

# Design Modelling Control and Simulation of DC/DC Power Buck Converter

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**Abstract**—The power buck converter is the most widely used DC/DC converter topology. They have a very large application area such as DC motor drives, photovoltaic power system which require fast transient responses and high efficiency over a wide range of load current. This work proposes, the modelling of DC/DC power buck converter using state-space averaging method and the current-mode control using a proportional-integral controller. The efficiency of the proposed model and control loop are evaluated with operating point changes. The simulation results proved the effectiveness of the linear model of DC/DC power buck converter.

**Keywords**—DC/DC power buck converter, Linear current control, State-space averaging method.

## NOMENCLATURE

|            |  |
|------------|--|
| $C$        | Capacitance of the capacitor;                            |
| $\alpha$   | Duty cycle;  |
| $i_c$      | Capacitor current;                                       |
| $i_{in}$   | Input current;   |
| $i_L$      | Inductor current;  |
| $I_o$      | Output current (load current);                           |
| $L$        | Inductance of the inductor;                              |
| $R$        | Resistance of the load resistor;                         |
| $T$        | Switching period;  |
| $f$        | Switching frequency;                                     |
| $V_e$      | Input voltage;   |
| $V_s$      | Output voltage;  |
| $V_{sref}$ | Output voltage reference;                                |
| $v_k$      | Switching voltage;                                       |
| $v_D$      | Diode voltage;   |
| $V_c$      | Voltage control signal of Gate Turn-Off thyristor (GTO). |

## I. INTRODUCTION

THE DC/DC power buck converters convert high level DC voltage signal to low level stabilized DC voltage signal. They have received an increasing deal of interest in many areas. This is due to their high conversion efficiency and flexible output voltage. The DC/DC power buck converters are extensively used in the applications like appliance control such as DC motor drives, photovoltaic power system....

Due to the switching characteristics of the DC/DC power buck converter, the control problem poses a challenge for researchers. The objective of the switching control in DC/DC power buck converter is to realize high power transfer efficiency and good tracking of output voltage. Many control methods are used for control of switch mode DC/DC power

buck converter and the simple and low cost controller structure is always in demand for most industrial applications [1]-[3]. The control method that gives the best performances under any conditions is always in demand. Conventionally, the DC/DC power buck converters have been controlled by linear voltage mode and current mode control methods such as proportional integral (PI) and proportional integral derivative (PID) controllers [4].

To obtain high performance control of converters, a good model of the converter should be designed. The DC/DC power buck converter modelling is described in [1]-[7].

In this paper, a linear approach for modelling and control of the DC/DC power converter are proposed. On the basis of the state-space averaging method, we calculated the output transfer function of power buck converter which can be further utilized for designing of a PI controller. The performance of the linear model and control loop are tested with operating point changes. The numerical simulations demonstrate the efficiency of proposed approach.

This paper is organized as following: after the introduction and short description of DC/DC power buck converter circuit, we present in the third section the components design and waveforms converter analysis. The fourth section gives the state-space averaging model of power buck converter. The fifth section presents a brief exposition of the PI linear control loop. In the sixth section, the simulation results and discussion are presented. Finally, the obtained results are commented in the last section.

## II. POWER BUCK CONVERTER CIRCUIT TOPOLOGY

A large number of DC/DC converter circuits are known that can increase or decrease the magnitude of DC voltage and/or invert its polarity. Fig. 1 illustrates the power buck converter circuit topology.

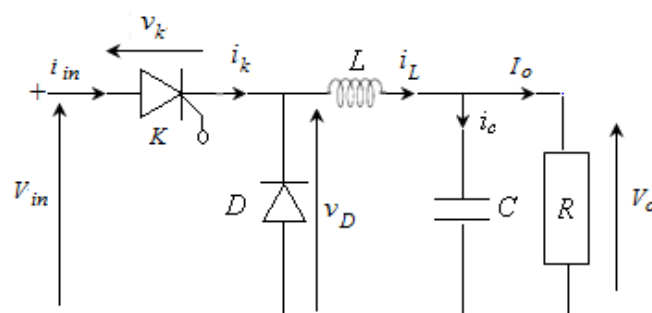


Fig. 1 Power buck converter circuit

The switch  $K$  is commonly realized using a power Gate

Turn-Off thyristor (GTO) and diode. However, other semiconductor switches such as Bipolar Transistor, Insulated Gate Bipolar Transistor (IGBT) and Metal–Oxide–Semiconductor Field-Effect Transistor (MOSFET), can be substituted if desired.

### III. COMPONENTS DESIGN AND WAVEFORMS POWER BUCK CONVERTER ANALYSIS

To obtain the differential equations describing the power buck converter, we consider the ideal topology shown in Fig. 1. The system of differential equations describing the dynamics of the power buck converter is obtained through the direct application of Kirchoff's rules laws for each one of the possible circuit topologies arising from the assumed particular switch position function value.

#### A. Current and Voltage Components

During  $t_{on}$ : ( $0 < t < \alpha T$  (K: ON, D: OFF)  $\alpha = t_{on}/T$ )

$$v_k(t) \approx 0, i_m(t) = i_o(t), i_D(t) = 0 \text{ and } v_D(t) = -V_{in}$$

$$V_o(t) = V_o = V_R + v_L(t) \text{ and } v_L(t) = L \frac{di_o(t)}{dt} \quad (1)$$

During  $t_{off}$  ( $\alpha T < t < T$  (K: OFF, D: ON))

$$v_D(t) \approx 0, i_m(t) = 0$$

$$i_D(t) = i_L(t) \text{ and } V_R + v_L(t) = 0 \quad (2)$$

#### B. Differential Equations

During  $t_{on}$

$$V_{in}(t) = V_o(t) + L \frac{di_L(t)}{dt} \quad (3)$$

The solution of (3) is given by:

$$i_L(t) = \frac{V_{in}(t) - V_o(t)}{L} t + i_L(0) \quad (4)$$

with  $i_L(0) = I_{o\min}$

During  $t_{off}$ :

$$V_o(t) + L \frac{di_L(t)}{dt} = 0 \quad (5)$$

The solution of (5) is given by:

$$i_L(t) = \frac{-V_o(t)}{L} (t - \alpha T) + i_L(\alpha T) \quad (6)$$

with

$$i_L(\alpha T) = I_{o\max} = \frac{V_{in}(t) - V_o(t)}{L} \alpha T + I_{o\min}$$

#### C. Average Value of the Output Voltage and Output Current

$$V_o = \frac{1}{T} \int_0^{\alpha T} V_o(t) dt + \frac{1}{T} \int_{\alpha T}^T V_o(t) dt = \alpha V_{in} \quad (7)$$

with  $V_o = \alpha V_{in} \leq V_{in}$

$$I_o = \frac{1}{TR} \int_0^T V_o(t) dt = \alpha \frac{V_{in}}{R} = \frac{I_{o\max} + I_{o\min}}{2} \quad (8)$$

#### D. Output current Ripple

During  $t_{on}$

The average value of (3) is given by:

$$V_{in} - V_o = L \frac{\Delta i_o}{\alpha T} = V_{in} (1 - \alpha) \geq 0 \quad (9)$$

⇒ The current creases in the L.

During  $t_{off}$

The average value of (5) is given by:

$$-V_o = L \frac{\Delta i_o}{(1 - \alpha) T} \leq 0 \quad (10)$$

⇒ The current decreases in L.

Finally, the output current ripple can be expressed by:

$$\Delta i_o = \frac{V_{in}}{L f} (1 - \alpha) \alpha = I_{o\max} - I_{o\min} \quad (11)$$

#### E. Average Value of the Input Current

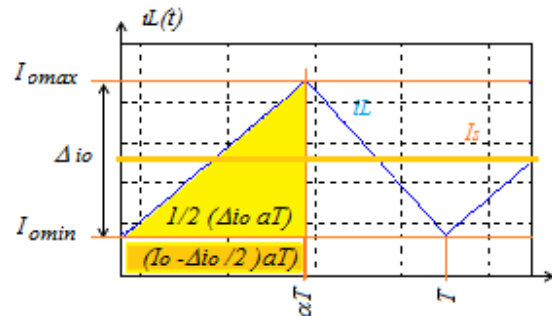


Fig. 2 Inductance current

Based on Fig. 2, the average value of the input current can be calculated by:

$$I_{in} = \frac{1}{T} \int_0^T i_{in}(t) dt = \alpha I_o \quad (12)$$

#### F. Output Voltage Ripple

The capacitor current, given by (13), is equivalent to the alternative component of output current given by:

$$i_c(t) = C \frac{dv_c(t)}{dt} = C \frac{dV_o(t)}{dt} \quad (13)$$

Assumed the switching frequency value is sufficiently high, the output current form is composed of straight portions. Consequently, the capacitor voltage (output voltage) is composed of parable portions:

$$V_o(t) = \frac{1}{C} \int i_c(t) dt \quad (14)$$

Fig. 3 shows the output voltage and the capacitor current.

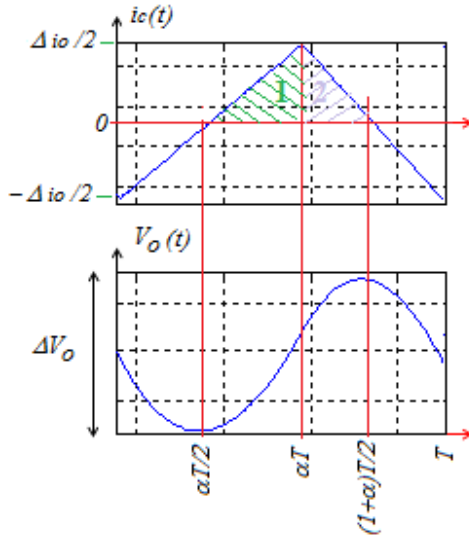


Fig. 3 Capacitor current and output voltage

$V_{omin}$  and  $V_{omax}$  respectively corresponding to  $\alpha T/2$  and  $(1+\alpha)T/2$ . The output voltage ripple can be calculated by:

$$\Delta V_o = \frac{1}{C} \int_{\alpha T/2}^{(1+\alpha)T/2} i_c(t) dt = \frac{\Delta i_o}{8Cf} \quad (14)$$

Replace the expression of the output current ripple given by (11) in (14). The output voltage ripple can be finally expressed by:

$$\Delta V_o = \frac{V_{in}}{8LCf^2} \alpha(1-\alpha) \quad (15)$$

#### G.LC Filter Design

The inductance and capacitor of LC filter can be designed using (11) and (15). The inductance should be designed considering the maximal value of output current ripple corresponding to  $\alpha=0.5$ . Thus, the output current ripple will never exceed the specified value:

$$L = \frac{V_{in}}{\Delta i_o f} (0,5)^2 \quad (16)$$

The switching frequency value can be increased to reduce the inductance value if sizing, based on the  $\Delta i_o$ , leads to excessive values. The output current ripple value decreases with the increase of the inductance or the switching frequency values.

In the steady state the waveforms power buck converter are given in Fig. 4.

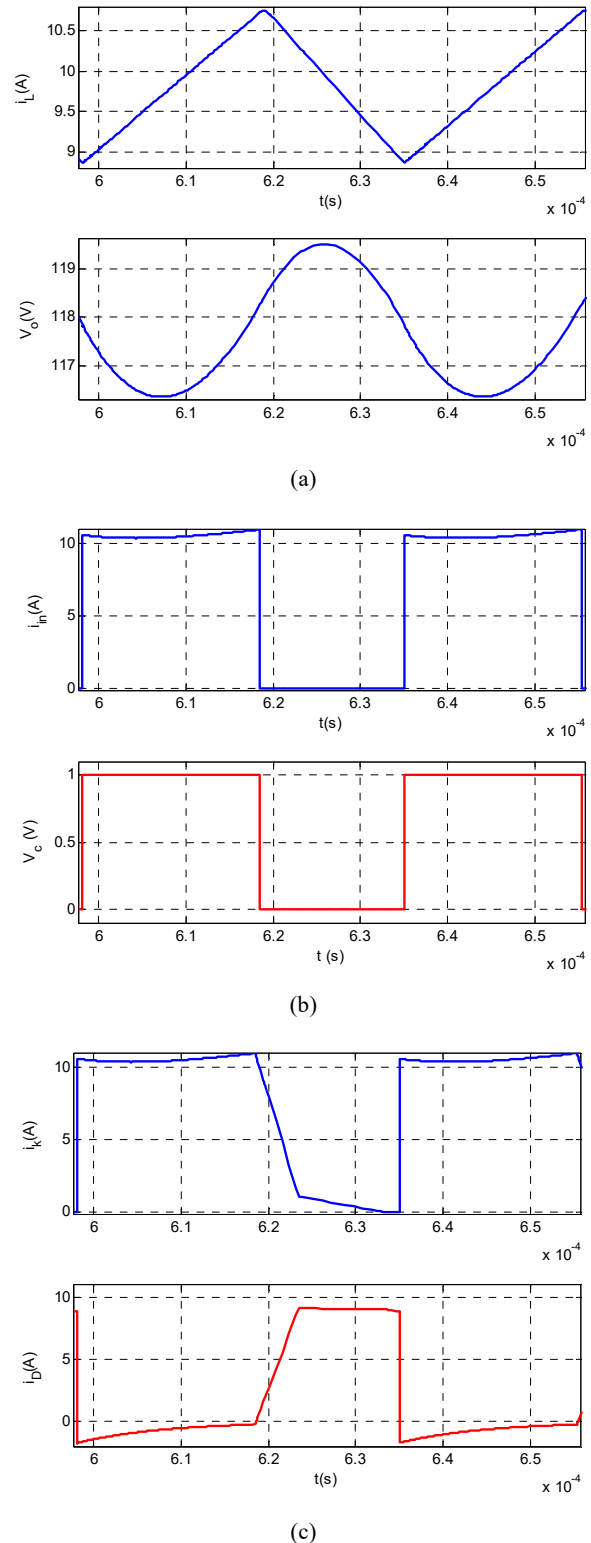


Fig. 4 (a) output current and output voltage, (b) input current and voltage control signal of GTO, (c) GTO and diode currents

#### IV. STATE-SPACE AVERAGING MODEL OF POWER BUCK CONVERTER

The DC/DC power buck converter can be described as switching between different time-invariant systems and is

subsequently a time-variant system. The state-space descriptions of the different time-invariant systems are used as a starting point in the state-space averaging method. The control-to-output transfer function can be obtained by applying the state-space averaging method to the DC/DC power buck converter [8].

The components of the DC/DC power buck converter are assumed ideal (equivalent circuit given in Fig. 1). The waveforms of the signals in the circuit are as shown in Fig. 4 and they are obtained from a matlab simulation. Steady state is reached and, therefore, the control signal  $V_c(t)$ , consists of pulses with constant width.

- While the thyristor is ON, the voltage across the diode is equal to the input voltage and the circuit in Fig. 5 (a) can be used as a model.
- While the thyristor is OFF, the voltage across the diode is equal to zero and the circuit in Fig. 5 (b) can be used as a model.

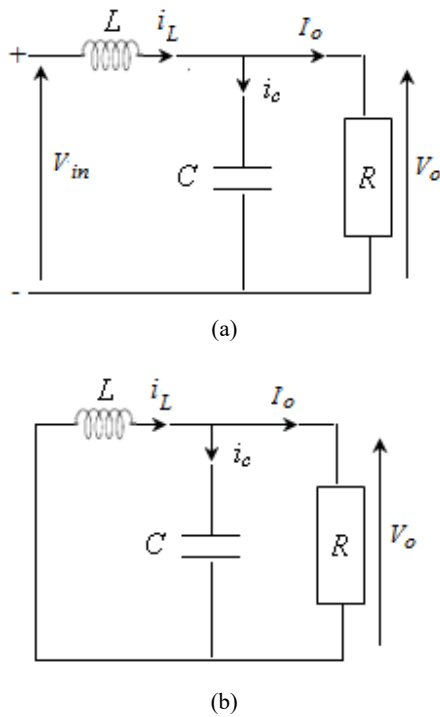


Fig. 5 The circuit of the DC/DC power buck converter: (a) switch ON during  $t_{on}$  and (b) switch OFF during  $t_{off}$

The state-space descriptions of the circuits in Figs. 5 (a) and (b) are given in (17) and (18).

$$\begin{cases} \frac{dx(t)}{dt} = A_1 x(t) + B_1 u(t) \\ y(t) = C_1 x(t) + E_1 u(t) \end{cases} \quad (17)$$

$$\begin{cases} \frac{dx(t)}{dt} = A_2 x(t) + B_2 u(t) \\ y(t) = C_2 x(t) + E_2 u(t) \end{cases} \quad (18)$$

with:

$$x(t) = \begin{bmatrix} i_L(t) \\ V_s(t) \end{bmatrix}, u(t) = \begin{bmatrix} V_{in}(t) \\ i_{in}(t) \end{bmatrix} \text{ and } y(t) = V_o(t) \quad (19)$$

The two systems are different linear time-invariant. In state-space averaging method, these two systems are first averaged with respect to their duration in the switching period:

$$\begin{cases} \frac{dx(t)}{dt} = [d(t)A_1 + (1-d(t))A_2]x(t) + [d(t)B_1 + (1-d(t))B_2]u(t) \\ y(t) = [d(t)C_1 + (1-d(t))C_2]x(t) + [d(t)E_1 + (1-d(t))E_2]u(t) \end{cases} \quad (20)$$

The system equation (20) is an approximation of the time-variant system and new variable names should formally have been used. To limit the number of variable names, this is not made. The duty cycle  $d(t)$  is an additional input signal. A new input vector is therefore defined:

$$u'(t) = \begin{bmatrix} u(t) \\ d(t) \end{bmatrix} \quad (21)$$

This is not made in traditional presentations of state-space averaging, where the control signal  $d(t)$  is kept separate from the disturbance signals  $V_{in}(t)$  and  $i_L(t)$ . However, in system theory, all control signals and disturbance signals are put in an input vector.

Since the duty cycle can be considered to be a discrete-time signal with switching frequency  $f$ , one cannot expect the system in the last differential equation to be valid for frequencies higher than half the switching frequency.

The second step in state-space averaging method is linearization of the nonlinear time-invariant system. The deviations from an operating point are defined as:

$$x(t) = X + \hat{x}(t), u'(t) = U' + \hat{u}'(t), y(t) = Y + \hat{y}(t) \quad (22)$$

$X$ ,  $U'$  and  $Y$  denote the operating-point (DC, steady-state) values;  $\hat{x}$ ,  $\hat{u}'$  and  $\hat{y}$  denotes the perturbation (AC) signals.

The result is a linearized (AC, small-signal) system:

$$\begin{cases} \frac{d\hat{x}(t)}{dt} = A' \hat{x}(t) + B' \hat{u}'(t) \\ \hat{y}(t) = C' \hat{x}(t) + E' \hat{u}'(t) \end{cases} \quad (23)$$

Besides the AC model given in (23), a DC model can be obtained from (20) by setting the perturbation components to zero. One can extract the transfer function of power buck converter from the Laplace transform of (23).

$$H(p) = \frac{\hat{V}_o(p)}{\hat{d}(p)} = H_0 \frac{1}{1 + 2 \frac{\xi}{\omega_n} p + \frac{p^2}{\omega_n^2}} \quad (24)$$

with

$$\omega_n = \frac{1}{\sqrt{LC}}, \xi = \frac{\omega_n L}{2 R} \text{ and } H_0 = V_{in}$$

The operating-point value of  $d(t)$  is denoted  $D$  and  $D'=1- D$ .

### V. CURRENT-MODE CONTROL

In a current-mode control, an additional inner control loop is used as shown in Fig. 6, where the control voltage directly controls the output inductor current that feeds the output stage and thus the output voltage. Ideally, the control voltage should act to directly control the average value of the inductor for the faster response. The fact that the current feeding the output stage is controlled directly in a current-mode control has a profound effect on the dynamic behavior of the negative feedback control loop [4].

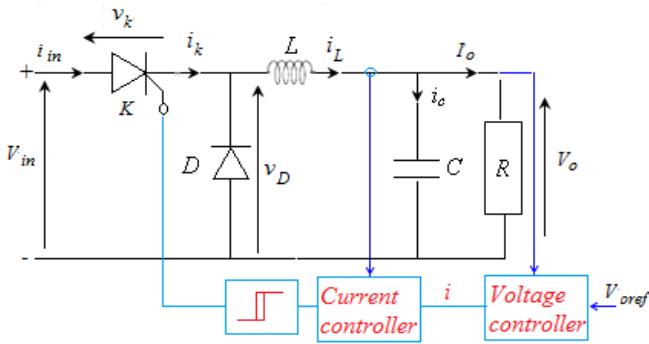


Fig. 6 The DC/DC power buck converter with a current controller and a voltage controller

### VI. SIMULATION RESULTS

The values of the capacitor and inductor will vary. In this paper, we consider an ideal DC/DC power buck converter, but they do provide us some rough values to start the designing and analysis of our system. All diode, thyristor and LC filter are ideal components, the input voltage is also a pure DC value; and the duty cycle remains approximately constant.

For an input voltage  $V_{in} = 209V$ , output voltage  $V_o = 125.4V$ , current ripple  $\Delta i_o = 5\% I_o$  and the voltage ripple  $\Delta V_o = 2.68\% V_o$ :

The duty cycle is given by:

$$\alpha = \frac{V_o}{V_{in}} \approx 55\%$$

The output current can be calculated by:

$$I_o = \frac{I_{in}}{\alpha} = \frac{5,5}{0,55} \approx 10A$$

LC filter value can be calculated by:

$$L = \frac{V_{in}}{\Delta i_o f} (1 - \alpha) \alpha \approx 1mH \text{ and } C = \frac{V_{in}}{8\Delta V_o L f^2} (1 - \alpha) \alpha = 2.8\mu F$$

The simulation parameters are summered in Table I

Based upon these calculated values the corresponding open loop and closed Bode Plots are given in Figs. 7 and 8.

Fig. 9 depicts the dynamic response of the DC/DC power

buck converter output voltage to the output voltage reference step change from 120V to 80V at 0.3ms and from 80V to 120V at 0.6ms.

TABLE I  
 THE SIMULATION PARAMETER VALUES OF THE POWER BUCK CONVERTER

| Variable        | Parameter                 | Value   |
|-----------------|---------------------------|---------|
| L               | Inductance                | 1mH     |
| C               | Output filter capacitance | 2.8μF   |
| V <sub>in</sub> | Input voltage             | 210V    |
| V <sub>o</sub>  | Out put voltage           | 115.5V  |
| P <sub>n</sub>  | Constant power load       | 1.2kW   |
| f               | Switching frequency       | 27.4kHz |
| R               | load                      | 10Ω     |

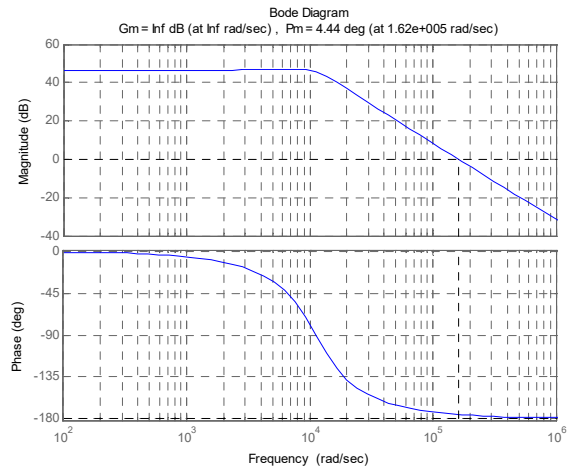


Fig. 7 Bode plot of DC/DC power buck converter transfer function

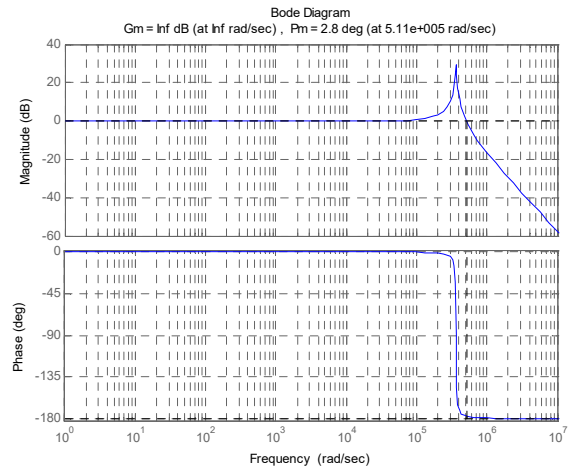


Fig. 8 Bode plot of DC/DC power buck converter closed loop transfer function

Fig. 10 shows the dynamic analysis of output inductance current  $i_L$  and output current  $I_o$  of feedback control of DC/DC power buck converter.

Fig. 11 shows the phase portrait. The reference voltage is 120V and the load is 10 Ω. The controller makes the trajectory to form a stable limit cycle. The limit cycle width on the  $V_o$ -axis is equal to the voltage ripple seen on the output of the converter. The two phases of the dynamics can easily be

observed. This is due to the fact that control input is not in infinite frequency.

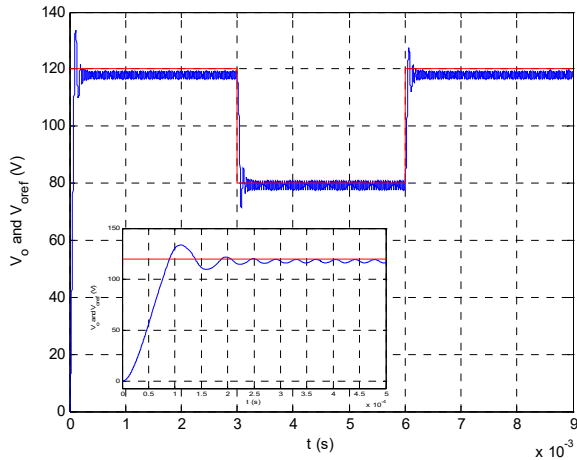


Fig. 9 Dynamic analysis of output voltage feedback control of DC/DC power buck converter

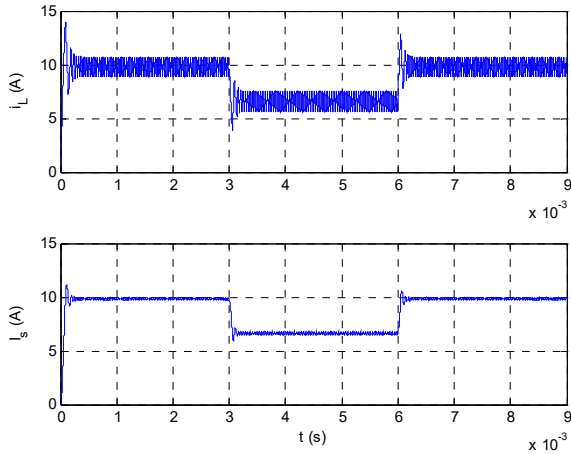


Fig. 10 Dynamic analysis of output inductance current  $i_L$  and output current  $I_o$  of feedback control of DC/DC power buck converter

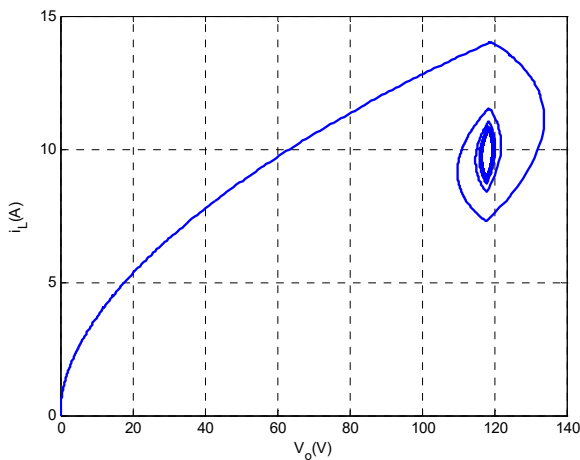


Fig. 11 Phase portrait (output voltage, current waveforms)

The step response characteristics of DC/DC power buck

converter are summered in Table II.

TABLE II  
 STEP RESPONSE CHARACTERISTICS OF DC/DC POWER BUCK CONVERTER

| Parameter   | Value   |
|---|---------|
| Rise time   | 0.06ms  |
| Settling time   | 0.98ms  |
| Minimum value of y once the response has risen          | 107.33V |
| Maximum value of y once the response has risen          | 133.75V |
| Percentage overshoot (relative to output voltage final) | 12.59V  |
| Percentage undershoot                                   | 0       |
| Peak absolute value of output voltage                   | 133.75V |
| Time at which this peak is reached                      | 0.11ms  |

## VII. CONCLUSION

In this paper, the linear model of DC/DC power buck converter is developed on the basis of the state-space averaging method. The output voltage of the power buck converter is regulated using a PI controller. The performance of the proposed model and control loop of DC/DC power buck converter are tested with operating point changes. The numerical simulations demonstrate the efficiency, offers a fast dynamic response and it is robust against the variations of the output voltage of DC/DC power buck converter.

In the future work, we will study the design, modelling, liner control of DC/DC real power buck converter considering the Equivalent Series Resistance (ESR) of the capacitor, the Resistance in the diode while it conducts, and the Resistance in the inductor.

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