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Optical and Double Folding Model Analysis for Alpha Particles Elastically Scattered from ^{9}Be and ^{11}B Nuclei at Different Energies

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***Abstract*—**Elastic scattering of α-particles from ^{9}Be and ^{11}B nuclei at different alpha energies have been analyzed. Optical model parameters (OMPs) of α-particles elastic scattering by these nuclei at different energies have been obtained. In the present calculations, the real part of the optical potential are derived by folding of nucleon-nucleon (NN) interaction into nuclear matter density distribution of the projectile and target nuclei using computer code FRESCO. A density-dependent version of the M3Y interaction (CDM3Y6), which is based on the G-matrix elements of the Paris NN potential, has been used. Volumetric integrals of the real and imaginary potential depth (J_R, J_W) have been calculated and found to be energy dependent. Good agreement between the experimental data and the theoretical predictions in the whole angular range. In double folding (DF) calculations, the obtained normalization coefficient Nr is in the range 0.70–1.32.

***Keywords*—**Elastic scattering of α-particles, optical model parameters, double folding model, nucleon-nucleon interaction.

I. Introduction

IT is known that the interaction between two nuclei A1, A2 is a many body problem, but within the framework of optical model (OM) we can reduce the many body problem into one-body problem of reduced mass μ in a potential well V (r) created by all the other nucleons exit in the projectile and target nuclei. There are various ambiguities associated with the interaction potential parameters derived by the phenomenological method (OM), such as discrete and continuous ambiguity. So, it is better to derive the interaction potential using microscopic method as double folding (DF) model [1]. Elastic alpha-nucleus scattering processes are generally described by the optical model employing complex Woods-Saxon potentials whose parameters are adjusted to reproduce the scattering data. Usually reasonable fits to the experimental data are obtained and the observed energy and mass dependences are well described [2]. During the past decades, the double-folding model [3] has been widely used to generate the real parts of both theα -nucleus and heavy-ion (HI) optical potentials. It is simple to see that folding model generates the first order term in the expression for the microscopic optical potential that is derived from Feshbach's theory of nuclear reactions [4]. The success of this approach in describing the observed elastic scattering of many systems suggests that the first-order term of the microscopic optical potential is indeed the dominant part of the real HI optical potential [5]. The basic inputs for a folding calculation are the nuclear densities of the colliding nuclei and the effective nucleon-nucleon (*NN*) interaction. A popular choice for the effective *NN* interaction has been one of the M3Y interactions which were designed to reproduce the *G*-matrix elements of the Reid [6] and Paris [7] *NN* potentials in an oscillator basis. These density independent M3Y interactions have been used with some success in folding model calculations of the HI optical potential at relatively low energies [3], where the data are sensitive to the potential only at the surface. However, in cases of refractive nuclear scattering, characterized by the observation of rainbow features [8]-[13], the scattering is sensitive to the optical potential over a wider radial domain and the simple M3Y-type interaction failed to give a good description of the data. This has motivated the inclusion of an explicit density dependence into the original M3Y interactions [14], to account for the reduction in the attractive strength of the effective *NN* interaction that occurs as the density of the nuclear medium increases. A Hartree - Fock study of nuclear matter (NM) [15], [16] has also shown that, as expected [17], the original density independent M3Y interaction [4], [5] failed to saturate cold NM, leading to collapse. Therefore, several parameterizations of the density dependence (DD) for the M3Y interactions were introduced [15], [16], [18] in order to reproduce the observed NM saturation density and binding energy. Although different versions of the (DD) give, by design, the same saturation values, they do result in different values of the nuclear incompressibility *K*. These density dependences of the M3Y interaction have been carefully tested in the folding analysis of refractive α-nucleus and light HI elastic scattering [15], [16], [18], [19], and one was able to conclude from these studies that *K* values ranging from 240 to 270 MeV are the most appropriate for the cold nuclear matter [18]. N. Burtebaev et al. [20] measured the differential cross sections for elastic and inelastic scattering of α- particles on ^{11}B nuclei at energies of 40 and 50 MeV in the entire angular range. The measured angular distributions were analyzed in terms of the optical model, the distorted-wave born approximation (DWBA), and the coupled-channel method. Optical model potentials and quadrupole (β2) and hexadecapole (β4) deformation parameters were found from this analysis. The rise in the cross sections at backward angles was shown to be associated with the transfer mechanism of the heavy ^{7}Li cluster. H. Abele et al. [2] measured the elastic α-scattering on some light nuclei in the mass region A=11-24 at

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incident energies E_α =48.7 and54.1 MeV. The data were analyzed using different phenomenological optical potentials of the Woods-Saxon and Michel type. Special emphasis is paid to the application of the double-folding concept. This procedure determines the real part of the potential. For the imaginary part, we reduce the constraint imposed on the potential shape by a "model-independent" analysis, expressing this term in sums of Fourier Bessel functions. J. D. Goss et al. [21] studied the elastic scattering of α-particles by ^{9}Be in the bombarding energy range of 1.7- 6.2 MeV. The aim of the present work is establishing reliable values for the parameters of interaction potentials for the elastic scattering of nuclear systems ^{4}He with ^{9}Be and ^{11}B at different energies and also due to the role to be played by these nuclei in nuclear technology, nuclear energy, and astrophysics. The data analyzed using both phenomenological optical potential and also double folding (DF) with normalization coefficient for the CDM3Y6 microscopic potential.

II. Theoretical Calculations

In a nuclear reaction, the form of a potential, which represents interaction between the projectile and the target nucleus, must be appropriate to the elastic scattering and the reactions take place between the projectile and the target. The real part and also imaginary volume part of the optical potential in this phenomenological analysis of the following systems assumed to be taken the woods Saxon form factor. Thus, the optical potential can be written as

$$U_{op}(r) = V_C(r) - V(r) - iW_V(r) \tag{1}$$

The first term is the Coulomb potential was assumed to be that between two uniform charge distributions with radii consistent with electron scattering.

$$V_C(r) = \frac{z_p z_t e^2}{2R_C}\left(3 - \frac{r^2}{R_C}\right) \text{ for } r \le R_C$$

$$V_C(r) = \frac{z_p z_t e^2}{r} \text{ for } r \succ R_C \tag{2}$$

The real part has the following form:

$$V(r) f(r, r_v, a_v) = V_o\left[1 + \exp\left(\frac{r - r_v}{a_v}\right)\right]^{-1} \tag{3}$$

The imaginary volume part has the following form:

$$W_V(r) f(r, r_w, a_w) = -W_o\left[1 + \exp\left(\frac{r - r_w}{a_w}\right)\right]^{-1} \tag{4}$$

So, the interaction potential can be rewritten as

$$U_{op}(r) = V_C(r) - V_o\left[1 + \exp\left(\frac{r - r_v}{a_v}\right)\right]^{-1} - iW_o\left[1 + \exp\left(\frac{r - r_w}{a_w}\right)\right]^{-1} \tag{5}$$

A. Double-Folding Model Calculations

Effective nucleon-nucleon interactions have been used to generate microscopic real potentials which, associated with phenomenological imaginary terms, successfully describe the elastic-scattering data at low and intermediate energies. The degree of success of the model is indicated by the potential renormalization required to give an optimum fit to the measurements. This renormalization should be close to unity. The real part of the optical potential is calculated from a more fundamental basis by the folding method in which the NN interaction $V_{NN}(r)$ is folded into the densities of both the projectile and target nuclei [3].

$$V^{DF}(R) = N_r \int \rho_t(r_2)\rho_P(r_1) V_{NN}(r_{pt}) dr_P dr_t \tag{6}$$

where N_r is a free renormalization factor, $\rho_p(r_1)$ and $\rho_t(r_2)$ are the nuclear matter density distributions of both the projectile and target nuclei, respectively, and V_{NN} is the NN potential , $r_{pt}= R+r_2-r_1$. A popular choice for the effective NN-interaction has been one of the M3Y-interactions. In the present folding calculation, the effective NN-interaction is taken according to the form of radial shape of the M3Y-Paris interaction which is given in terms of three Yukawa's [7] as:

$$V_D(S) = 11061.625\frac{\exp(-4S)}{4S} - 2537.5\frac{(-2.5S)}{2.5S} \tag{7}$$

$$V_{EX}(S) = -1524.25\frac{\exp(-4S)}{4S} - 518.75\frac{\exp(-2.5S)}{2.5S} - 7.8474\frac{\exp(-0.7072\,S)}{0.7072\,S} \tag{8}$$

The nuclear matter density distribution of ^{4}He is expressed in a modified form of the Gaussian shape as:

$$\rho_P(r) = \rho_O\left(1 + wr^2\right)\exp\left(-\beta r^2\right) \tag{9}$$

Where W=0.0, β=0.7024fm^{-2} and ρ_o = 0.4229 fm^{-3} for ^{4}He.

The nuclear density distribution for ^{9}Be and ^{11}B was calculated using the harmonic oscillator model (HO), where $\rho_t(r)$ was calculated from the relation [22]:

$$\rho_t(r) = \rho_O\left(1 + \alpha\left(\frac{r^2}{a^2}\right)\right)\exp\left(-\left(\frac{r^2}{a^2}\right)\right) \tag{10}$$

where (a=1.791, α=0.611 and ρ_o= 0.20909 fm^{-3}) for ^{9}Be and (a=1.69, α=0.811 and ρ_o= 0.1818 fm^{-3}) for ^{11}B.

In order to satu rate the nuclear matter (NM) these density-independent M3Y interaction equations (7) and (8) should be scaled with an explicit density-dependent function F (ρ) according to the following relation:

$$V_{D(EX)}(\rho,S) = F(\rho)\, V_{D(EX)}(S) \tag{11}$$

where V_D and $V_{(EX)}$ are the direct and exchange terms, respectively, derived from the M3Y interactions [6], [7] and s is the inter-nucleon separation; ρ is the density of the surrounding nuclear medium in which the two nucleons are embedded.

In this paper, the density-dependent version of the M3Y interaction (CDM3Y6), which is based on the G-matrix elements of the Paris NN potential, has been used and the density dependent function was taken in the following form:

$$F(\rho) = C\left[1 + \alpha \exp(-\beta\rho) - \gamma \rho^{n}\right] \quad (12)$$

has been proven to give good fits to a large set of elastic scattering data (including also elastic α -nucleus scattering) and it generates (in a Hartree–Fock scheme) the nuclear matter incompressibility K = 252 MeV.

The parameters C, α, β, γ and N given in Table I are taken from [18] and [23].

TABLE I
PARAMETERS OF DENSITY –DEPENDENCE FUNCTION F(P)

Model	C	α	β $_{(fm)^3}$	γ $_{(fm)^{3n}}$	N
CDM3Y6	0.2658	3.8033	1.4099	4.0	1

III. RESULTS AND DISCUSSIONS

A. Phenomenological and Semi-Microscopic Analysis of ^{9}Be (^{4}He,^{4}He)^{9}Be

The elastic scattering of α-particles on ^{9}Be at energies (28 MeV [24], (29, 40, 45, 50.5) MeV [25], 48 MeV [26]) from literature is shown in Fig. 1 is analyzed in order to obtain the global optical potential parameters, which could fairly reproduce the experimental measurements. For data analysis, both phenomenological approach and semi-microscopic approach using code FRESCO were used. The optical model analysis of the experimental data was performed by using woods-Saxon (WS) forms for both real and imaginary parts of the potential where the radii r_C=1.3fm, r_V=1.245fm and r_D=1.57fmwere fixed taken from [27], and only the four remaining parameters (V, W, a_V, a_W) were varied. The semi-microscopic analysis was performed by obtaining the real part from the folding procedure and using it with a WS term for the imaginary potential. The real part of optical potential is calculated from a more fundamental basis by the folding model in which effective nucleon-nucleon NN interaction is folded into the densities of both ^{4}He and ^{9}Be nuclei. Using code FRESCO [28] with normalization coefficient close to unity, $R=r_oA^{1/3}$.

TABLE II
OPTICAL AND DOUBLE FOLDING POTENTIAL PARAMETERS FOR ^{4}HE+^{9}BE NUCLEAR SYSTEM USING FRESCO CODE

E_α (MeV)	Model	V (MeV)	a_v (fm)	Nr	W_i (MeV)	r_i (fm)	a_i (fm)	J_R MeV.fm^3	J_W MeV.fm^3
28	OM	91.420	0.897		14.027	1.57	0.86	406.054	95.891
	DF			0.70	16.727		0.65		
29	OM	91.189	0.867		14.099	1.57	0.916	390.213	101.066
	DF			0.734	15.744		0.836		
40	OM	99.156	0.799		16.192	1.57	0.928	390.007	118.181
	DF			0.886	21.853		0.771		
45	OM	109.132	0.748		17.596	1.57	0.88	403.127	122.616
	DF			0.95	21.946		0.776		
48	OM	95.552	0.809		18.341	1.57	0.79	380.511	117.357
	DF			0.848	24.79		0.695		
50.5	OM	148.887	0.745		22.895	1.57	0.817	547.961	150.255
	DF			1.254	31.059		0.748		

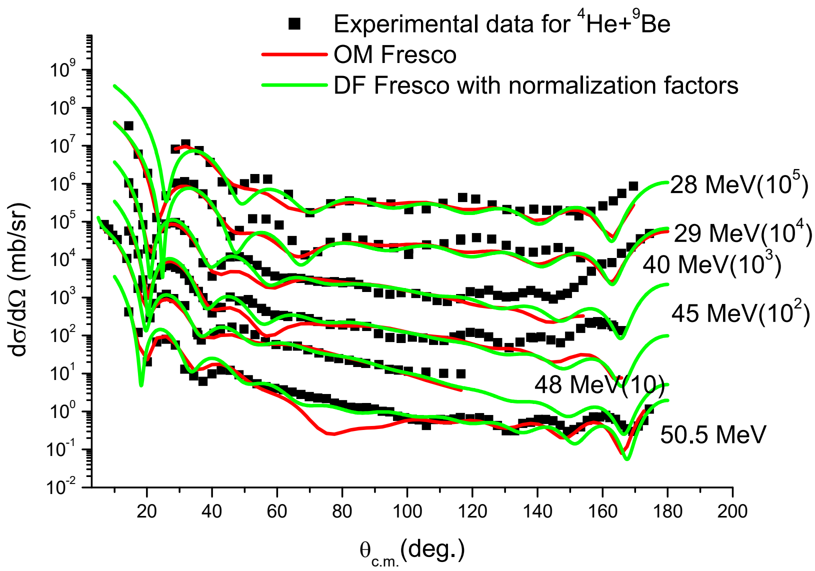

Fig. 1 Angular distribution for ^{4}He+^{9}Be elastic scattering at different energies and calculations using OM (red line) and DF (green line) FRESCO code where experimental data (dots)

We have calculated the real volume integral using:

$$J_R(E) = -\left(\frac{1}{A_p A_t}\right) \int V(r) 4\pi r^2 dr, \quad (13)$$

where A_P and A_t mass values of the incident particle and the target nucleus. The volumetric integral of the imaginary part of the optical potential, determined as:

$$J_W(E) = -\left(\frac{1}{A_p A_t}\right) \int [W_v(E.r) + W_s(E.r)] dr \quad (14)$$

As expected the relation between volume integral of both real potential J_R and imaginary potential J_W with α-particles energy $E_\alpha$is linear as shown in Figs. 2 and 3 respectively according to:

$$J_R(E) = 424.319 - 0.899 E_\alpha,$$
$$J_W(E) = 47.756 + 1.74 E_\alpha$$

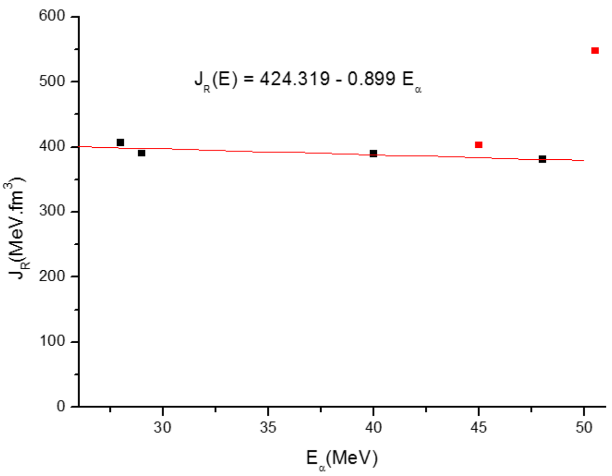

Fig. 2 Relation between volume integral of real part of optical potential and energy

B. Phenomenological and Semi-Microscopic Analysis of ^{11}B *(*^4He*,* ^4He*)*^{11}B

The comparison between the experimental data for α-particles elastically scattering on ^{11}B at energies (24 MeV [27], (40, 50.5) MeV [20] (48.7, 54.1) MeV [2]) is shown in Fig. 4. These angular distributions are analyzed phenomenological by optical model using woods-Saxon shape potential with imaginary volume potential where the radii r_C=1.28fm, r_V=1.245fm and r_D=1.57fm were fixed taken from [20], and only the four remaining parameters (V, W, a_V, a_W) were varied. Also analyzed microscopically within the framework of double folding model by folding the densities of both ^4He and ^{11}B nuclei with effective nucleon-nucleon NN interaction using code FRESCO [28] with normalization coefficient Nr also listed, $R=r_oA^{1/3}$. As we see from the figures, the diffraction structure, which clearly shows up in the forward hemisphere, decreases with increasing angle. At backward angles, the differential cross sections exhibit a rise that is the most pronounced at given energies, where the angular distributions show a rainbow pattern that manifests itself in a broad maximum near 70◦ followed by an exponential decrease up to 150◦.

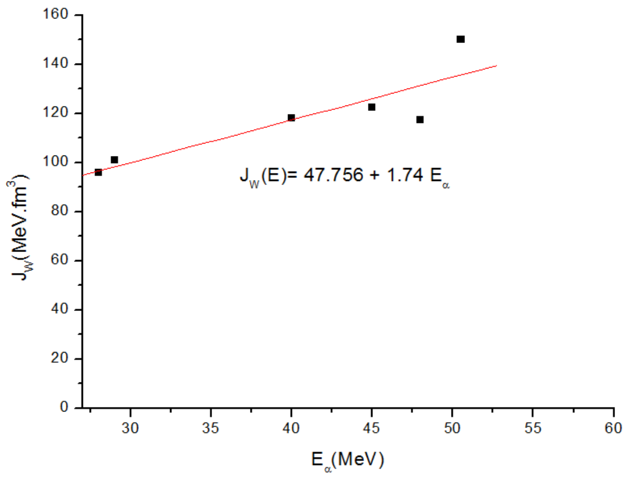

Fig. 3 Relation between volume integral of imaginary part of optical potential and energy

TABLE III
OPTICAL AND DOUBLE FOLDING POTENTIAL PARAMETERS FOR ^{4}HE+^{11}B NUCLEAR SYSTEM USING FRESCO CODE

E_α (MeV)	Model	V_o (MeV)	a_o (fm)	Nr	W_i (MeV)	r_i (fm)	a_i (fm)	J_R MeV.fm^3	J_W MeV.fm^3
24	OM	141.866	0.8379		12.189	1.57	0.50	547.55	59.39
	DF			1.287	15.038	1.41	0.5		
40	OM	136.8736	0.756		18.548	1.57	0.628	480.98	99.18
	DF			1.318	28.00	1.416	0.607		
48.7	OM	123.014	0.7406		18.919	1.57	0.6234	424.804	100.8
	DF			1.305	22.326	1.61	0.523		
50.5	OM	124.366	0.751		19.386	1.57	0.6859	434.56	108.51
	DF			1.302	23.803	1.563	0.602		
54.1	OM	125.080	0.7459		19.564	1.57	0.6285	434.54	104.65
	DF			1.2246	42.394	1.187	0.717		

As expected the relation between volume integral of both real potential J_R and imaginary potential J_W with α-particles energy $E_\alpha$is linear as shown in Figs. 5 and 6 respectively according to:

$$J_R(E) = 644.847 - 4.15 E_\alpha,$$
$$J_W(E) = 26.671 + 1.561 E_\alpha$$

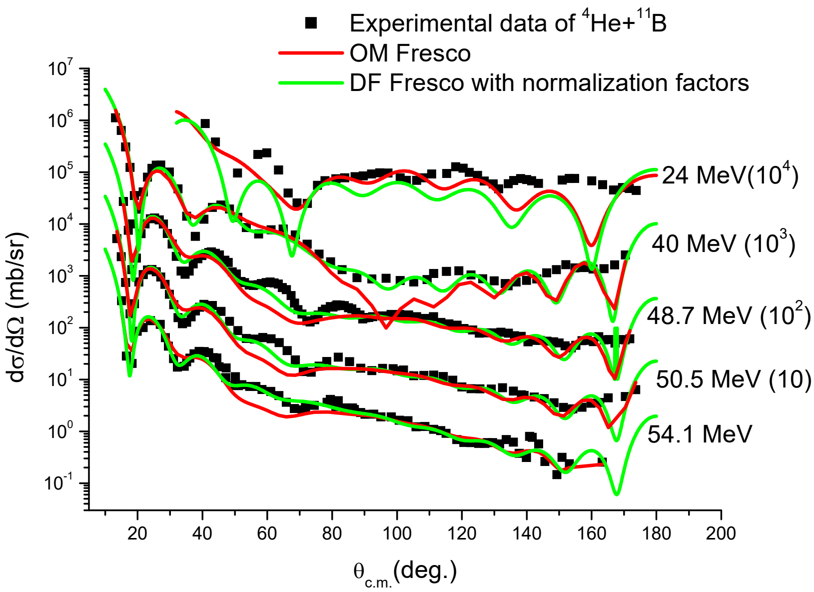

Fig. 4 Angular distribution for $^{4}He+^{11}B$ elastic scattering at different energies and calculations using OM (red line) and DF (green line) FRESCO code where experimental data (dots)

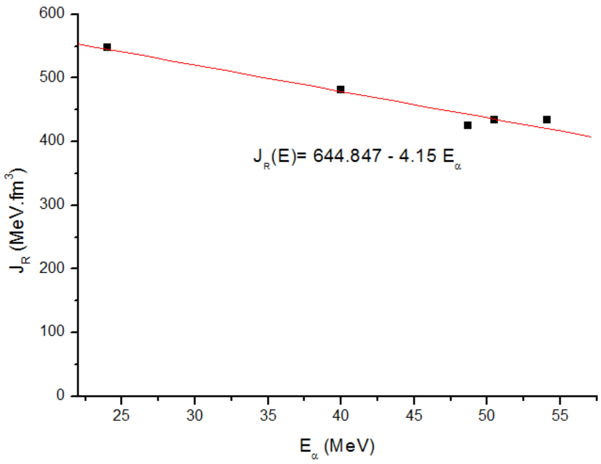

Fig. 5 Relation between volume integral of real part of optical potential and energy

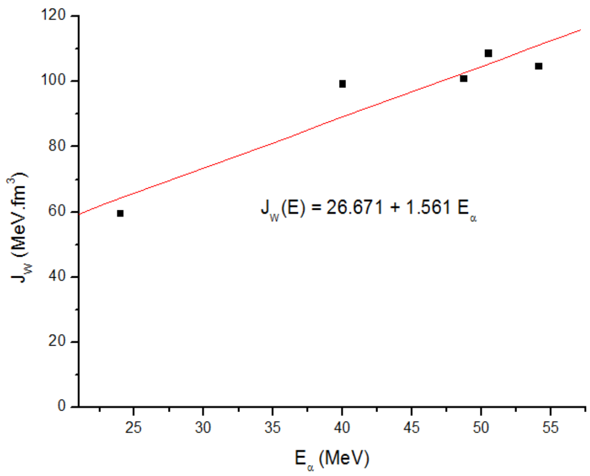

Fig. 6 Relation between volume integral of imaginary part of optical potential and energy

IV. Conclusion

The elastic scattering of α-particles from ^{9}Be at energies (28, 29, 40, 45, 48, 50.5) MeV and from ^{11}B at energies (24, 40, 48.7, 50.5, 54.1) MeV is analyzed within the framework of optical and double folding model using code FRESCO. The diffraction structure, which clearly shows up in the forward hemisphere, decreases with increasing angle is observed. At backward angles, the differential cross sections exhibit a rise that is the most pronounced at given energies, where the angular distributions show a rainbow pattern that manifests itself in a broad maximum near 70◦ followed by an exponential decrease up to 150◦. Volumetric integrals of the real and imaginary potential depth found to be energy dependent.

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