

#### VI. MONTE CARLO SIMULATION METHOD

Monte Carlo simulation has been successfully used for the reliability analysis of different structures and infrastructure [18], [24], [35]-[37]. Monte Carlo simulation techniques involve sampling at random to artificially simulate a large number of experiments and to observe the results. To use this method in structural reliability analysis, a value for each

random variable is selected randomly ( $\hat{x}_i$ ) and the limit state function ( $G(\hat{x})$ ) is checked. If the limit state function is violated (i.e.  $G(\hat{x}) \leq 0$ ), the structure or the system has failed. The experiment is repeated many times, each time with randomly chosen variables. If  $N$  trials are conducted, the probability of failure then can be estimated by dividing the number of failures to the total number of iterations:

$$P_f \approx \frac{n(G(\hat{x}) \leq 0)}{N} \quad (24)$$

The accuracy of Monte Carlo simulation results depends on the sample size generated and, in cases when the probability of failure is estimated, on the value of the probability (the smaller the probability of failure, the larger the sample size needed to ensure the same accuracy). The accuracy of the failure probability estimates can be checked by calculating their coefficient of variation (e.g., [24]).

In order to improve the accuracy of estimating the probability of ultimate strength failure, while keeping the computation time within reasonable limits, variance reduction techniques (e.g., importance sampling, Latin hypercube, and directional simulation) can be employed. However, in cases that the main emphasis is on serviceability failure, which can be estimated by a crude Monte Carlo simulation with very good accuracy within a relatively short computation time, it is not necessary to use such techniques [38].

Importance sampling is a variance reduction technique that can be used in the Monte Carlo method [24]. The idea behind importance sampling is that certain values of the input random variables in a simulation have more impact on the parameter being estimated than others. If these "important" values are emphasised by sampling more frequently, then the estimator variance can be reduced. Hence, the basic methodology in importance sampling is to choose a distribution which "encourages" the important values. The use of "biased" distributions will result in a biased estimator if it is applied directly in the simulation. However, the simulation outputs are weighted to correct use of the biased distribution, and this ensures that the new importance sampling estimator is unbiased.

The fundamental issue in implementing importance sampling simulation is the choice of the biased distribution which encourages the important regions of the input variables. Choosing or designing a good biased distribution is the "art" of importance sampling. The rewards for a good distribution can be significant run-time savings; the penalty for a bad distribution can be longer run times than for a general Monte Carlo simulation without importance sampling.

Details of the Monte Carlo method including sampling techniques can be found in [24], [39] and [40].

## VII. SENSITIVITY ANALYSIS METHODS

Sensitivity analysis is widely accepted as a necessary part of the reliability analysis of structures and infrastructure. The effect of variables on the reliability of a pipeline can be analysed by undertaking a comprehensive sensitivity analysis.

Sensitivity analysis should be carried out to provide quantitative information necessary for classifying random variables according to their importance. These measures are essential for the reliability-based service life prediction of deteriorating materials and structures.

Sensitivity analysis provides the degree of variation of limit state functions or measures at a specific point characterised by a realisation of all random variables. Similar to the conventional sensitivity measure in the reliability approaches, the sensitivity measure,  $S$ , can be defined as follows [41]:

$$S_{G(X)}(X_i) = \frac{\partial G(X)}{\partial X_i} = \lim_{\epsilon \rightarrow 0} \frac{G(X+\epsilon) - G(X)}{\epsilon} \quad (25)$$

where  $G$  is a performance function of  $X$ ;  $X$  and  $\epsilon$  are vectors; and  $\epsilon$  is a small perturbation. An element  $X_i$  of  $X$  can be any type of variable or parameter. For instance, it can be a mean or a standard deviation of a variable, or a deterministic parameter. For a complex system, the sensitivity measure can be computed by using the numerical differentiation method rather than by an analytical approach [41].

Sensitivity analysis is the study of how the variation in the output of a model (numerical or otherwise) can be apportioned, qualitatively or quantitatively, to different sources of variation [42]. Among the reasons for using sensitivity analysis are:

- To identify the factors that have the most influence on the reliability of the pipe.
- To identify factors that may need more research to improve confidence in the analysis.
- To identify factors that are insignificant to the reliability analysis and can be eliminated from further analysis.
- To identify which, if any, factors or groups of factors interact with each other.

Different sensitivity indexes have been introduced. In this paper, those which have been used in reliability analysis of concrete sewers [43] are discussed.

### Relative Contribution

A sensitivity index that can be used in a comprehensive reliability analysis is the relative contribution of each variable in limit state function. The relative contribution ( $\alpha_x^2$ ) of each random variable ( $x$ ) to the variance of the limit state function is introduced as follows [44]:

$$\alpha_x^2 = \frac{\left(\frac{\partial G}{\partial x} \sigma_x\right)^2}{\sigma_G^2} \quad (26)$$

where  $\sigma_x$  is standard deviation of the random variable  $x$  and  $\sigma_G^2$  is the variance of the limit state function. Variables with higher values of  $\alpha_x^2$  contribute more in limit state function than other variables; therefore more focus and study needs to be carried out to determine the accurate values for such variables.

### Sensitivity Ratio (SR)

A method of sensitivity analysis applied in many different models in science, engineering, and economics is the Sensitivity Ratio (SR), also known as the elasticity equation. The ratio is equal to the percentage change in output (e.g.,

probability of failure) divided by the percentage change in input for a specific input variable, as shown in the following equation [45]:

$$SR = \frac{\left(\frac{Y_2 - Y_1}{Y_1}\right) \times 100\%}{\left(\frac{X_2 - X_1}{X_1}\right) \times 100\%} \quad (27)$$

where,  $Y_1$  = the baseline value of the output variable using baseline values of input variables

$Y_2$  = the value of the output variable after changing the value of one input variable

$X_1$  = the baseline point estimate for an input variable

$X_2$  = the value of the input variable after changing  $X_1$

Risk estimates are considered most sensitive to input variables that yield the highest absolute value for SR. The basis for this equation can be understood by examining the fundamental concepts associated with partial derivatives. In fact, SR is equivalent to normalised partial derivatives. Variables with higher values of sensitivity ratios are more effective on the limit state function or the probability of failure [45].

Alani and Mahmoodian (2014), [43], conducted a comprehensive study on the sensitivity analysis of a corrosion affected concrete sewer in the UK. Their study showed that among eight random variables, alkalinity of concrete and relative depth of the flow has the most effect on the probability of sewer failure. The analysis showed a less significant contribution of other variables in failure functions. Results of sensitivity analysis confirmed that it would not be necessary to consider those parameters as random variables and they can be treated as deterministic constant values for further studies.

### VIII. CONCLUSION

While the process of corrosion is time dependent, the reliability analysis methods that are used for corrosion affected pipes should also be time dependent. Three appropriate methods for the reliability analysis of concrete sewers were discussed in this paper.

A critical analysis of methods for the reliability analysis and service life prediction of concrete sewers was presented. The evaluation of the contributions of various uncertain parameters associated with pipeline life assessment can be carried out by using sensitivity analysis techniques. The effectiveness and contribution of corrosion parameters on the service life of concrete sewers can be analysed by using sensitivity indexes.

The output of an accurate and comprehensive reliability analysis will help infrastructure managers and design professionals to predict the service life of pipeline systems and to optimise material selection and design parameters for designing pipelines with longer service life.

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