

Membership Surface and Arithmetic Operations of Imprecise Matrix

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Abstract—In this paper, a method has been developed to construct the membership surfaces of row and column vectors and arithmetic operations of imprecise matrix. A matrix with imprecise elements would be called an imprecise matrix. The membership surface of imprecise vector has been already shown based on Randomness-Impreciseness Consistency Principle. The Randomness-Impreciseness Consistency Principle leads to defining a normal law of impreciseness using two different laws of randomness. In this paper, the author has shown row and column membership surfaces and arithmetic operations of imprecise matrix and demonstrated with the help of numerical example.

Keywords—Imprecise number, Imprecise vector, Membership surface, Imprecise matrix.

I. INTRODUCTION

BARUAH ([1]-[4]) has defined an imprecise number $X = [a, b, c]$ with membership function

$$\mu_{X|Y}(x, y) = \begin{cases} L(x) & a \leq x \leq b, p \leq y \leq r \\ R(x) & b \leq x \leq c, p \leq y \leq r \\ 0 & \text{otherwise.} \end{cases}$$

$L(x)$ being a continuous non-decreasing function in the interval $[a, b]$, and $R(x)$ being a continuous non-increasing function in the interval $[b, c]$, with $L(a) = R(c) = 0$ and $L(b) = R(b) = 1$. A continuous non-decreasing function of this type is also called a distribution function with reference to Lebesgue-Stieltjes measure ([5]). Das and Baruah ([6]-[8]) have shown the construction method of the membership surface of imprecise vector. If (X, Y) is an imprecise vector, where X and Y both are imprecise numbers represented by $X = [a, b, c]$ and $Y = [p, q, r]$ respectively and if the membership function of X and Y be

$$\mu_{X|Y}(x, y) = \begin{cases} L(x) & a \leq x \leq b, p \leq y \leq r \\ R(x) & b \leq x \leq c, p \leq y \leq r \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Y|X}(x, y) = \begin{cases} L(y) & a \leq x \leq c, p \leq y \leq q \\ R(y) & a \leq x \leq c, q \leq y \leq r \\ 0 & \text{otherwise.} \end{cases}$$

Then the membership surface of the imprecise vector (X, Y) can be obtained [3] as:

$$\mu_{X,Y}(x, y) = \begin{cases} L(x)L(y) & a \leq x \leq b, p \leq y \leq q \\ L(x)R(y) & a \leq x \leq b, q \leq y \leq r \\ R(x)L(y) & b \leq x \leq c, p \leq y \leq q \\ R(x)R(y) & b \leq x \leq c, q \leq y \leq r \\ 0 & \text{otherwise.} \end{cases}$$

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Here it is assumed that X and Y are independently distributed. In a matrix all rows and columns are nothing but row vectors and column vectors. So, we may obtain the row membership surfaces and column membership surfaces of an imprecise matrix. If the elements of the matrix are imprecise then we can construct the membership surfaces of row vectors and column vectors of that matrix.

II. MEMBERSHIP SURFACES OF ROW AND COLUMN VECTORS OF AN IMPRECISE MATRIX

Consider a 2×2 matrix

$$\begin{pmatrix} X & Y \\ Z & W \end{pmatrix}$$

where all the elements of the matrix X, Y, Z and W are imprecise. Assume that all the elements X, Y, Z and W are triangular imprecise numbers represented by $[x_1, x_2, x_3], [y_1, y_2, y_3], [z_1, z_2, z_3]$ and $[w_1, w_2, w_3]$ respectively. We may consider (X, Y) and (Z, W) as row vectors and (X, Z) and (Y, W) as column vectors. The membership functions of X, Y, Z and W for row vectors are as:

$$\mu_{X|Y}(x, y) = \begin{cases} \frac{x - x_1}{x_2 - x_1} & x_1 \leq x \leq x_2, \\ & y_1 \leq y \leq y_3 \\ \frac{x - x_3}{x_2 - x_3} & x_2 \leq x \leq x_3, \\ & y_1 \leq y \leq y_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Y|X}(x, y) = \begin{cases} \frac{y - y_1}{y_2 - y_1} & x_1 \leq x \leq x_3, \\ & y_1 \leq y \leq y_2 \\ \frac{y - y_3}{y_2 - y_3} & x_1 \leq x \leq x_3, \\ & y_2 \leq y \leq y_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z|W}(z, w) = \begin{cases} \frac{z - z_1}{z_2 - z_1} & w_1 \leq w \leq w_3, \\ & z_1 \leq z \leq z_2 \\ \frac{z - z_3}{z_2 - z_3} & w_1 \leq w \leq w_3, \\ & z_2 \leq z \leq z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{W|Z}(z, w) = \begin{cases} \frac{w - w_1}{w_2 - w_1} & w_1 \leq w \leq w_2, \\ & z_1 \leq z \leq z_3 \\ \frac{w - w_3}{w_2 - w_3} & w_2 \leq w \leq w_3, \\ & z_1 \leq z \leq z_3 \\ 0 & \text{otherwise.} \end{cases}$$

Now the membership surfaces of row vectors of the imprecise matrix will be as:

$$\mu_R(x, y) = \begin{cases} \frac{(x-x_1)(y-y_1)}{(x_2-x_1)(y_2-y_1)} & x_1 \leq x \leq x_2, y_1 \leq y \leq y_2 \\ \frac{(x-x_1)(y-y_3)}{(x_2-x_1)(y_2-y_3)} & x_1 \leq x \leq x_2, y_2 \leq y \leq y_3 \\ \frac{(x_2-x_1)(y_2-y_3)}{(x-x_3)(y-y_1)} & x_2 \leq x \leq x_3, y_1 \leq y \leq y_2 \\ \frac{(x_2-x_3)(y_2-y_1)}{(x-x_3)(y-y_3)} & x_2 \leq x \leq x_3, y_2 \leq y \leq y_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_R(z, w) = \begin{cases} \frac{(z-z_1)(w-w_1)}{(z_2-z_1)(w_2-w_1)} & z_1 \leq z \leq z_2, w_1 \leq w \leq w_2 \\ \frac{(z_2-z_1)(w_2-w_1)}{(z-z_1)(w-w_3)} & z_1 \leq z \leq z_2, w_2 \leq w \leq w_3 \\ \frac{(z_2-z_1)(w_2-w_3)}{(z-z_3)(w-w_1)} & z_2 \leq z \leq z_3, w_1 \leq w \leq w_2 \\ \frac{(z_2-z_3)(w_2-w_1)}{(z-z_3)(w-w_3)} & z_2 \leq z \leq z_3, w_2 \leq w \leq w_3 \\ 0 & \text{otherwise.} \end{cases}$$

Again, the membership functions of X, Y, Z and W for column vectors are as:

$$\mu_{X|Z}(x, z) = \begin{cases} \frac{x-x_1}{x_2-x_1} & x_1 \leq x \leq x_2, z_1 \leq z \leq z_3 \\ \frac{x-x_3}{x_2-x_3} & x_2 \leq x \leq x_3, z_1 \leq z \leq z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Y|W}(y, w) = \begin{cases} \frac{y-y_1}{y_2-y_1} & y_1 \leq y \leq y_2, w_1 \leq w \leq w_3 \\ \frac{y-y_3}{y_2-y_3} & y_2 \leq y \leq y_3, w_1 \leq w \leq w_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z|X}(z, x) = \begin{cases} \frac{z-z_1}{z_2-z_1} & x_1 \leq x \leq x_3, z_1 \leq z \leq z_2 \\ \frac{z-z_3}{z_2-z_3} & x_1 \leq x \leq x_3, z_2 \leq z \leq z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{W|Y}(w, y) = \begin{cases} \frac{w-w_1}{w_2-w_1} & w_1 \leq w \leq w_2, y_1 \leq y \leq y_3 \\ \frac{w-w_3}{w_2-w_3} & w_2 \leq w \leq w_3, y_1 \leq y \leq y_3 \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, the membership surfaces of column vectors of the imprecise matrix will be as:

$$\mu_C(x, z) = \begin{cases} \frac{(x-x_1)(z-z_1)}{(x_2-x_1)(z_2-z_1)} & x_1 \leq x \leq x_2, z_1 \leq z \leq z_2 \\ \frac{(x-x_1)(z-z_3)}{(x_2-x_1)(z_2-z_3)} & x_1 \leq x \leq x_2, z_2 \leq z \leq z_3 \\ \frac{(x_2-x_1)(z_2-z_3)}{(x-x_3)(z-z_1)} & x_2 \leq x \leq x_3, z_1 \leq z \leq z_2 \\ \frac{(x_2-x_3)(z_2-z_1)}{(x-x_3)(z-z_3)} & x_2 \leq x \leq x_3, z_2 \leq z \leq z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_C(y, w) = \begin{cases} \frac{(y-y_1)(w-w_1)}{(y_2-y_1)(w_2-w_1)} & y_1 \leq y \leq y_2, w_1 \leq w \leq w_2 \\ \frac{(y-y_1)(w-w_3)}{(y_2-y_1)(w_2-w_3)} & y_1 \leq y \leq y_2, w_2 \leq w \leq w_3 \\ \frac{(y_2-y_1)(w_2-w_3)}{(y-y_3)(w-w_1)} & y_2 \leq y \leq y_3, w_1 \leq w \leq w_2 \\ \frac{(y_2-y_3)(w_2-w_1)}{(y-y_3)(w-w_3)} & y_2 \leq y \leq y_3, w_2 \leq w \leq w_3 \\ 0 & \text{otherwise.} \end{cases}$$

In the same way, for a 3×3 matrix

$$\begin{pmatrix} X & Y & P \\ Z & W & Q \\ M & N & R \end{pmatrix}$$

where all the elements of the matrix X, Y, P, Z, W, Q, M, N and R are imprecise. Assume that all the elements X, Y, P, Z, W, Q, M, N and R are triangular imprecise numbers represented by $[x_1, x_2, x_3], [y_1, y_2, y_3], [p_1, p_2, p_3], [z_1, z_2, z_3], [w_1, w_2, w_3], [q_1, q_2, q_3], [m_1, m_2, m_3], [n_1, n_2, n_3]$ and $[r_1, r_2, r_3]$ respectively. We may consider as $(X, Y, P), (Z, W, Q)$ and (M, N, R) as row vectors and $(X, Z, M), (Y, W, N)$ and (P, Q, R) as column vectors. The membership functions of X, Y, P, Z, W, Q, M, N and R for row vectors are as:

$$\mu_{X|Y,P}(x, y, p) = \begin{cases} \frac{x-x_1}{x_2-x_1} & x_1 \leq x \leq x_2, y_1 \leq y \leq y_3, \\ & p_1 \leq p \leq p_3 \\ \frac{x-x_3}{x_2-x_3} & x_2 \leq x \leq x_3, y_1 \leq y \leq y_3, \\ & p_1 \leq p \leq p_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Y|X,P}(x, y, p) = \begin{cases} \frac{y-y_1}{y_2-y_1} & x_1 \leq x \leq x_3, y_1 \leq y \leq y_2, \\ & p_1 \leq p \leq p_3 \\ \frac{y-y_3}{y_2-y_3} & x_1 \leq x \leq x_3, y_2 \leq y \leq y_3, \\ & p_1 \leq p \leq p_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{P|X,Y}(x, y, p) = \begin{cases} \frac{p-p_1}{p_2-p_1} & x_1 \leq x \leq x_3, y_1 \leq y \leq y_3, \\ & p_1 \leq p \leq p_2 \\ \frac{p-p_3}{p_2-p_3} & x_1 \leq x \leq x_3, y_1 \leq y \leq y_3, \\ & p_2 \leq p \leq p_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z|W,Q}(z, w, q) = \begin{cases} \frac{z-z_1}{z_2-z_1} & z_1 \leq z \leq z_2, w_1 \leq w \leq w_3, \\ & q_1 \leq q \leq q_3 \\ \frac{z-z_3}{z_2-z_3} & z_2 \leq z \leq z_3, w_1 \leq w \leq w_3, \\ & q_1 \leq q \leq q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{W|Q,Z}(z, w, q) = \begin{cases} \frac{w-w_1}{w_2-w_1} & z_1 \leq z \leq z_3, w_1 \leq w \leq w_2, \\ & q_1 \leq q \leq q_3 \\ \frac{w-w_3}{w_2-w_3} & z_1 \leq z \leq z_3, w_2 \leq w \leq w_3, \\ & q_1 \leq q \leq q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Q|Z,W}(z, w, q) = \begin{cases} \frac{q-q_1}{q_2-q_1} & z_1 \leq z \leq z_3, w_1 \leq w \leq w_3, \\ & q_1 \leq q \leq q_2 \\ \frac{q-q_3}{q_2-q_3} & z_1 \leq z \leq z_3, w_1 \leq w \leq w_3, \\ & q_2 \leq q \leq q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{M|N,R}(m, n, r) = \begin{cases} \frac{m-m_1}{m_2-m_1} & m_1 \leq m \leq m_2, n_1 \leq n \leq n_3, \\ & r_1 \leq r \leq r_3 \\ \frac{m-m_3}{m_2-m_3} & m_2 \leq m \leq m_3, n_1 \leq n \leq n_3, \\ & r_1 \leq r \leq r_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{N|M,R}(m, n, r) = \begin{cases} \frac{n-n_1}{n_2-n_1} & m_1 \leq m \leq m_3, n_1 \leq n \leq n_2, \\ & r_1 \leq r \leq r_3 \\ \frac{n-n_3}{n_2-n_3} & m_1 \leq m \leq m_3, n_2 \leq n \leq n_3, \\ & r_1 \leq r \leq r_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{R|M,N}(m, n, r) = \begin{cases} \frac{r-r_1}{r_2-r_1} & m_1 \leq m \leq m_3, n_1 \leq n \leq n_3, \\ & r_1 \leq r \leq r_2 \\ \frac{r-r_3}{r_2-r_3} & m_1 \leq m \leq m_3, n_1 \leq n \leq n_3, \\ & r_2 \leq r \leq r_3 \\ 0 & \text{otherwise.} \end{cases}$$

Then the membership surfaces of row vectors of the imprecise matrix will be as:

$$\mu_R(x, y, p) = \begin{cases} \frac{(x-x_1)(y-y_1)(p-p_1)}{(x_2-x_1)(y_2-y_1)(p_2-p_1)} & x_1 \leq x \leq x_2, \\ & y_1 \leq y \leq y_2, p_1 \leq p \leq p_2 \\ \frac{(x-x_1)(y-y_3)(p-p_1)}{(x_2-x_1)(y_2-y_3)(p_2-p_1)} & x_1 \leq x \leq x_2, \\ & y_2 \leq y \leq y_3, p_1 \leq p \leq p_2 \\ \frac{(x-x_3)(y-y_1)(p-p_1)}{(x_2-x_3)(y_2-y_1)(p_2-p_1)} & x_2 \leq x \leq x_3, \\ & y_1 \leq y \leq y_2, p_1 \leq p \leq p_2 \\ \frac{(x-x_3)(y-y_3)(p-p_1)}{(x_2-x_3)(y_2-y_3)(p_2-p_1)} & x_2 \leq x \leq x_3, \\ & y_2 \leq y \leq y_3, p_1 \leq p \leq p_2 \\ \frac{(x-x_1)(y-y_1)(p-p_3)}{(x_2-x_1)(y_2-y_1)(p_2-p_3)} & x_1 \leq x \leq x_2, \\ & y_1 \leq y \leq y_2, p_2 \leq p \leq p_3 \\ \frac{(x-x_1)(y-y_3)(p-p_3)}{(x_2-x_1)(y_2-y_3)(p_2-p_3)} & x_1 \leq x \leq x_2, \\ & y_2 \leq y \leq y_3, p_2 \leq p \leq p_3 \\ \frac{(x-x_3)(y-y_1)(p-p_3)}{(x_2-x_3)(y_2-y_1)(p_2-p_3)} & x_2 \leq x \leq x_3, \\ & y_1 \leq y \leq y_2, p_2 \leq p \leq p_3 \\ \frac{(x-x_3)(y-y_3)(p-p_3)}{(x_2-x_3)(y_2-y_3)(p_2-p_3)} & x_2 \leq x \leq x_3, \\ & y_2 \leq y \leq y_3, p_2 \leq p \leq p_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_R(z, w, q) = \begin{cases} \frac{(z-z_1)(w-w_1)(q-q_1)}{(z_2-z_1)(w_2-w_1)(q_2-q_1)} & z_1 \leq z \leq z_2, \\ & w_1 \leq w \leq w_2, q_1 \leq q \leq q_2 \\ \frac{(z-z_1)(w-w_3)(q-q_1)}{(z_2-z_1)(w_2-w_3)(q_2-q_1)} & z_1 \leq z \leq z_2, \\ & w_2 \leq w \leq w_3, q_1 \leq q \leq q_2 \\ \frac{(z-z_3)(w-w_1)(q-q_1)}{(z_2-z_3)(w_2-w_1)(q_2-q_1)} & z_2 \leq z \leq z_3, \\ & w_1 \leq w \leq w_2, q_1 \leq q \leq q_2 \\ \frac{(z-z_3)(w-w_3)(q-q_1)}{(z_2-z_3)(w_2-w_3)(q_2-q_1)} & z_2 \leq z \leq z_3, \\ & w_2 \leq w \leq w_3, q_1 \leq q \leq q_2 \\ \frac{(z-z_1)(w-w_1)(q-q_3)}{(z_2-z_1)(w_2-w_1)(q_2-q_3)} & z_1 \leq z \leq z_2, \\ & w_1 \leq w \leq w_2, q_2 \leq q \leq q_3 \\ \frac{(z-z_1)(w-w_3)(q-q_3)}{(z_2-z_1)(w_2-w_3)(q_2-q_3)} & z_1 \leq z \leq z_2, \\ & w_2 \leq w \leq w_3, q_2 \leq q \leq q_3 \\ \frac{(z-z_3)(w-w_1)(q-q_3)}{(z_2-z_3)(w_2-w_1)(q_2-q_3)} & z_2 \leq z \leq z_3, \\ & w_1 \leq w \leq w_2, q_2 \leq q \leq q_3 \\ \frac{(z-z_3)(w-w_3)(q-q_3)}{(z_2-z_3)(w_2-w_3)(q_2-q_3)} & z_2 \leq z \leq z_3, \\ & w_2 \leq w \leq w_3, q_2 \leq q \leq q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_R(m, n, r) = \begin{cases} \frac{(m-m_1)(n-n_1)(r-r_1)}{(m_2-m_1)(n_2-n_1)(r_2-r_1)} & m_1 \leq m \leq m_2, \\ & n_1 \leq n \leq n_2, r_1 \leq r \leq r_2 \\ \frac{(m-m_1)(n-n_3)(r-r_1)}{(m_2-m_1)(n_2-n_3)(r_2-r_1)} & m_1 \leq m \leq m_2, \\ & n_2 \leq n \leq n_3, r_1 \leq r \leq r_2 \\ \frac{(m-m_3)(n-n_1)(r-r_1)}{(m_2-m_3)(n_2-n_1)(r_2-r_1)} & m_2 \leq m \leq m_3, \\ & n_1 \leq n \leq n_2, r_1 \leq r \leq r_2 \\ \frac{(m-m_3)(n-n_3)(r-r_1)}{(m_2-m_3)(n_2-n_3)(r_2-r_1)} & m_2 \leq m \leq m_3, \\ & n_2 \leq n \leq n_3, r_1 \leq r \leq r_2 \\ \frac{(m-m_1)(n-n_1)(r-r_3)}{(m_2-m_1)(n_2-n_1)(r_2-r_3)} & m_1 \leq m \leq m_2, \\ & n_1 \leq n \leq n_2, r_2 \leq r \leq r_3 \\ \frac{(m-m_1)(n-n_3)(r-r_3)}{(m_2-m_1)(n_2-n_3)(r_2-r_3)} & m_1 \leq m \leq m_2, \\ & n_2 \leq n \leq n_3, r_2 \leq r \leq r_3 \\ \frac{(m-m_3)(n-n_1)(r-r_3)}{(m_2-m_3)(n_2-n_1)(r_2-r_3)} & m_2 \leq m \leq m_3, \\ & n_1 \leq n \leq n_2, r_2 \leq r \leq r_3 \\ \frac{(m-m_3)(n-n_3)(r-r_3)}{(m_2-m_3)(n_2-n_3)(r_2-r_3)} & m_2 \leq m \leq m_3, \\ & n_2 \leq n \leq n_3, r_2 \leq r \leq r_3 \\ 0 & \text{otherwise.} \end{cases}$$

The membership functions of X, Z, M, Y, W, N, P, Q and R for column vectors are as:

$$\mu_{X|M,Z}(x, m, z) = \begin{cases} \frac{x-x_1}{x_2-x_1} & x_1 \leq x \leq x_2, \\ & m_1 \leq m \leq m_3, z_1 \leq z \leq z_3 \\ \frac{x-x_3}{x_2-x_3} & x_2 \leq x \leq x_3, \\ & m_1 \leq m \leq m_3, z_1 \leq z \leq z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z|X,M}(x, m, z) = \begin{cases} \frac{z-z_1}{z_2-z_1} & z_1 \leq z \leq z_2, \\ & x_1 \leq x \leq x_3, m_1 \leq m \leq m_3 \\ \frac{z-z_3}{z_2-z_3} & z_2 \leq z \leq z_3, \\ & x_1 \leq x \leq x_3, m_1 \leq m \leq m_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{M|X,Z}(x, m, z) = \begin{cases} \frac{m-m_1}{m_2-m_1} & m_1 \leq m \leq m_2, \\ & x_1 \leq x \leq x_3, z_1 \leq z \leq z_3 \\ \frac{m-m_3}{m_2-m_3} & m_2 \leq m \leq m_3, \\ & x_1 \leq x \leq x_3, z_1 \leq z \leq z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Y|N,W}(y, n, w) = \begin{cases} \frac{y-y_1}{y_2-y_1} & y_1 \leq y \leq y_2, \\ & n_1 \leq n \leq n_3, w_1 \leq w \leq w_3 \\ \frac{y-y_3}{y_2-y_3} & y_2 \leq y \leq y_3, \\ & n_1 \leq n \leq n_3, w_1 \leq w \leq w_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{W|N,Y}(w, n, y) = \begin{cases} \frac{w-w_1}{w_2-w_1} & w_1 \leq w \leq w_2, y_1 \leq y \leq y_3 \\ \frac{w-w_3}{w_2-w_3} & w_2 \leq w \leq w_3, y_1 \leq y \leq y_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{N|M,Y}(m, n, y) = \begin{cases} \frac{n-n_1}{n_2-n_1} & m_1 \leq m \leq m_3, \\ & n_1 \leq n \leq n_2, y_1 \leq y \leq y_3 \\ \frac{n-n_3}{n_2-n_3} & m_1 \leq m \leq m_3, \\ & n_2 \leq n \leq n_3, y_1 \leq y \leq y_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{P|Q,R}(p, q, r) = \begin{cases} \frac{p-p_1}{p_2-p_1} & p_1 \leq p \leq p_2, \\ & q_1 \leq q \leq q_3, r_1 \leq r \leq r_3 \\ \frac{p-p_3}{p_2-p_3} & p_2 \leq p \leq p_3, \\ & q_1 \leq q \leq q_3, r_1 \leq r \leq r_3, \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Q|P,R}(p, q, r) = \begin{cases} \frac{q-q_1}{q_2-q_1} & p_1 \leq p \leq p_3, \\ & q_1 \leq q \leq q_2, r_1 \leq r \leq r_3, \\ \frac{q-q_3}{q_2-q_3} & p_1 \leq p \leq p_3, \\ & q_2 \leq q \leq q_3, r_1 \leq r \leq r_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{R|P,Q}(p, q, r) = \begin{cases} \frac{r-r_1}{r_2-r_1} & p_1 \leq p \leq p_3, \\ & q_1 \leq q \leq q_3, r_1 \leq r \leq r_2 \\ \frac{r-r_3}{r_2-r_3} & p_1 \leq p \leq p_3, \\ & q_1 \leq q \leq q_3, r_2 \leq r \leq r_3 \\ 0 & \text{otherwise.} \end{cases}$$

The membership surfaces of column vectors of the imprecise matrix will be as:

$$\mu_C(x, z, m) = \begin{cases} \frac{(x-x_1)(z-z_1)(m-m_1)}{(x_2-x_1)(z_2-z_1)(m_2-m_1)} & x_1 \leq x \leq x_2, \\ & z_1 \leq z \leq z_2, m_1 \leq m \leq m_2 \\ \frac{(x-x_1)(z-z_3)(m-m_1)}{(x_2-x_1)(z_2-z_3)(m_2-m_1)} & x_1 \leq x \leq x_2, \\ & z_2 \leq z \leq z_3, m_1 \leq m \leq m_2 \\ \frac{(x-x_3)(z-z_1)(m-m_1)}{(x_2-x_3)(z_2-z_1)(m_2-m_1)} & x_2 \leq x \leq x_3, \\ & z_1 \leq z \leq z_2, m_1 \leq m \leq m_2 \\ \frac{(x-x_3)(z-z_3)(m-m_1)}{(x_2-x_3)(z_2-z_3)(m_2-m_1)} & x_2 \leq x \leq x_3, \\ & z_2 \leq z \leq z_3, m_1 \leq m \leq m_2 \\ \frac{(x-x_1)(z-z_1)(m-m_3)}{(x_2-x_1)(z_2-z_1)(m_2-m_3)} & x_1 \leq x \leq x_2, \\ & z_1 \leq z \leq z_2, m_2 \leq m \leq m_3 \\ \frac{(x-x_1)(z-z_3)(m-m_3)}{(x_2-x_1)(z_2-z_3)(m_2-m_3)} & x_1 \leq x \leq x_2, \\ & z_2 \leq z \leq z_3, m_2 \leq m \leq m_3 \\ \frac{(x-x_3)(z-z_1)(m-m_3)}{(x_2-x_3)(z_2-z_1)(m_2-m_3)} & x_2 \leq x \leq x_3, \\ & z_1 \leq z \leq z_2, m_2 \leq m \leq m_3 \\ \frac{(x-x_3)(z-z_3)(m-m_3)}{(x_2-x_3)(z_2-z_3)(m_2-m_3)} & x_2 \leq x \leq x_3, \\ & z_2 \leq z \leq z_3, m_2 \leq m \leq m_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_C(y, w, n) = \begin{cases} \frac{(y-y_1)(w-w_1)(n-n_1)}{(y_2-y_1)(w_2-w_1)(n_2-n_1)} & y_1 \leq y \leq y_2, \\ & w_1 \leq w \leq w_2, n_1 \leq n \leq n_2 \\ \frac{(y-y_1)(w-w_3)(n-n_1)}{(y_2-y_1)(w_2-w_3)(n_2-n_1)} & y_1 \leq y \leq y_2, \\ & w_2 \leq w \leq w_3, n_1 \leq n \leq n_2 \\ \frac{(y-y_3)(w-w_1)(n-n_1)}{(y_2-y_3)(w_2-w_1)(n_2-n_1)} & y_2 \leq y \leq y_3, \\ & w_1 \leq w \leq w_2, n_1 \leq n \leq n_2 \\ \frac{(y-y_3)(w-w_3)(n-n_1)}{(y_2-y_3)(w_2-w_3)(n_2-n_1)} & y_2 \leq y \leq y_3, \\ & w_2 \leq w \leq w_3, n_1 \leq n \leq n_2 \\ \frac{(y-y_1)(w-w_1)(n-n_3)}{(y_2-y_1)(w_2-w_1)(n_2-n_3)} & y_1 \leq y \leq y_2, \\ & w_1 \leq w \leq w_2, n_2 \leq n \leq n_3 \\ \frac{(y-y_1)(w-w_3)(n-n_3)}{(y_2-y_1)(w_2-w_3)(n_2-n_3)} & y_1 \leq y \leq y_2, \\ & w_2 \leq w \leq w_3, n_2 \leq n \leq n_3 \\ \frac{(y-y_3)(w-w_1)(n-n_3)}{(y_2-y_3)(w_2-w_1)(n_2-n_3)} & y_2 \leq y \leq y_3, \\ & w_1 \leq w \leq w_2, n_2 \leq n \leq n_3 \\ \frac{(y-y_3)(w-w_3)(n-n_3)}{(y_2-y_3)(w_2-w_3)(n_2-n_3)} & y_2 \leq y \leq y_3, \\ & w_2 \leq w \leq w_3, n_2 \leq n \leq n_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_C(p, q, r) = \begin{cases} \frac{(p-p_1)(q-q_1)(r-r_1)}{(p_2-p_1)(q_2-q_1)(r_2-r_1)} & p_1 \leq p \leq p_2, \\ & q_1 \leq q \leq q_2, r_1 \leq r \leq r_2 \\ \frac{(p-p_1)(q-q_3)(r-r_1)}{(p_2-p_1)(q_2-q_3)(r_2-r_1)} & p_1 \leq p \leq p_2, \\ & q_2 \leq q \leq q_3, r_1 \leq r \leq r_2 \\ \frac{(p-p_3)(q-q_1)(r-r_1)}{(p_2-p_3)(q_2-q_1)(r_2-r_1)} & p_2 \leq p \leq p_3, \\ & q_1 \leq q \leq q_2, r_1 \leq r \leq r_2 \\ \frac{(p-p_3)(q-q_3)(r-r_1)}{(p_2-p_3)(q_2-q_3)(r_2-r_1)} & p_2 \leq p \leq p_3, \\ & q_2 \leq q \leq q_3, r_1 \leq r \leq r_2 \\ \frac{(p-p_1)(q-q_1)(r-r_3)}{(p_2-p_1)(q_2-q_1)(r_2-r_3)} & p_1 \leq p \leq p_2, \\ & q_1 \leq q \leq q_2, r_2 \leq r \leq r_3 \\ \frac{(p-p_1)(q-q_3)(r-r_3)}{(p_2-p_1)(q_2-q_3)(r_2-r_3)} & p_1 \leq p \leq p_2, \\ & q_2 \leq q \leq q_3, r_2 \leq r \leq r_3 \\ \frac{(p-p_3)(q-q_1)(r-r_3)}{(p_2-p_3)(q_2-q_1)(r_2-r_3)} & p_2 \leq p \leq p_3, \\ & q_1 \leq q \leq q_2, r_2 \leq r \leq r_3 \\ \frac{(p-p_3)(q-q_3)(r-r_3)}{(p_2-p_3)(q_2-q_3)(r_2-r_3)} & p_2 \leq p \leq p_3, \\ & q_2 \leq q \leq q_3, r_2 \leq r \leq r_3 \\ 0 & \text{otherwise.} \end{cases}$$

III. ARITHMETIC OPERATIONS OF IMPRECISE MATRIX

Consider two matrices

$$A = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix}$$

and

$$B = \begin{pmatrix} P & Q \\ R & S \end{pmatrix}$$

where the row membership surfaces of A and B are as:

$$\mu_{A_R}(x, y) = \begin{cases} \frac{(x-x_1)(y-y_1)}{(x_2-x_1)(y_2-y_1)} & x_1 \leq x \leq x_2, y_1 \leq y \leq y_2 \\ \frac{(x-x_1)(y-y_3)}{(x_2-x_1)(y_2-y_3)} & x_1 \leq x \leq x_2, y_2 \leq y \leq y_3 \\ \frac{(x-x_3)(y-y_1)}{(x_2-x_3)(y_2-y_1)} & x_2 \leq x \leq x_3, y_1 \leq y \leq y_2 \\ \frac{(x-x_3)(y-y_3)}{(x_2-x_3)(y_2-y_3)} & x_2 \leq x \leq x_3, y_2 \leq y \leq y_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{A_R}(z, w) = \begin{cases} \frac{(z-z_1)(w-w_1)}{(z_2-z_1)(w_2-w_1)} & z_1 \leq z \leq z_2, w_1 \leq w \leq w_2 \\ \frac{(z-z_1)(w-w_3)}{(z_2-z_1)(w_2-w_3)} & z_1 \leq z \leq z_2, w_2 \leq w \leq w_3 \\ \frac{(z-z_3)(w-w_1)}{(z_2-z_3)(w_2-w_1)} & z_2 \leq z \leq z_3, w_1 \leq w \leq w_2 \\ \frac{(z-z_3)(w-w_3)}{(z_2-z_3)(w_2-w_3)} & z_2 \leq z \leq z_3, w_2 \leq w \leq w_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{B_R}(p, q) = \begin{cases} \frac{(p-p_1)(q-q_1)}{(p_2-p_1)(q_2-q_1)} & p_1 \leq p \leq p_2, q_1 \leq q \leq q_2 \\ \frac{(p-p_1)(q-q_3)}{(p_2-p_1)(q_2-q_3)} & p_1 \leq p \leq p_2, q_2 \leq q \leq q_3 \\ \frac{(p-p_3)(q-q_1)}{(p_2-p_3)(q_2-q_1)} & p_2 \leq p \leq p_3, q_1 \leq q \leq q_2 \\ \frac{(p-p_3)(q-q_3)}{(p_2-p_3)(q_2-q_3)} & p_2 \leq p \leq p_3, q_2 \leq q \leq q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{B_R}(r, s) = \begin{cases} \frac{(r-r_1)(s-s_1)}{(r_2-r_1)(s_2-s_1)} & r_1 \leq r \leq r_2, s_1 \leq s \leq s_2 \\ \frac{(r-r_1)(s-s_3)}{(r_2-r_1)(s_2-s_3)} & r_1 \leq r \leq r_2, s_2 \leq s \leq s_3 \\ \frac{(r-r_3)(s-s_1)}{(r_2-r_3)(s_2-s_1)} & r_2 \leq r \leq r_3, s_1 \leq s \leq s_2 \\ \frac{(r-r_3)(s-s_3)}{(r_2-r_3)(s_2-s_3)} & r_2 \leq r \leq r_3, s_2 \leq s \leq s_3 \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the column membership surfaces of A and B are as:

$$\mu_{A_C}(x, z) = \begin{cases} \frac{(x-x_1)(z-z_1)}{(x_2-x_1)(z_2-z_1)} & x_1 \leq x \leq x_2, z_1 \leq z \leq z_2 \\ \frac{(x-x_1)(z-z_3)}{(x_2-x_1)(z_2-z_3)} & x_1 \leq x \leq x_2, z_2 \leq z \leq z_3 \\ \frac{(x-x_3)(z-z_1)}{(x_2-x_3)(z_2-z_1)} & x_2 \leq x \leq x_3, z_1 \leq z \leq z_2 \\ \frac{(x-x_3)(z-z_3)}{(x_2-x_3)(z_2-z_3)} & x_2 \leq x \leq x_3, z_2 \leq z \leq z_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{A_C}(y, w) = \begin{cases} \frac{(y-y_1)(w-w_1)}{(y_2-y_1)(w_2-w_1)} & y_1 \leq y \leq y_2, w_1 \leq w \leq w_2 \\ \frac{(y-y_1)(w-w_3)}{(y_2-y_1)(w_2-w_3)} & y_1 \leq y \leq y_2, w_2 \leq w \leq w_3 \\ \frac{(y-y_3)(w-w_1)}{(y_2-y_3)(w_2-w_1)} & y_2 \leq y \leq y_3, w_1 \leq w \leq w_2 \\ \frac{(y-y_3)(w-w_3)}{(y_2-y_3)(w_2-w_3)} & y_2 \leq y \leq y_3, w_2 \leq w \leq w_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{B_C}(p, r) = \begin{cases} \frac{(p-p_1)(r-r_1)}{(p_2-p_1)(r_2-r_1)} & p_1 \leq p \leq p_2, r_1 \leq r \leq r_2 \\ \frac{(p-p_1)(r-r_3)}{(p_2-p_1)(r_2-r_3)} & p_1 \leq p \leq p_2, r_2 \leq r \leq r_3 \\ \frac{(p-p_3)(r-r_1)}{(p_2-p_3)(r_2-r_1)} & p_2 \leq p \leq p_3, r_1 \leq r \leq r_2 \\ \frac{(p-p_3)(r-r_3)}{(p_2-p_3)(r_2-r_3)} & p_2 \leq p \leq p_3, r_2 \leq r \leq r_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{BC}(q, s) = \begin{cases} \frac{(q - q_1)(s - s_1)}{(q_2 - q_1)(s_2 - s_1)} & q_1 \leq q \leq q_2, s_1 \leq s \leq s_2 \\ \frac{(q - q_1)(s - s_3)}{(q_2 - q_1)(s_2 - s_3)} & q_1 \leq q \leq q_2, s_2 \leq s \leq s_3 \\ \frac{(q_2 - q_3)(s - s_1)}{(q - q_3)(s - s_1)} & q_2 \leq q \leq q_3, s_1 \leq s \leq s_2 \\ \frac{(q_2 - q_3)(s_2 - s_1)}{(q - q_3)(s - s_3)} & q_2 \leq q \leq q_3, s_2 \leq s \leq s_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{CC}(l, n) = \begin{cases} \frac{\{(x+p) - (x_1 + p_1)\}\{(z+r) - (z_1 + r_1)\}}{\{(x_2 + p_2) - (x_1 + p_1)\}\{(z_2 + r_2) - (z_1 + r_1)\}} & x_1 + p_1 \leq x + p \leq x_2 + p_2, \\ & z_1 + r_1 \leq z + r \leq z_2 + r_2 \\ \frac{\{(x+p) - (x_1 + p_1)\}\{(z+r) - (z_3 + r_3)\}}{\{(x_2 + p_2) - (x_1 + p_1)\}\{(z_2 + r_2) - (z_3 + r_3)\}} & x_1 + p_1 \leq x + p \leq x_2 + p_2, \\ & z_2 + r_2 \leq z + r \leq z_3 + r_3 \\ \frac{\{(x+p) - (x_3 + p_3)\}\{(z+r) - (z_1 + r_1)\}}{\{(x_2 + p_2) - (x_3 + p_3)\}\{(z_2 + r_2) - (z_1 + r_1)\}} & x_2 + p_2 \leq x + p \leq x_3 + p_3, \\ & z_1 + r_1 \leq z + r \leq z_2 + r_2 \\ \frac{\{(x+p) - (x_3 + p_3)\}\{(z+r) - (z_3 + r_3)\}}{\{(x_2 + p_2) - (x_3 + p_3)\}\{(z_2 + r_2) - (z_3 + r_3)\}} & x_2 + p_2 \leq x + p \leq x_3 + p_3, \\ & z_2 + r_2 \leq z + r \leq z_3 + r_3 \\ 0 & \text{otherwise.} \end{cases}$$

Here, for simplicity and without any loss of generality it is assumed that $X = [x_1, x_2, x_3]$, $Y = [y_1, y_2, y_3]$, $Z = [z_1, z_2, z_3]$, $W = [w_1, w_2, w_3]$ and $P = [p_1, p_2, p_3]$, $Q = [q_1, q_2, q_3]$, $R = [r_1, r_2, r_3]$, $S = [s_1, s_2, s_3]$ are triangular imprecise numbers.

A. Addition of Imprecise Matrix

Addition of two imprecise matrices A and B will be as:

$$A + B = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix} + \begin{pmatrix} P & Q \\ R & S \end{pmatrix} = \begin{pmatrix} L & M \\ N & K \end{pmatrix}$$

where $L = X + P = [x_1 + p_1, x_2 + p_2, x_3 + p_3]$, $M = Y + Q = [y_1 + q_1, y_2 + q_2, y_3 + q_3]$, $N = Z + R = [z_1 + r_1, z_2 + r_2, z_3 + r_3]$ and $K = W + S = [w_1 + s_1, w_2 + s_2, w_3 + s_3]$. Now, the row membership surfaces of the matrix

$$C = \begin{pmatrix} L & M \\ N & K \end{pmatrix}$$

will be as follows

$$\mu_{CR}(l, m) = \begin{cases} \frac{\{(x+p) - (x_1 + p_1)\}\{(y+q) - (y_1 + q_1)\}}{\{(x_2 + p_2) - (x_1 + p_1)\}\{(y_2 + q_2) - (y_1 + q_1)\}} & x_1 + p_1 \leq x + p \leq x_2 + p_2, \\ & y_1 + q_1 \leq y + q \leq y_2 + q_2 \\ \frac{\{(x+p) - (x_1 + p_1)\}\{(y+q) - (y_3 + q_3)\}}{\{(x_2 + p_2) - (x_1 + p_1)\}\{(y_2 + q_2) - (y_3 + q_3)\}} & x_1 + p_1 \leq x + p \leq x_2 + p_2, \\ & y_2 + q_2 \leq y + q \leq y_3 + q_3 \\ \frac{\{(x+p) - (x_3 + p_3)\}\{(y+q) - (y_1 + q_1)\}}{\{(x_2 + p_2) - (x_3 + p_3)\}\{(y_2 + q_2) - (y_1 + q_1)\}} & x_2 + p_2 \leq x + p \leq x_3 + p_3, \\ & y_1 + q_1 \leq y + q \leq y_2 + q_2 \\ \frac{\{(x+p) - (x_3 + p_3)\}\{(y+q) - (y_3 + q_3)\}}{\{(x_2 + p_2) - (x_3 + p_3)\}\{(y_2 + q_2) - (y_3 + q_3)\}} & x_2 + p_2 \leq x + p \leq x_3 + p_3, \\ & y_2 + q_2 \leq y + q \leq y_3 + q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{CR}(n, k) =$$

$$\begin{cases} \frac{\{(z+r) - (z_1 + r_1)\}\{(w+s) - (w_1 + s_1)\}}{\{(z_2 + r_2) - (z_1 + r_1)\}\{(w_2 + s_2) - (w_1 + s_1)\}} & z_1 + r_1 \leq z + r \leq z_2 + r_2, \\ & w_1 + s_1 \leq w + s \leq w_2 + s_2 \\ \frac{\{(z+r) - (z_1 + r_1)\}\{(w+s) - (w_3 + s_3)\}}{\{(z_2 + r_2) - (z_1 + r_1)\}\{(w_2 + s_2) - (w_3 + s_3)\}} & z_1 + r_1 \leq z + r \leq z_2 + r_2, \\ & w_2 + s_2 \leq w + s \leq w_3 + s_3 \\ \frac{\{(z+r) - (z_3 + r_3)\}\{(w+s) - (w_1 + s_1)\}}{\{(z_2 + r_2) - (z_3 + r_3)\}\{(w_2 + s_2) - (w_1 + s_1)\}} & z_2 + r_2 \leq z + r \leq z_3 + r_3, \\ & w_1 + s_1 \leq w + s \leq w_2 + s_2 \\ \frac{\{(z+r) - (z_3 + r_3)\}\{(w+s) - (w_3 + s_3)\}}{\{(z_2 + r_2) - (z_3 + r_3)\}\{(w_2 + s_2) - (w_3 + s_3)\}} & z_2 + r_2 \leq z + r \leq z_3 + r_3, \\ & w_2 + s_2 \leq w + s \leq w_3 + s_3 \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the column membership surfaces of the matrix C will be as:

$$\mu_{CC}(m, k) = \begin{cases} \frac{\{(y+q) - (y_1 + q_1)\}\{(w+s) - (w_1 + s_1)\}}{\{(y_2 + q_2) - (y_1 + q_1)\}\{(w_2 + s_2) - (w_1 + s_1)\}} & y_1 + q_1 \leq y + q \leq y_2 + q_2, \\ & w_1 + s_1 \leq w + s \leq w_2 + s_2 \\ \frac{\{(y+q) - (y_1 + q_1)\}\{(w+s) - (w_3 + s_3)\}}{\{(y_2 + q_2) - (y_1 + q_1)\}\{(w_2 + s_2) - (w_3 + s_3)\}} & y_1 + q_1 \leq y + q \leq y_2 + q_2, \\ & w_2 + s_2 \leq w + s \leq w_3 + s_3 \\ \frac{\{(y+q) - (y_3 + q_3)\}\{(w+s) - (w_1 + s_1)\}}{\{(y_2 + q_2) - (y_3 + q_3)\}\{(w_2 + s_2) - (w_1 + s_1)\}} & y_2 + q_2 \leq y + q \leq y_3 + q_3, \\ & w_1 + s_1 \leq w + s \leq w_2 + s_2 \\ \frac{\{(y+q) - (y_3 + q_3)\}\{(w+s) - (w_3 + s_3)\}}{\{(y_2 + q_2) - (y_3 + q_3)\}\{(w_2 + s_2) - (w_3 + s_3)\}} & y_2 + q_2 \leq y + q \leq y_3 + q_3, \\ & w_2 + s_2 \leq w + s \leq w_3 + s_3 \\ 0 & \text{otherwise.} \end{cases}$$

B. Subtraction of Imprecise Matrix

Subtraction of two imprecise matrices A and B will be

$$A - B = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix} - \begin{pmatrix} P & Q \\ R & S \end{pmatrix} = \begin{pmatrix} L & M \\ N & K \end{pmatrix}$$

where $L = X - P = [x_1 - p_3, x_2 - p_2, x_3 - p_1]$, $M = Y - Q = [y_1 - q_3, y_2 - q_2, y_3 - q_1]$, $N = Z - R = [z_1 - r_3, z_2 - r_2, z_3 - r_1]$ and $K = W - S = [w_1 - s_3, w_2 - s_2, w_3 - s_1]$ respectively. Now, the row membership surfaces of the matrix

$$C = \begin{pmatrix} L & M \\ N & K \end{pmatrix}$$

will be as follows

$$\mu_{CR}(l, m) = \begin{cases} \frac{\{(x-p) - (x_1 - p_3)\}\{(y-q) - (y_1 - q_3)\}}{\{(x_2 - p_2) - (x_1 - p_3)\}\{(y_2 - q_2) - (y_1 - q_3)\}} & x_1 - p_3 \leq x - p \leq x_2 - p_2, \\ & y_1 - q_3 \leq y - q \leq y_2 - q_2 \\ \frac{\{(x-p) - (x_1 - p_3)\}\{(y-q) - (y_3 - q_1)\}}{\{(x_2 - p_2) - (x_1 - p_3)\}\{(y_2 - q_2) - (y_3 - q_1)\}} & x_1 - p_3 \leq x - p \leq x_2 - p_2, \\ & y_2 - q_2 \leq y - q \leq y_3 - q_1 \\ \frac{\{(x-p) - (x_3 - p_1)\}\{(y-q) - (y_1 - q_3)\}}{\{(x_2 - p_2) - (x_3 - p_1)\}\{(y_2 - q_2) - (y_1 - q_3)\}} & x_2 - p_2 \leq x - p \leq x_3 - p_1, \\ & y_1 - q_3 \leq y - q \leq y_2 - q_2 \\ \frac{\{(x-p) - (x_3 - p_1)\}\{(y-q) - (y_3 - q_1)\}}{\{(x_2 - p_2) - (x_3 - p_1)\}\{(y_2 - q_2) - (y_3 - q_1)\}} & x_2 - p_2 \leq x - p \leq x_3 - p_1, \\ & y_2 - q_2 \leq y - q \leq y_3 - q_1 \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\mu_{CR}(n, k) = \begin{cases} \frac{\{(z-r) - (z_1-r_3)\}\{(w-s) - (w_1-s_3)\}}{\{(z_2-r_2) - (z_1-r_3)\}\{(w_2-s_2) - (w_1-s_3)\}} & z_1-r_3 \leq z-r \leq z_2-r_2, \\ & w_1-s_3 \leq w-s \leq w_2-s_2 \\ \frac{\{(z-r) - (z_1-r_3)\}\{(w-s) - (w_3-s_1)\}}{\{(z_2-r_2) - (z_1-r_3)\}\{(w_2-s_2) - (w_3-s_1)\}} & z_1-r_3 \leq z-r \leq z_2-r_2, \\ & w_2-s_2 \leq w-s \leq w_3-s_1 \\ \frac{\{(z-r) - (z_3-r_1)\}\{(w-s) - (w_1-s_3)\}}{\{(z_2-r_2) - (z_3-r_1)\}\{(w_2-s_2) - (w_1-s_3)\}} & z_2-r_2 \leq z-r \leq z_3-r_1, \\ & w_1-s_3 \leq w-s \leq w_2-s_2 \\ \frac{\{(z-r) - (z_3-r_1)\}\{(w-s) - (w_3-s_1)\}}{\{(z_2-r_2) - (z_3-r_1)\}\{(w_2-s_2) - (w_3-s_1)\}} & z_2-r_2 \leq z-r \leq z_3-r_1, \\ & w_2-s_2 \leq w-s \leq w_3-s_1 \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the column membership surfaces of the matrix C will be as:

$$\mu_{CC}(l, n) = \begin{cases} \frac{\{(x-p) - (x_1-p_3)\}\{(z-r) - (z_1-r_3)\}}{\{(x_2-p_2) - (x_1-p_3)\}\{(z_2-r_2) - (z_1-r_3)\}} & x_1-p_3 \leq x-p \leq x_2-p_2, \\ & z_1-r_3 \leq z-r \leq z_2-r_2 \\ \frac{\{(x-p) - (x_1-p_3)\}\{(z-r) - (z_3-r_1)\}}{\{(x_2-p_2) - (x_1-p_3)\}\{(z_2-r_2) - (z_3-r_1)\}} & x_1-p_3 \leq x-p \leq x_2-p_2, \\ & z_2-r_2 \leq z-r \leq z_3-r_1 \\ \frac{\{(x-p) - (x_3-p_1)\}\{(z-r) - (z_1-r_3)\}}{\{(x_2-p_2) - (x_3-p_1)\}\{(z_2-r_2) - (z_1-r_3)\}} & x_2-p_2 \leq x-p \leq x_3-p_1, \\ & z_1-r_3 \leq z-r \leq z_2-r_2 \\ \frac{\{(x-p) - (x_3-p_1)\}\{(z-r) - (z_3-r_1)\}}{\{(x_2-p_2) - (x_3-p_1)\}\{(z_2-r_2) - (z_3-r_1)\}} & x_2-p_2 \leq x-p \leq x_3-p_1, \\ & z_2-r_2 \leq z-r \leq z_3-r_1 \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\mu_{CC}(m, k) = \begin{cases} \frac{\{(y-q) - (y_1-q_3)\}\{(w-s) - (w_1-s_3)\}}{\{(y_2-q_2) - (y_1-q_3)\}\{(w_2-s_2) - (w_1-s_3)\}} & y_1-q_3 \leq y-q \leq y_2-q_2, \\ & w_1-s_3 \leq w-s \leq w_2-s_2 \\ \frac{\{(y-q) - (y_1-q_3)\}\{(w-s) - (w_3-s_1)\}}{\{(y_2-q_2) - (y_1-q_3)\}\{(w_2-s_2) - (w_3-s_1)\}} & y_1-q_3 \leq y-q \leq y_2-q_2, \\ & w_2-s_2 \leq w-s \leq w_3-s_1 \\ \frac{\{(y-q) - (y_3-q_1)\}\{(w-s) - (w_1-s_3)\}}{\{(y_2-q_2) - (y_3-q_1)\}\{(w_2-s_2) - (w_1-s_3)\}} & y_2-q_2 \leq y-q \leq y_3-q_1, \\ & w_1-s_3 \leq w-s \leq w_2-s_2 \\ \frac{\{(y-q) - (y_3-q_1)\}\{(w-s) - (w_3-s_1)\}}{\{(y_2-q_2) - (y_3-q_1)\}\{(w_2-s_2) - (w_3-s_1)\}} & y_2-q_2 \leq y-q \leq y_3-q_1, \\ & w_2-s_2 \leq w-s \leq w_3-s_1 \\ 0 & \text{otherwise.} \end{cases}$$

C. Multiplication of imprecise matrix

Multiplication of two imprecise matrices A and B will be as:

$$A.B = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix} \cdot \begin{pmatrix} P & Q \\ R & S \end{pmatrix} = \begin{pmatrix} L & M \\ N & K \end{pmatrix}$$

where

$$L = XP + YR = [x_1p_1, x_2p_2, x_3p_3]$$

$$M = XQ + YS = [y_1q_1, y_2q_2, y_3q_3]$$

$$N = ZP + WR = [z_1r_1, z_2r_2, z_3r_3]$$

$$K = ZQ + WS = [w_1s_1, w_2s_2, w_3s_3]$$

The membership functions of L, M, N and K are as:

$$\mu_{L|M}(l, m) = \begin{cases} \frac{1}{2(x_2-x_3)(p_2-p_3)} \{-x_1p_2 - 2x_1p_1 + x_2p_1\} + \sqrt{(x_1p_2 - 2x_1p_1 + x_2p_1)^2 - 4(x_1p_1 - l)(x_2-x_1)(p_2-p_1)} & x_1p_1 \leq l \leq x_2p_2, \\ & y_1q_1 \leq m \leq y_3q_3 \\ \frac{1}{2(x_2-x_3)(p_2-p_3)} \{-x_3p_2 - 2x_3p_3 + x_2p_3\} - \sqrt{(x_3p_2 - 2x_3p_3 + x_2p_3)^2 - 4(x_3p_3 - l)(x_2-x_3)(p_2-p_3)} & x_2p_2 \leq l \leq x_3p_3, \\ & y_1q_1 \leq m \leq y_3q_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{M|L}(l, m) = \begin{cases} \frac{1}{2(y_2-y_3)(q_2-q_3)} \{-y_1q_2 - 2y_1q_1 + y_2q_1\} + \sqrt{(y_1q_2 - 2y_1q_1 + y_2q_1)^2 - 4(y_1q_1 - m)(y_2-y_1)(q_2-q_1)} & y_1q_1 \leq m \leq y_2q_2, \\ & x_1p_1 \leq l \leq x_3p_3 \\ \frac{1}{2(y_2-y_3)(q_2-q_3)} \{-y_3q_2 - 2y_3q_3 + y_2q_3\} - \sqrt{(y_3q_2 - 2y_3q_3 + y_2q_3)^2 - 4(y_3q_3 - m)(y_2-y_3)(q_2-q_3)} & y_2q_2 \leq m \leq y_3q_3, \\ & x_1p_1 \leq l \leq x_3p_3 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{N|K}(n, k) = \begin{cases} \frac{1}{2(z_2-z_3)(r_2-r_3)} \{-z_1r_2 - 2z_1r_1 + z_2r_1\} + \sqrt{(z_1r_2 - 2z_1r_1 + z_2r_1)^2 - 4(z_1r_1 - n)(z_2-z_1)(r_2-r_1)} & z_1r_1 \leq n \leq z_2r_2, \\ & w_1s_1 \leq k \leq w_3s_3 \\ \frac{1}{2(z_2-z_3)(r_2-r_3)} \{-z_3r_2 - 2z_3r_3 + z_2r_3\} - \sqrt{(z_3r_2 - 2z_3r_3 + z_2r_3)^2 - 4(z_3r_3 - n)(z_2-z_3)(r_2-r_3)} & z_2r_2 \leq n \leq z_3r_3, \\ & w_1s_1 \leq k \leq w_3s_3 \\ 0 & \text{otherwise.} \end{cases}$$

and

$$\mu_{K|N}(n, k) = \begin{cases} \frac{1}{2(w_2-w_3)(s_2-s_3)} \{-w_1s_2 - 2w_1s_1 + w_2s_1\} + \sqrt{(w_1s_2 - 2w_1s_1 + w_2s_1)^2 - 4(w_1s_1 - k)(w_2-w_1)(s_2-s_1)} & w_1s_1 \leq k \leq w_2s_2, \\ & z_1r_1 \leq n \leq z_3r_3 \\ \frac{1}{2(w_2-w_3)(s_2-s_3)} \{-w_3s_2 - 2w_3s_3 + w_2s_3\} - \sqrt{(w_3s_2 - 2w_3s_3 + w_2s_3)^2 - 4(w_3s_3 - k)(w_2-w_3)(s_2-s_3)} & w_2s_2 \leq k \leq w_3s_3, \\ & z_1r_1 \leq n \leq z_3r_3 \\ 0 & \text{otherwise.} \end{cases}$$

Now, the row and column membership surfaces of the matrix

$$C = A.B = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix} \cdot \begin{pmatrix} P & Q \\ R & S \end{pmatrix} = \begin{pmatrix} L & M \\ N & K \end{pmatrix}$$

will be as shown in (1)-(4).

$$\mu_{C_R}(l, m) = \begin{cases} \frac{1}{4(x_2 - x_3)(p_2 - p_3)(y_2 - y_3)(q_2 - q_3)} \{-(x_1 p_2 - 2x_1 p_1 + x_2 p_1) + \sqrt{(x_1 p_2 - 2x_1 p_1 + x_2 p_1)^2 - 4(x_1 p_1 - l)(x_2 - x_1)(p_2 - p_1)}\} \{-(y_1 q_2 - 2y_1 q_1 + y_2 q_1) + \sqrt{(y_1 q_2 - 2y_1 q_1 + y_2 q_1)^2 - 4(y_1 q_1 - m)(y_2 - y_1)(q_2 - q_1)}\} & x_1 p_1 \leq l \leq x_2 p_2, y_1 q_1 \leq m \leq y_2 q_2 \\ \frac{1}{4(x_2 - x_3)(p_2 - p_3)(y_2 - y_3)(q_2 - q_3)} \{-(x_1 p_2 - 2x_1 p_1 + x_2 p_1) + \sqrt{(x_1 p_2 - 2x_1 p_1 + x_2 p_1)^2 - 4(x_1 p_1 - l)(x_2 - x_1)(p_2 - p_1)}\} \{-(y_3 q_2 - 2y_3 q_3 + y_2 q_3) - \sqrt{(y_3 q_2 - 2y_3 q_3 + y_2 q_3)^2 - 4(y_3 q_3 - m)(y_2 - y_3)(q_2 - q_3)}\} & x_1 p_1 \leq l \leq x_2 p_2; y_2 q_2 \leq m \leq y_3 q_3 \\ \frac{1}{4(x_2 - x_3)(p_2 - p_3)(y_2 - y_3)(q_2 - q_3)} \{-(x_3 p_2 - 2x_3 p_3 + x_2 p_3) - \sqrt{(x_3 p_2 - 2x_3 p_3 + x_2 p_3)^2 - 4(x_3 p_3 - l)(x_2 - x_3)(p_2 - p_3)}\} \{-(y_1 q_2 - 2y_1 q_1 + y_2 q_1) + \sqrt{(y_1 q_2 - 2y_1 q_1 + y_2 q_1)^2 - 4(y_1 q_1 - m)(y_2 - y_1)(q_2 - q_1)}\} & x_2 p_2 \leq l \leq x_3 p_3; y_1 q_1 \leq m \leq y_2 q_2 \\ \frac{1}{4(x_2 - x_3)(p_2 - p_3)(y_2 - y_3)(q_2 - q_3)} \{-(x_3 p_2 - 2x_3 p_3 + x_2 p_3) - \sqrt{(x_3 p_2 - 2x_3 p_3 + x_2 p_3)^2 - 4(x_3 p_3 - l)(x_2 - x_3)(p_2 - p_3)}\} \{-(y_3 q_2 - 2y_3 q_3 + y_2 q_3) - \sqrt{(y_3 q_2 - 2y_3 q_3 + y_2 q_3)^2 - 4(y_3 q_3 - m)(y_2 - y_3)(q_2 - q_3)}\} & x_2 p_2 \leq l \leq x_3 p_3; y_2 q_2 \leq m \leq y_3 q_3 \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

$$\mu_{C_R}(n, k) = \begin{cases} \frac{1}{4(z_2 - z_3)(r_2 - r_3)(w_2 - w_3)(s_2 - s_3)} \{-(z_1 r_2 - 2z_1 r_1 + z_2 r_1) + \sqrt{(z_1 r_2 - 2z_1 r_1 + z_2 r_1)^2 - 4(z_1 r_1 - n)(z_2 - z_1)(r_2 - r_1)}\} \{-(w_1 s_2 - 2w_1 s_1 + w_2 s_1) + \sqrt{(w_1 s_2 - 2w_1 s_1 + w_2 s_1)^2 - 4(w_1 s_1 - k)(w_2 - w_1)(s_2 - s_1)}\} & z_1 r_1 \leq n \leq z_2 r_2; w_1 s_1 \leq k \leq w_2 s_2 \\ \frac{1}{4(z_2 - z_3)(r_2 - r_3)(w_2 - w_3)(s_2 - s_3)} \{-(z_1 r_2 - 2z_1 r_1 + z_2 r_1) + \sqrt{(z_1 r_2 - 2z_1 r_1 + z_2 r_1)^2 - 4(z_1 r_1 - n)(z_2 - z_1)(r_2 - r_1)}\} \{-(w_3 s_2 - 2w_3 s_3 + w_2 s_3) - \sqrt{(w_3 s_2 - 2w_3 s_3 + w_2 s_3)^2 - 4(w_3 s_3 - k)(w_2 - w_3)(s_2 - s_3)}\} & z_1 r_1 \leq n \leq z_2 r_2; w_2 s_2 \leq k \leq w_3 s_3 \\ \frac{1}{4(z_2 - z_3)(r_2 - r_3)(w_2 - w_3)(s_2 - s_3)} \{-(z_3 r_2 - 2z_3 r_3 + z_2 r_3) - \sqrt{(z_3 r_2 - 2z_3 r_3 + z_2 r_3)^2 - 4(z_3 r_3 - n)(z_2 - z_3)(r_2 - r_3)}\} \{-(w_1 s_2 - 2w_1 s_1 + w_2 s_1) + \sqrt{(w_1 s_2 - 2w_1 s_1 + w_2 s_1)^2 - 4(w_1 s_1 - k)(w_2 - w_1)(s_2 - s_1)}\} & z_2 r_2 \leq n \leq z_3 r_3; w_1 s_1 \leq k \leq w_2 s_2 \\ \frac{1}{4(z_2 - z_3)(r_2 - r_3)(w_2 - w_3)(s_2 - s_3)} \{-(z_3 r_2 - 2z_3 r_3 + z_2 r_3) - \sqrt{(z_3 r_2 - 2z_3 r_3 + z_2 r_3)^2 - 4(z_3 r_3 - n)(z_2 - z_3)(r_2 - r_3)}\} \{-(w_3 s_2 - 2w_3 s_3 + w_2 s_3) - \sqrt{(w_3 s_2 - 2w_3 s_3 + w_2 s_3)^2 - 4(w_3 s_3 - k)(w_2 - w_3)(s_2 - s_3)}\} & z_2 r_2 \leq n \leq z_3 r_3; w_2 s_2 \leq k \leq w_3 s_3 \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

$$\mu_{C_C}(l, n) = \begin{cases} \frac{1}{4(x_2 - x_3)(p_2 - p_3)(z_2 - z_3)(r_2 - r_3)} \{-(x_1 p_2 - 2x_1 p_1 + x_2 p_1) + \sqrt{(x_1 p_2 - 2x_1 p_1 + x_2 p_1)^2 - 4(x_1 p_1 - l)(x_2 - x_1)(p_2 - p_1)}\} \{-(z_1 r_2 - 2z_1 r_1 + z_2 r_1) + \sqrt{(z_1 r_2 - 2z_1 r_1 + z_2 r_1)^2 - 4(z_1 r_1 - n)(z_2 - z_1)(r_2 - r_1)}\} & x_1 p_1 \leq l \leq x_2 p_2; z_1 r_1 \leq n \leq z_2 r_2 \\ \frac{1}{4(x_2 - x_3)(p_2 - p_3)(z_2 - z_3)(r_2 - r_3)} \{-(x_1 p_2 - 2x_1 p_1 + x_2 p_1) + \sqrt{(x_1 p_2 - 2x_1 p_1 + x_2 p_1)^2 - 4(x_1 p_1 - l)(x_2 - x_1)(p_2 - p_1)}\} \{-(z_3 r_2 - 2z_3 r_3 + z_2 r_3) - \sqrt{(z_3 r_2 - 2z_3 r_3 + z_2 r_3)^2 - 4(z_3 r_3 - n)(z_2 - z_3)(r_2 - r_3)}\} & x_1 p_1 \leq l \leq x_2 p_2; z_2 r_2 \leq n \leq z_3 r_3 \\ \frac{1}{4(x_2 - x_3)(p_2 - p_3)(z_2 - z_3)(r_2 - r_3)} \{-(x_3 p_2 - 2x_3 p_3 + x_2 p_3) - \sqrt{(x_3 p_2 - 2x_3 p_3 + x_2 p_3)^2 - 4(x_3 p_3 - l)(x_2 - x_3)(p_2 - p_3)}\} \{-(z_1 r_2 - 2z_1 r_1 + z_2 r_1) + \sqrt{(z_1 r_2 - 2z_1 r_1 + z_2 r_1)^2 - 4(z_1 r_1 - n)(z_2 - z_1)(r_2 - r_1)}\} & x_2 p_2 \leq l \leq x_3 p_3; z_1 r_1 \leq n \leq z_2 r_2 \\ \frac{1}{4(x_2 - x_3)(p_2 - p_3)(z_2 - z_3)(r_2 - r_3)} \{-(x_3 p_2 - 2x_3 p_3 + x_2 p_3) - \sqrt{(x_3 p_2 - 2x_3 p_3 + x_2 p_3)^2 - 4(x_3 p_3 - l)(x_2 - x_3)(p_2 - p_3)}\} \{-(z_3 r_2 - 2z_3 r_3 + z_2 r_3) - \sqrt{(z_3 r_2 - 2z_3 r_3 + z_2 r_3)^2 - 4(z_3 r_3 - n)(z_2 - z_3)(r_2 - r_3)}\} & x_2 p_2 \leq l \leq x_3 p_3; z_2 r_2 \leq n \leq z_3 r_3 \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

$$\mu_{C_R}(n, k) = \begin{cases} \frac{1}{4(y_2 - y_3)(q_2 - q_3)(w_2 - w_3)(s_2 - s_3)} \{-(y_1 q_2 - 2y_1 q_1 + y_2 q_1) + \sqrt{(y_1 q_2 - 2y_1 q_1 + y_2 q_1)^2 - 4(y_1 q_1 - m)(y_2 - y_1)(q_2 - q_1)}\} \{-(w_1 s_2 - 2w_1 s_1 + w_2 s_1) + \sqrt{(w_1 s_2 - 2w_1 s_1 + w_2 s_1)^2 - 4(w_1 s_1 - k)(w_2 - w_1)(s_2 - s_1)}\} & y_1 q_1 \leq m \leq y_2 q_2; w_1 s_1 \leq k \leq w_2 s_2 \\ \frac{1}{4(y_2 - y_3)(q_2 - q_3)(w_2 - w_3)(s_2 - s_3)} \{-(y_1 q_2 - 2y_1 q_1 + y_2 q_1) + \sqrt{(y_1 q_2 - 2y_1 q_1 + y_2 q_1)^2 - 4(y_1 q_1 - m)(y_2 - y_1)(q_2 - q_1)}\} \{-(w_3 s_2 - 2w_3 s_3 + w_2 s_3) - \sqrt{(w_3 s_2 - 2w_3 s_3 + w_2 s_3)^2 - 4(w_3 s_3 - k)(w_2 - w_3)(s_2 - s_3)}\} & y_1 q_1 \leq m \leq y_2 q_2; w_2 s_2 \leq k \leq w_3 s_3 \\ \frac{1}{4(y_2 - y_3)(q_2 - q_3)(w_2 - w_3)(s_2 - s_3)} \{-(y_3 q_2 - 2y_3 q_3 + y_2 q_3) - \sqrt{(y_3 q_2 - 2y_3 q_3 + y_2 q_3)^2 - 4(y_3 q_3 - m)(y_2 - y_3)(q_2 - q_3)}\} \{-(w_1 s_2 - 2w_1 s_1 + w_2 s_1) + \sqrt{(w_1 s_2 - 2w_1 s_1 + w_2 s_1)^2 - 4(w_1 s_1 - k)(w_2 - w_1)(s_2 - s_1)}\} & y_2 q_2 \leq m \leq y_3 q_3; w_1 s_1 \leq k \leq w_2 s_2 \\ \frac{1}{4(y_2 - y_3)(q_2 - q_3)(w_2 - w_3)(s_2 - s_3)} \{-(y_3 q_2 - 2y_3 q_3 + y_2 q_3) - \sqrt{(y_3 q_2 - 2y_3 q_3 + y_2 q_3)^2 - 4(y_3 q_3 - m)(y_2 - y_3)(q_2 - q_3)}\} \{-(w_3 s_2 - 2w_3 s_3 + w_2 s_3) - \sqrt{(w_3 s_2 - 2w_3 s_3 + w_2 s_3)^2 - 4(w_3 s_3 - k)(w_2 - w_3)(s_2 - s_3)}\} & y_2 q_2 \leq m \leq y_3 q_3; w_2 s_2 \leq k \leq w_3 s_3 \\ 0 & \text{otherwise.} \end{cases} \quad (4)$$

IV. NUMERICAL EXAMPLE

Consider two matrices

$$A = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix}$$

and

$$B = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}$$

Where $X_{11} = [1, 2, 3]$, $X_{12} = [2, 3, 4]$, $X_{21} = [3, 4, 5]$, $X_{22} = [5, 6, 7]$ and $Y_{11} = [2, 3, 4]$, $Y_{12} = [6, 7, 8]$, $Y_{21} = [8, 9, 10]$, $Y_{22} = [1, 2, 3]$ are imprecise numbers. For simplicity we have assumed that all are triangular imprecise numbers.

A. Addition of two imprecise matrices

Addition of two imprecise matrices A and B will be as follows

$$A + B = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} + \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

where $Z_{11} = [3, 5, 7]$, $Z_{12} = [8, 10, 12]$, $Z_{21} = [11, 13, 15]$ and $Z_{22} = [6, 8, 10]$ respectively. The membership functions of Z_{11} , Z_{12} , Z_{21} and Z_{22} for row vectors are as follows

$$\mu_{Z_{11}|Z_{12}}(z_{11}, z_{12}) = \begin{cases} \frac{z_{11}-3}{2} & 3 \leq z_{11} \leq 5, \\ & 8 \leq z_{12} \leq 12 \\ \frac{7-z_{11}}{2} & 5 \leq z_{11} \leq 7, \\ & 8 \leq z_{12} \leq 12 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{12}|Z_{11}}(z_{11}, z_{12}) = \begin{cases} \frac{z_{12}-8}{2} & 8 \leq z_{12} \leq 10, \\ & 3 \leq z_{11} \leq 7 \\ \frac{12-z_{12}}{2} & 10 \leq z_{12} \leq 12, \\ & 3 \leq z_{11} \leq 7 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{21}|Z_{22}}(Z_{21}, Z_{22}) = \begin{cases} \frac{z_{21}-11}{2} & 11 \leq z_{21} \leq 13, \\ & 6 \leq z_{22} \leq 10 \\ \frac{15-z_{21}}{2} & 13 \leq z_{21} \leq 15, \\ & 6 \leq z_{22} \leq 10 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{22}|Z_{21}}(Z_{21}, Z_{22}) = \begin{cases} \frac{z_{22}-6}{2} & 6 \leq z_{22} \leq 8, \\ & 11 \leq z_{21} \leq 15 \\ \frac{10-z_{22}}{2} & 8 \leq z_{22} \leq 10, \\ & 11 \leq z_{21} \leq 15 \\ 0 & \text{otherwise.} \end{cases}$$

Now, the row membership surfaces of the matrix

$$C = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

will be as follows

$$\mu_R(z_{11}, z_{12}) = \begin{cases} \frac{(z_{11}-3)(z_{12}-8)}{4} & 3 \leq z_{11} \leq 5, \\ & 8 \leq z_{12} \leq 10 \\ \frac{(z_{11}-3)(12-z_{12})}{4} & 3 \leq z_{11} \leq 5, \\ & 10 \leq z_{12} \leq 12 \\ \frac{(7-z_{11})(z_{12}-8)}{4} & 5 \leq z_{11} \leq 7, \\ & 8 \leq z_{12} \leq 10 \\ \frac{(7-z_{11})(12-z_{12})}{4} & 5 \leq z_{11} \leq 7, \\ & 10 \leq z_{12} \leq 12 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_R(z_{21}, z_{22}) = \begin{cases} \frac{(z_{21}-11)(z_{22}-6)}{4} & 11 \leq z_{21} \leq 13, \\ & 6 \leq z_{22} \leq 8 \\ \frac{(z_{21}-11)(10-z_{22})}{4} & 11 \leq z_{21} \leq 13, \\ & 8 \leq z_{22} \leq 10 \\ \frac{(15-z_{21})(z_{22}-6)}{4} & 13 \leq z_{21} \leq 15, \\ & 6 \leq z_{22} \leq 8 \\ \frac{(15-z_{21})(10-z_{22})}{4} & 13 \leq z_{21} \leq 15, \\ & 8 \leq z_{22} \leq 10 \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the column membership surfaces of the matrix C will be as follows

$$\mu_C(z_{11}, z_{21}) = \begin{cases} \frac{(z_{11}-3)(z_{21}-11)}{4} & 3 \leq z_{11} \leq 5, \\ & 11 \leq z_{21} \leq 13 \\ \frac{(z_{11}-3)(15-z_{21})}{4} & 3 \leq z_{11} \leq 5, \\ & 13 \leq z_{21} \leq 15 \\ \frac{(7-z_{11})(z_{21}-11)}{4} & 5 \leq z_{11} \leq 7, \\ & 11 \leq z_{21} \leq 13 \\ \frac{(7-z_{11})(15-z_{21})}{4} & 5 \leq z_{11} \leq 7, \\ & 13 \leq z_{21} \leq 15 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_C(z_{12}, z_{22}) = \begin{cases} \frac{(z_{12}-8)(z_{22}-6)}{4} & 8 \leq z_{12} \leq 10, \\ & 6 \leq z_{22} \leq 8 \\ \frac{(z_{12}-8)(10-z_{22})}{4} & 8 \leq z_{12} \leq 10, \\ & 8 \leq z_{22} \leq 10 \\ \frac{(12-z_{12})(z_{22}-6)}{4} & 10 \leq z_{12} \leq 12, \\ & 6 \leq z_{22} \leq 8 \\ \frac{(12-z_{12})(10-z_{22})}{4} & 10 \leq z_{12} \leq 12, \\ & 8 \leq z_{22} \leq 10 \\ 0 & \text{otherwise.} \end{cases}$$

B. Subtraction of Two Imprecise Matrices

Subtraction of two imprecise matrices A and B will be

$$A - B = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} - \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

where $Z_{11} = [-3, -1, 1]$, $Z_{12} = [-6, -4, -2]$, $Z_{21} = [-7, -5, -3]$ and $Z_{22} = [2, 4, 6]$ respectively. The membership functions of Z_{11} , Z_{12} , Z_{21} and Z_{22} for row vectors are as follows

$$\mu_{Z_{11}|Z_{12}}(z_{11}, z_{12}) = \begin{cases} \frac{z_{11}+3}{2} & -3 \leq z_{11} \leq -1, \\ & -6 \leq z_{12} \leq -2 \\ \frac{1-z_{11}}{2} & -1 \leq z_{11} \leq 1, \\ & -6 \leq z_{12} \leq -2 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{12}|Z_{11}}(z_{11}, z_{12}) = \begin{cases} \frac{z_{12} + 6}{2} & -6 \leq z_{12} \leq -4, \\ \frac{-2 - z_{12}}{2} & -3 \leq z_{11} \leq 1 \\ 0 & -4 \leq z_{12} \leq -2, \\ & -3 \leq z_{11} \leq 1 \\ & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{21}|Z_{22}}(z_{21}, z_{22}) = \begin{cases} \frac{z_{21} + 7}{2} & -7 \leq z_{21} \leq -5, \\ \frac{-3 - z_{21}}{2} & 2 \leq z_{22} \leq 6 \\ 0 & -5 \leq z_{21} \leq -3, \\ & 2 \leq z_{22} \leq 6 \\ & \text{otherwise.} \end{cases}$$

$$\mu_{Z_{22}|Z_{21}}(z_{21}, z_{22}) = \begin{cases} \frac{z_{22} - 2}{2} & 2 \leq z_{22} \leq 4, \\ \frac{6 - z_{22}}{2} & -7 \leq z_{21} \leq -3 \\ 0 & 4 \leq z_{22} \leq 6, \\ & -7 \leq z_{21} \leq -3 \\ & \text{otherwise.} \end{cases}$$

Now, the row membership surfaces of the matrix

$$C = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

will be as follows

$$\mu_R(z_{11}, z_{12}) = \begin{cases} \frac{(z_{11} + 3)(z_{12} + 6)}{4} & -3 \leq z_{11} \leq -1, \\ \frac{(z_{11} + 3)(-2 - z_{12})}{4} & -6 \leq z_{12} \leq -4 \\ \frac{(1 - z_{11})(z_{12} + 6)}{4} & -3 \leq z_{11} \leq -1, \\ \frac{(1 - z_{11})(-2 - z_{12})}{4} & -4 \leq z_{12} \leq -2 \\ 0 & -1 \leq z_{11} \leq 1, \\ & -6 \leq z_{12} \leq -4 \\ & -1 \leq z_{11} \leq 1, \\ & -4 \leq z_{12} \leq -2 \\ & \text{otherwise.} \end{cases}$$

$$\mu_R(z_{21}, z_{22}) = \begin{cases} \frac{(z_{21} + 7)(z_{22} - 2)}{4} & -7 \leq z_{21} \leq -5, \\ \frac{(z_{21} + 7)(6 - z_{22})}{4} & 2 \leq z_{22} \leq 4 \\ \frac{(-3 - z_{21})(z_{22} - 2)}{4} & -7 \leq z_{21} \leq -5, \\ \frac{(-3 - z_{21})(6 - z_{22})}{4} & 4 \leq z_{22} \leq 6 \\ 0 & -5 \leq z_{21} \leq -3, \\ & 2 \leq z_{22} \leq 4 \\ & -5 \leq z_{21} \leq -3, \\ & 2 \leq z_{22} \leq 4 \\ & \text{otherwise.} \end{cases}$$

Similarly, the column membership surfaces of the matrix C will be

$$\mu_C(z_{11}, z_{21}) = \begin{cases} \frac{(z_{11} + 3)(z_{21} + 7)}{4} & -3 \leq z_{11} \leq -1, \\ \frac{(z_{11} + 3)(-3 - z_{21})}{4} & -7 \leq z_{21} \leq -5 \\ \frac{(1 - z_{11})(z_{21} + 7)}{4} & -3 \leq z_{11} \leq -1, \\ \frac{(1 - z_{11})(-3 - z_{21})}{4} & -5 \leq z_{21} \leq -3 \\ 0 & -1 \leq z_{11} \leq 1, \\ & -7 \leq z_{21} \leq -5 \\ & -1 \leq z_{11} \leq 1, \\ & -5 \leq z_{21} \leq -3 \\ & \text{otherwise.} \end{cases}$$

$$\mu_C(z_{12}, z_{22}) = \begin{cases} \frac{(z_{12} + 6)(z_{22} - 2)}{4} & -6 \leq z_{12} \leq -4, \\ \frac{(z_{12} + 6)(6 - z_{22})}{4} & 2 \leq z_{22} \leq 4 \\ \frac{(-2 - z_{12})(z_{22} - 2)}{4} & -6 \leq z_{12} \leq -4, \\ \frac{(-2 - z_{12})(6 - z_{22})}{4} & 4 \leq z_{22} \leq 6 \\ 0 & -4 \leq z_{12} \leq -2, \\ & 2 \leq z_{22} \leq 4 \\ & -4 \leq z_{12} \leq -2, \\ & 4 \leq z_{22} \leq 6 \\ & \text{otherwise.} \end{cases}$$

C. Multiplication of Two Imprecise Matrices

Multiplication of two imprecise matrices A and B will be

$$A.B = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix} \cdot \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

where $Z_{11} = [18, 33, 52]$, $Z_{12} = [8, 20, 36]$, $Z_{21} = [46, 66, 90]$ and $Z_{22} = [23, 40, 61]$ respectively. The membership functions of Z_{11}, Z_{12}, Z_{21} and Z_{22} for row vectors are as follows

$$\mu_{z_{11}|z_{12}}(z_{11}, z_{12}) = \begin{cases} \frac{-13 + \sqrt{25 + 8z_{11}}}{4} & 18 \leq z_{11} \leq 33, \\ \frac{21 - \sqrt{25 + 8z_{11}}}{4} & 8 \leq z_{12} \leq 36 \\ 0 & 33 \leq z_{11} \leq 52, \\ & 8 \leq z_{12} \leq 36 \\ & \text{otherwise.} \end{cases}$$

$$\mu_{z_{12}|z_{11}}(z_{11}, z_{12}) = \begin{cases} \frac{-5 + \sqrt{9 + 2z_{12}}}{2} & 8 \leq z_{12} \leq 20, \\ \frac{9 - \sqrt{9 + 2z_{12}}}{2} & 18 \leq z_{11} \leq 52 \\ 0 & 20 \leq z_{12} \leq 36, \\ & 18 \leq z_{11} \leq 52 \\ & \text{otherwise.} \end{cases}$$

$$\mu_{z_{21}|z_{22}}(z_{21}, z_{22}) = \begin{cases} \frac{-9 + \sqrt{2z_{21} - 11}}{2} & 46 \leq z_{21} \leq 66, \\ \frac{13 - \sqrt{2z_{21} - 11}}{2} & 23 \leq z_{22} \leq 61 \\ 0 & 66 \leq z_{21} \leq 90, \\ & 23 \leq z_{22} \leq 61 \\ & \text{otherwise.} \end{cases}$$

$$\mu_{z_{22}|z_{21}}(z_{21}, z_{22}) = \begin{cases} \frac{-15 + \sqrt{41 + 8z_{22}}}{4} & 23 \leq z_{22} \leq 40, \\ \frac{23 - \sqrt{41 + 8z_{22}}}{4} & 46 \leq z_{21} \leq 90 \\ 0 & 40 \leq z_{22} \leq 61, \\ & 46 \leq z_{21} \leq 90 \\ & \text{otherwise.} \end{cases}$$

Now, the row membership surfaces of the matrix

$$C = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$$

will be as follows

$$\mu_R(z_{11}, z_{12}) = \begin{cases} \frac{(-13 + \sqrt{25 + 8z_{11}})(-5 + \sqrt{9 + 2z_{12}})}{8} & 18 \leq z_{11} \leq 33, \\ \frac{(-13 + \sqrt{25 + 8z_{11}})(9 - \sqrt{9 + 2z_{12}})}{8} & 8 \leq z_{12} \leq 20 \\ \frac{(21 - \sqrt{25 + 8z_{11}})(-5 + \sqrt{9 + 2z_{12}})}{8} & 18 \leq z_{11} \leq 33, \\ \frac{(21 - \sqrt{25 + 8z_{11}})(9 - \sqrt{9 + 2z_{12}})}{8} & 20 \leq z_{12} \leq 36 \\ 0 & 33 \leq z_{11} \leq 52, \\ & 8 \leq z_{12} \leq 20 \\ & 33 \leq z_{11} \leq 52, \\ & 20 \leq z_{12} \leq 36 \\ & \text{otherwise.} \end{cases}$$

$$\mu_R(z_{21}, z_{22}) = \begin{cases} \frac{(-9 + \sqrt{2z_{21} - 11})(-15 + \sqrt{41 + 8z_{22}})}{8} & 46 \leq z_{21} \leq 66, \\ & 23 \leq z_{22} \leq 40 \\ \frac{(-9 + \sqrt{2z_{21} - 11})(23 - \sqrt{41 + 8z_{22}})}{8} & 46 \leq z_{21} \leq 66, \\ & 40 \leq z_{22} \leq 61 \\ \frac{(13 - \sqrt{2z_{21} - 11})(-15 + \sqrt{41 + 8z_{22}})}{8} & 66 \leq z_{21} \leq 90, \\ & 23 \leq z_{22} \leq 40 \\ \frac{(13 - \sqrt{2z_{21} - 11})(23 - \sqrt{41 + 8z_{22}})}{8} & 66 \leq z_{21} \leq 90, \\ & 40 \leq z_{22} \leq 61 \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the column membership surfaces of the matrix C will be as follows

$$\mu_C(z_{11}, z_{21}) = \begin{cases} \frac{(-13 + \sqrt{25 + 8z_{11}})(-9 + \sqrt{2z_{21} - 11})}{8} & 18 \leq z_{11} \leq 33, \\ & 46 \leq z_{21} \leq 66 \\ \frac{(-13 + \sqrt{25 + 8z_{11}})(13 - \sqrt{2z_{21} - 11})}{8} & 18 \leq z_{11} \leq 33, \\ & 66 \leq z_{21} \leq 90 \\ \frac{(21 - \sqrt{25 + 8z_{11}})(-9 + \sqrt{2z_{21} - 11})}{8} & 33 \leq z_{11} \leq 52, \\ & 46 \leq z_{21} \leq 66 \\ \frac{(21 - \sqrt{25 + 8z_{11}})(13 - \sqrt{2z_{21} - 11})}{8} & 33 \leq z_{11} \leq 52, \\ & 66 \leq z_{21} \leq 90 \\ 0 & \text{otherwise.} \end{cases}$$

$$\mu_C(z_{12}, z_{22}) = \begin{cases} \frac{(-5 + \sqrt{9 + 2z_{12}})(-15 + \sqrt{41 + 8z_{22}})}{8} & 8 \leq z_{12} \leq 20, \\ & 23 \leq z_{22} \leq 40 \\ \frac{(-5 + \sqrt{9 + 2z_{12}})(23 - \sqrt{41 + 8z_{22}})}{8} & 8 \leq z_{12} \leq 20, \\ & 40 \leq z_{22} \leq 61 \\ \frac{(9 - \sqrt{9 + 2z_{12}})(-15 + \sqrt{41 + 8z_{22}})}{8} & 20 \leq z_{12} \leq 36, \\ & 23 \leq z_{22} \leq 40 \\ \frac{(9 - \sqrt{9 + 2z_{12}})(23 - \sqrt{41 + 8z_{22}})}{8} & 20 \leq z_{12} \leq 36, \\ & 40 \leq z_{22} \leq 61 \\ 0 & \text{otherwise.} \end{cases}$$

V. CONCLUSION

In this article, we have shown the membership surface and arithmetic operations of imprecise matrix. We have shown here addition, subtraction and product of imprecise matrices. The numerical example of arithmetic operations are given only for 2×2 matrices. But using this method the arithmetic operations can be performed for $n \times n$ matrices too and it is possible to obtain the row and column membership surfaces of the matrix.

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