Modeling of a Vehicle Wheel System Having a Built-in Suspension Structure Consisted of Radially Deployed Colloidal Spokes between Hub and Rim

Barenten Suciu

Abstract—In this work, by replacing the traditional solid spokes with colloidal spokes, a vehicle wheel with a built-in suspension structure is proposed. Following the background and description of the wheel system, firstly, a vibration model of the wheel equipped with colloidal spokes is proposed, and based on such model the equivalent damping coefficients and spring constants are identified. Then, a modified model of a quarter-vehicle moving on a rough pavement is proposed in order to estimate the transmissibility of vibration from the road roughness to vehicle body. In the end, the optimal design of the colloidal spokes and the optimum number of colloidal spokes are decided in order to minimize the transmissibility of vibration, i.e., to maximize the ride comfort of the vehicle.

Keywords—Built-in suspension, colloidal spoke, intrinsic spring, vibration analysis, wheel.

I. INTRODUCTION

ALTHOUGH classical wheels are able to provide for a buffering function, their elastic and dissipative properties are insufficient to ensure the desired level of ride comfort for the usual vehicles. Therefore, a suspension is customarily inserted between the wheel and the vehicle body. Suspension is consisted of a dashpot (e.g., hydro-pneumatic damper, etc.), with a damping coefficient of about 10 times larger than the tire's damping coefficient, connected in parallel with a spring (e.g., compression helical spring, air spring, etc.), with a spring constant of about 10 times lower than the tire's spring constant [1], [2]. On the other hand, classical wheels are employing radially deployed solid spokes between hub and rim to achieve a hollowed wheel structure of reduced weight, higher elasticity and improved cooling of the brake system [3].

Early spokes were designed with a slenderness ratio larger than 2.5 but smaller than 30, to transmit both compressive and tensile loads, but to buckle only under extreme loading conditions [4]. In order to further reduce the wheel stiffness, spokes were redesigned as sheet-like or wire-like elements with a slenderness ratio larger than 30 but smaller than 45, to be able to transmit tensile loads, but to allow buckling with substantially no compressive resistance when passing through the contact area with the ground [5], [6]. Similar effects were obtained by using chain-like spokes, made of rigid parts assembled together by hinges [7]. Unfortunately, a reduction of 50% of the wheel stiffness was accompanied by longer time of

reaction, and consequently, especially at high travelling speeds, such spoke buffers were unable to promptly regain the circular shape after exiting from the contact region with the ground [8].

In this work, an alternative design solution of the wheel is proposed. Concretely, traditional solid spokes are replaced by so-called *colloidal spokes*, radially deployed between hub and rim, in order to achieve a novel vehicle wheel with a built-in suspension structure. Following to detailed description of the proposed wheel system, firstly, a vibration model of the wheel equipped with colloidal spokes is proposed, and based on such model, the equivalent damping coefficients and spring constants are identified. Then, a modified model of a quarter-vehicle moving on a rough pavement is proposed in order to estimate the transmissibility of vibration from the road to vehicle body. In the end, the optimal design of the colloidal spokes and the optimum number of colloidal spokes are decided in order to minimize the transmissibility of vibration, i.e., to maximize the ride comfort of the vehicle.

II. DESCRIPTION OF THE PROPOSED WHEEL SYSTEM

A. Wheel with Built-in Suspension Structure

A wheel with built-in suspension structure can be achieved by deploying radially between the hub and rim a plurality of suspension-spokes (Fig. 1). Each spoke is designed to provide for elastic (spring-like) and/or dissipative (damper-like) functions. Accordingly, it can be modeled as an elastic element of stiffness k_{sp} [N/m] connected with a dashpot of damping coefficient c_{sp} [N·s/m], either in parallel (Kelvin-Voigt suspension) or in series (Maxwell suspension). Such cushioned wheel is adapted to absorb and dissipate the energy of shock and vibration from the rough ground, and can be used either for pneumatic or non-pneumatic tires. It can be employed for lightweight vehicles, such as bicycles, wheelchairs, carts, etc., which lack of other suspension systems, to increase the ride comfort and to reduce or inhibit the damage to the wheel itself. It can be also used for heavy vehicles, such as trucks, busses, automobiles, etc., in which case, traditionally inserted suspension between the wheel and body can be suppressed.

Suspension-spokes can be connected to rim and hub via joints at one or both ends. There is quite a large variety of design solutions concerning the suspension element, which can be simply incorporated as an ideal spring of constant k_{sp} [9], or as an ideal dashpot of damping coefficient c_{sp}, where the dashpot can be selected as an oil damper [10], air damper [11], [12], hydro-pneumatic damper [13], friction damper [14], etc.

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World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:10, No:1, 2016

Moreover, suspension element can be complexly incorporated as an elastic rubber element [15], as hydro-pneumatic absorbers working in parallel with elastic rubber elements [16], as friction dampers serially connected with helical compression springs [17], etc.
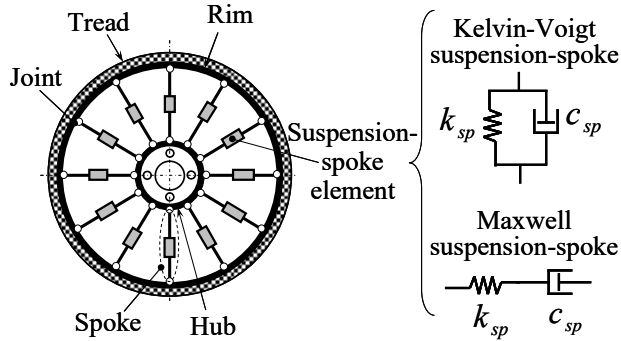

Fig. 1 Schematic view of a wheel with built-in suspension structure

B. Colloidal Spokes

Colloidal spokes proposed here are employing a traditional cylinder-piston-seal structure (Fig. 2), that accommodates a colloidal mixture of liquid (e.g., water) and liquid-repellent micro-particles of nanoporous silica gel. Colloidal spokes can be regarded as a particular application of the well-known colloidal dampers [18], which were recently proposed as potential candidates for a new generation of shock absorbers for automotive suspensions [1], [19], [20]. Colloidal spokes are designed based on the well-known fact that liquids are able to transmit compressive loads, but unable to transmit tensile loads [21], [22]. Such selective transmissibility of loads is opposite to that of solid spokes, which buckle under compression.
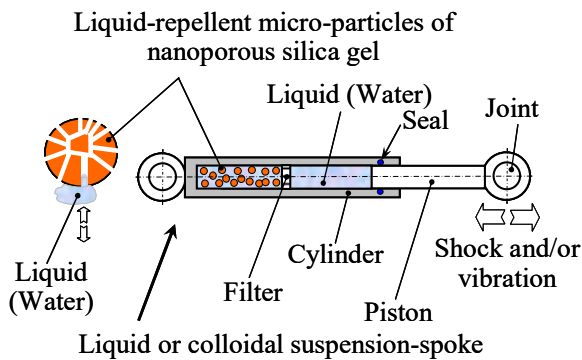

Fig. 2 Schematic view of a liquid or colloidal suspension-spoke

Thus, in the absence of silica gel particles, liquid suspension-spoke shown in Fig. 2 behaves as a classical liquid spring [21]. However, by mixing silica gel particles in water, beneficial reduction of the bulk modulus of elasticity of the resulting colloidal solution is accompanied by an advantageous dissipative effect, produced during the cyclical penetration/exudation of water in/from the liquid-repellent nanopores. As a result, the colloidal spoke displays a dual function, of absorber and compression spring [19].

III. EVALUATION OF THE TIRE STIFFNESS

Tires are usually consisted of four parts, as follows: a hub; a spoke region; a rim; and an elastomeric tread surrounding the rim and contacting the ground. It is generally preferable to design the hub and rim as rigid, but the spoke region as elastic. Thus, by using a rigid rim, the tire becomes more resistant to tear and abrasive wear, and consequently, its lifespan increases. Moreover, contact area with the ground decreases, leading to reduced consumption of fuel. On the other hand, by using an elastic spoke region that behaves as a buffer, the equivalent spring constant of the tire diminishes, leading to improved ride comfort of the vehicle.

Such design conditions cannot be achieved by the standard pneumatic tires, since the elastic buffer (pressurized tread tube) is placed at the outer part of the tire, and besides, the central spoke region is rigid. However, previous design conditions can be easily satisfied by non-pneumatic (airless) tires with radially deployed colloidal spokes, which might be connected through rigid or elastic joints to the rim and hub.

For the sake of simplicity, only rigid joints are considered in this work, and in such circumstances, the equivalent spring constant k_t of the tire can be written as:

$$\frac{1}{k_t} = \frac{1}{k_{tread}} + \frac{1}{k_{sb}} \tag{1}$$

here k_{tread} is the spring constant of the tread and k_{sb} is the spring constant of the whole spoke buffer, where the tread is serially connected to the spoke buffer. Usually, the elastomeric tread has a thickness of about 10 mm and integrally covers the rim; it has a grooved pattern on the outer circumferential face, similar to that of a classical pneumatic tire. Based on Hertz's theory, applied to a cylinder of elastomer contacting a rigid flat surface, the spring constant of the tread can be calculated as:

$$k_{tread} = \frac{F}{R - \sqrt{R^2 - \frac{4}{\pi} F \frac{R}{B} \frac{1 - \nu^2}{E}}} \tag{2}$$

here ν is the Poisson's ratio and E is the longitudinal modulus of elasticity of the elastomeric material, R and B are the radius and width of the tire, and $F = (M_b + M_w)g$ is the static load of the tire. For $R = 285$ mm and $B = 175$ mm, which correspond to a 175/70 R13 type tire equipping a 1,300 cc class autovehicle, for a static load of $F = 2,617$ N (values of the body mass $M_b = 240$ kg and the wheel mass $M_w = 27$ kg, are taken for a quarter vehicle model, according to [1]), and for an usual tread rubber with $\nu = 0.4$ and $E = 3$MPa, a tread spring constant of about 977 N/mm is obtained. Since tread spring constant of a non-pneumatic tire is about 30 times larger than spring constant of the proposed colloidal spokes, the equivalent spring constant of the tire can be fairly approximated as:

$$k_t = k_{sb} \tag{3}$$

World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:10, No:1, 2016

Thus, in order to evaluate the tire stiffness, i.e., its spring constant k_t, it is necessary to find the relationship between the spring constant of the whole spoke buffer k_{sb} and the spring constant of a single suspension-spoke k_{sp}. This design problem is solved in the next sections under the following assumptions:

1) A number of n spokes ($n = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12$) are evenly deployed in radial direction between the hub and rim. Accordingly, angle between two successive spokes becomes:

$$\theta = 2\pi / n \qquad (4)$$

2) Spokes are able to transmit compressive loads, but unable to transmit tensile loads.

A. Case of Rigid Spokes

In this section, it is supposed that suspension-spokes are not to be connected to rim and hub via joints at one or both ends, but are integrally linked to rim and hub. Consequently, spokes cannot rotate to accommodate a working piston stroke, and the center of the tire occurs as fixed. Such case of study is relevant for the ideal incompressible liquids, and it is also relevant for the standard pneumatic tires, where the central spoke region is rigid. Since in this case, the spoke buffer spring constant tends to infinity, the tire spring constant can be approximated as:

$$k_t = k_{tread} \qquad (5)$$

1. Contact of the Tire with Ground Directly on the Spoke

In this case, (Fig. 3), the number of spokes working in the compression region of a quarter tire is given by:

$$k = RoundDown(n / 4) \qquad (6)$$

Assuming a cosinusoidal loading distribution for the spokes ($F_i = F_0 \cos(i\theta)$, $i = \overline{1, k}$), from the equilibrium of forces along the vertical direction, the dimensionless load on each spoke can be calculated as:

$$\frac{F_i}{F} = \frac{\cos(i\theta)}{1 + k + \sum_{j=1}^{k} \cos(2j\theta)}, \quad i = \overline{1, k} \qquad (7)$$
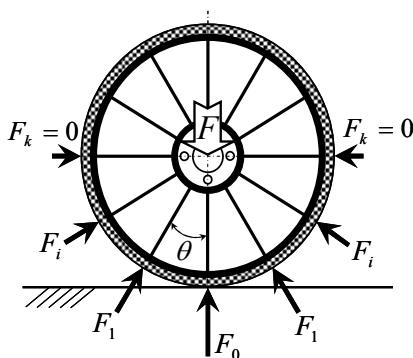

Fig. 3 Loading model of the rigid spokes (Contact of the tire with the ground is directly on the spoke)

Fig. 4 illustrates the variation of the dimensionless load versus the number of rigid spokes, when the contact of the tire with the ground is directly on the spoke. Note that central load F_0 / F is maximal and constant for $n = 2$-4, and then monotonically decreases against n. On the other hand, lateral load F_1 / F is nil for $n = 2$-4, then increases at its maximal value of 0.354 for $n = 8$, and after that decreases versus n. Lateral load F_2 / F is nil for $n = 2$-8, and then monotonically increases against n. Obviously, augmentation of the number of spokes leads to the evening of the loading distribution, but the tire cost increases due to the structural complexity.
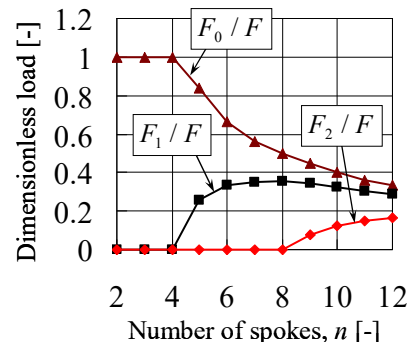

Fig. 4 Variation of dimensionless load versus the number of spokes (Contact of the tire with the ground is directly on the spoke)

2. Contact of the Tire with Ground in the Middle between Two Spokes

In this case, (Fig. 5), the number of spokes working in the compression region of a quarter tire is given by:

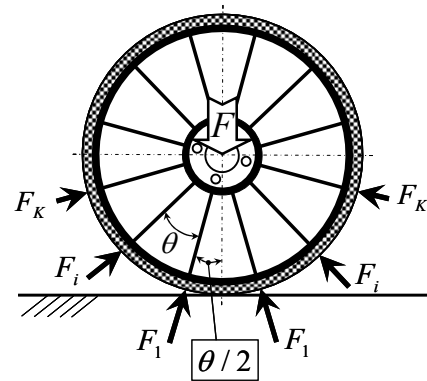$$K = RoundDown[1 + (n - 2) / 4] \qquad (8)$$

Fig. 5 Loading model of the rigid spokes (Contact of the tire with the ground is exactly in the middle between two spokes)

Assuming a cosinusoidal loading distribution for the spokes ($F_i = F_1 \cos[(2i - 1)\theta / 2]$, $i = \overline{2, K}$), from the equilibrium of forces along the vertical direction, the dimensionless load on each spoke can be calculated as:

World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:10, No:1, 2016

$$\frac{F_i}{F} = \frac{\cos[(2i-1)\theta/2]}{2\cos(\theta/2) + K - 1 + \sum_{j=2}^{K}\cos[(2j-1)\theta]}, \quad i = \overline{2, K} \quad (9)$$

Fig. 6 illustrates the variation of the dimensionless load versus the number of rigid spokes, when the contact of the tire with the ground is exactly in the middle between two spokes.

One observes that F_1/F monotonically decreases against n. On the other hand, F_2/F is nil for $n = 2\text{-}6$, then increases at its maximal value of 0.233 for $n = 11$, and after that decreases versus n. Load F_3/F is nil for $n = 2\text{-}10$, then monotonically increases against n. Again, augmentation of the number of spokes leads to the evening of the loading distribution, and this is in agreement with the result obtained in section A1.

Note that for $n = 2$, the dimensionless load F_1/F tends to infinity. Thus, number of spokes should be selected as $n \geq 3$ in order to avoid such catastrophic loading. Moreover, under the assumption that the spokes are rigid, for $n = 3$, spoke loading when the contact of the tire with the ground is directly on the spoke, equals the spoke loading when the contact is exactly in the middle between two spokes. Since such minimum number of spokes will provide constructive simplicity, reduced cost of manufacturing, and increased durability under constant loading of the tire, $n = 3$ appears as the optimal number of spokes.
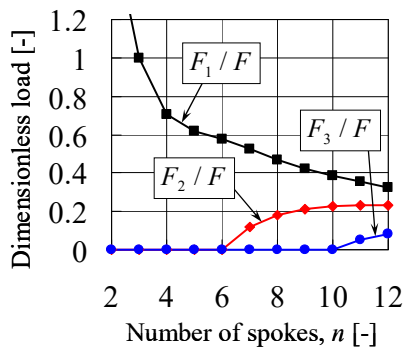

Fig. 6 Variation of dimensionless load versus the number of spokes (Contact of the tire with the ground is exactly in the middle between two spokes)

B. Case of Elastic Spokes

In this section, it is supposed that suspension-spokes are connected to rim and hub via rigid joints at one or both ends. Consequently, spokes are able to rotate and to accommodate a working piston stroke. As a result, under loading, the center of the tire can move along vertical direction until the equilibrium state is achieved. Thus, the equivalent stiffness k_{sb} of the spoke buffer can be defined as the ratio of the tire load F and the displacement δ in vertical direction of the center of the buffer (Figs. 7-9):

$$k_{sb} = F/\delta \quad (10)$$

On the other hand, spoke stiffness k_{sp} can be defined as the ratio of the actual force F_i acting on the spoke and the piston stroke $S = l_0 - l$ (difference between the initial length l_0 and the actual length l of the spoke, Figs. 7-9):

$$k_{sp} = F_i/S = F_i/(l_0 - l) \quad (11)$$

Then, the dimensionless stiffness $\overline{k_{sb}}$ (dimensionless spring constant) of the spoke buffer can be defined as:

$$\overline{k_{sb}} = k_{sb}/k_{sp} \quad (12)$$

1. Contact of the Tire with Ground Directly on the Spoke

For $n = 3$ and 4 (Fig. 7), only the lower vertical spoke is working in compression. Note that displacement δ in vertical direction of the center of the buffer equals the piston stroke S of the working spoke. Thus, from the force equilibrium ($F = F_0$) in vertical direction, one finds that the stiffness of the spoke buffer equals the stiffness of the spoke, and accordingly:
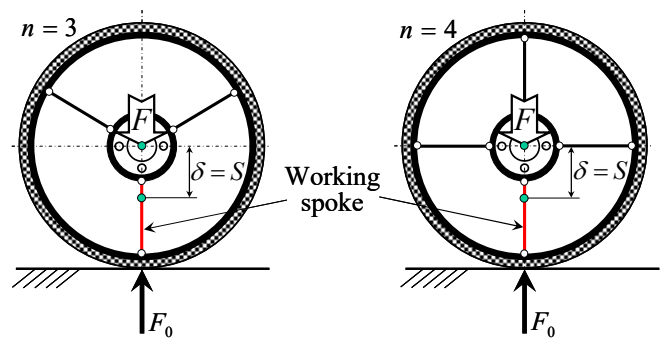
$$\overline{k_{sb}} = 1 \quad (13)$$

Fig. 7 Loading model of the elastic spokes for $n = 3$, and 4 (Contact of the tire with the ground is directly on the spoke)

On the other hand, for $n = 5$, 6, 7, and 8 (Fig. 8, for $n = 6$), there are three spokes working in compression.
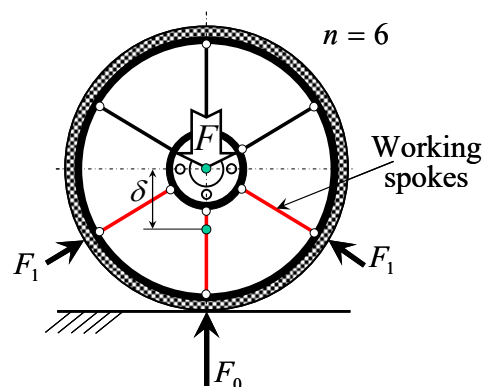

Fig. 8 Loading model of the elastic spokes for $n = 6$ (Contact of the tire with the ground is directly on the spoke)

From the similar force equilibrium condition along vertical direction ($F = F_0 + 2F_1 \cos(\theta + \Delta\theta)$), one arrives to a different

World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:10, No:1, 2016

expression for the spoke buffer stiffness:

$$\overline{k_{sb}} = 1 + 2\frac{\overline{S}}{\overline{\delta}}\cos(\theta + \Delta\theta) \qquad (14)$$

where $\overline{\delta} = \delta / l_0 \in [0; \cos\theta]$ is the dimensionless displacement of the center of the spoke buffer, $\overline{S} = (l_0 - l)/l_0 \in [0; 1 - \sin\theta]$ is the dimensionless stroke of the inclined spokes, and $\Delta\theta$ is the angle of rotation of the inclined spokes, allowed by the joints from the hub and rim. Analysis of the system geometry leads to the following expressions for the dimensionless stroke and angle of rotation:

$$\overline{S} = 1 - \sqrt{1 + \overline{\delta}^2 - 2\overline{\delta}\cos\theta} \quad ; \quad \Delta\theta = \sin^{-1}(\frac{\overline{\delta}}{1 - \overline{S}}\sin\theta) \quad (15)$$

Substituting (15) in (14), the spoke buffer stiffness becomes:

$$\overline{k_{sb}} = 1 + 2\frac{1 - \sqrt{1 + \overline{\delta}^2 - 2\overline{\delta}\cos\theta}}{\sqrt{1 + \overline{\delta}^2 - 2\overline{\delta}\cos\theta}}\frac{\cos\theta - \overline{\delta}}{\overline{\delta}} \qquad (16)$$

Note that (16) reduces to the following expression when the center of the spoke buffer approaches the origin O:

$$\overline{k_{sb}}(\overline{\delta} \to 0) = 1 + 2\cos^2\theta \qquad (17)$$

As shown also in Fig. 10, the dimensionless stiffness of the spoke buffer (16) is a function decreasing monotonically versus the displacement $\overline{\delta}$ from its maximal value given by (17).

2. Contact of the Tire with Ground in the Middle between Two Spokes

For $n = 3, 4, 5,$ and 6 (Fig. 9, for $n = 6$), there are two spokes working in compression.
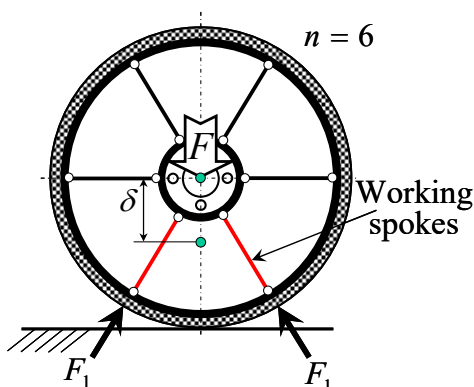

Fig. 9 Loading model of the elastic spokes for $n = 6$ (Contact of the tire with the ground is exactly in the middle between two spokes)

From the similar force equilibrium condition along vertical direction ($F = 2F_1\cos(\theta/2 + \Delta\theta)$), one finds the following expression for the spoke buffer stiffness:

$$\overline{k_{sb}} = 2\frac{\overline{S}}{\overline{\delta}}\cos(\theta/2 + \Delta\theta) \qquad (18)$$

where $\overline{\delta} = \delta / l_0 \in [0; \cos(\theta/2)]$ is dimensionless displacement of the center of spoke buffer, $\overline{S} = (l_0 - l)/l_0 \in [0; 1 - \sin(\theta/2)]$ is the dimensionless stroke of the inclined spokes, and $\Delta\theta$ is the angle of rotation of the inclined spokes, allowed by the joints from the hub and rim. Analysis of the system geometry leads to the following expressions for the dimensionless stroke and angle of rotation:

$$\overline{S} = 1 - \sqrt{1 + \overline{\delta}^2 - 2\overline{\delta}\cos\frac{\theta}{2}} \quad ; \quad \Delta\theta = \sin^{-1}(\frac{\overline{\delta}}{1 - \overline{S}}\sin\frac{\theta}{2}) \quad (19)$$

Substituting (19) in (18), the spoke buffer stiffness becomes:

$$\overline{k_{sb}} = 2\frac{1 - \sqrt{1 + \overline{\delta}^2 - 2\overline{\delta}\cos(\theta/2)}}{\sqrt{1 + \overline{\delta}^2 - 2\overline{\delta}\cos(\theta/2)}}\frac{\cos(\theta/2) - \overline{\delta}}{\overline{\delta}} \qquad (20)$$

Note that (20) reduces to the following expression when the center of the spoke buffer approaches the origin O:

$$\overline{k_{sb}}(\overline{\delta} \to 0) = 2\cos^2(\theta/2) \qquad (21)$$

As shown also in Fig. 10, the dimensionless stiffness of the spoke buffer (20) is a function decreasing monotonically versus the displacement $\overline{\delta}$ from its maximal value given by (21).

3. Necessary but Sufficient Number of Spokes

Next, elastic characteristics of the tire are evaluated based on the variation of the dimensionless stiffness $\overline{k_{sb}}$ of the spoke buffer versus the displacement $\overline{\delta}$ for various values of the number of spokes ($n = 3, 4, 5,$ and 6), in the cases when the contact of the tire with the ground is directly on the spoke, or in the middle between two spokes (Fig. 10). Appropriate design of the tire is achieved when the spoke buffer stiffness for the contact of the tire with the ground directly on the spoke equals the spoke buffer stiffness for the contact exactly in the middle between two spokes. Such design condition cannot be satisfied for $n = 3$ and 4 ((13), (16), (20), and Fig. 10). Oppositely, such design condition can be satisfied for tires using $n = 5$ or $n = 6$, if the dimensionless displacement is taken as $\overline{\delta} = 0.27$ or $\overline{\delta} = 0.41$, respectively.

Thus, although for a tire using rigid spokes, the necessary but sufficient number of spokes was found to be $n = 3$, for a tire with elastic spokes, the necessary but sufficient number of spokes should be increased to $n = 5$. In such circumstances, the dimensionless stiffness of the spoke buffer becomes $\overline{k_{sb}} = 1.014$, which is achieved for a dimensionless stroke of $\overline{S} = 0.048$ and an angle of rotation of the inclined spokes of $\Delta\theta = 15.7$ deg in the case of the contact directly on spoke, and which is also achieved for a dimensionless stroke of $\overline{S} = 0.203$ and an angle of rotation of the inclined spokes of $\Delta\theta = 11.5$ deg in the

World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:10, No:1, 2016

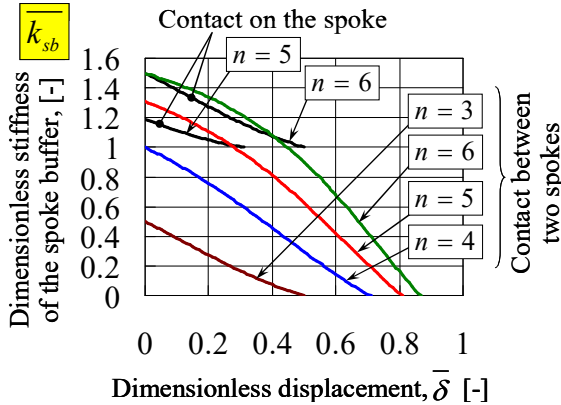case of the contact in the middle between two spokes.

Fig. 10 Variation of the dimensionless stiffness of the spoke buffer versus the dimensionless displacement for various values of the number of spokes, when the contact of the tire with the ground is directly on the spoke, or in the middle between two spokes

Note that initial length l_0 of the spoke can be calculated as:

$$l_0 = (1 - \overline{R}_{hub})R - t - h \tag{22}$$

where $R = 285$ mm is the radius of the tire, $\overline{R}_{hub} = 0.4$ is the dimensionless radius of the hub, $t = 10$ mm is the thickness of the tread, and $h = 5$ mm is the thickness of the rim. In these circumstances, the initial length of the spoke becomes $l_0 = 156$ mm, and accordingly, stroke of the colloidal spoke is expected to vary from a minimal value of $0.048 \times l_0 = 7.5$ mm (in the case of the contact directly on spoke) to a maximal value of $0.203 \times l_0 = 31.6$ mm (in the case of the contact in the middle between two spokes). Since such domain of stroke variation is easily achievable from a practical standpoint, one concludes that the colloidal spokes can be used to construct an efficient spoke buffer.

IV. EVALUATION OF THE SPOKE SPRING CONSTANT

As already stressed, in the absence of silica gel particles, the liquid suspension-spoke shown in Fig. 2 behaves as a classical liquid spring of stiffness $k_{l,sp}$ which can be calculated as [21]:

$$k_{l,sp} = \frac{A_p}{l_l} E_l \tag{23}$$

where A_p is the cross-sectional area of the piston, l_l is the length of the liquid column, and E_l is the liquid bulk modulus of elasticity, i.e., inverse of the liquid compressibility. Note that the liquid bulk modulus of elasticity is equivalent to the longitudinal modulus of elasticity (Young modulus) of the solid materials. Since some typical values are 1.8GPa for ISO 32VG mineral oil, 2.15GPa for water, 4.35GPa for glycerin, and 28.5GPa for mercury, one concludes that the liquid modulus of elasticity varies in the intermediate range, from the modulus of

elasticity of the elastomeric materials (0.01-0.1GPa), up to the modulus of elasticity of metallic materials (e.g., 45GPa for magnesium alloys, 70GPa for aluminum alloys, 105GPa for titan alloys), traditionally used in construction of solid spokes.

For a colloidal mixture of silica gel particles in water (Fig. 2), water spring occurs as serially connected to silica gel spring [20], and in such circumstances, the spring constant of resulting colloidal spoke can be written as:

$$\frac{1}{k_{sp}} = \frac{1}{k_w} + \frac{1}{k_{si}} \tag{24}$$

where k_w is the water spring constant as given by (23):

$$k_w = \frac{A_p}{l_w} E_w \tag{23'}$$

and k_{si} is the silica gel spring constant as given by [20]:

$$k_{si} = \frac{A_p}{x} E_{si} \quad ; \quad E_{si} = -\frac{2\sigma \cos\alpha}{r} \tag{25}$$

where l_w is the length of the water column, x is the piston displacement, $E_w = 2.15$GPa is the water bulk modulus of elasticity, E_{si} is the silica gel equivalent modulus of elasticity, $\sigma = 0.0728$ N/m is the surface tension of water, $r = 4$ nm is the mean radius of the nanopores, and $\alpha = 128$ deg is the contact angle at the interface of water and silica gel wall [18], [20]. In such circumstances, one obtains for the silica gel equivalent modulus of elasticity a typical value of $E_{si} = 0.022$GPa, which is 100 times smaller than the water bulk modulus of elasticity. Note that, during a compression-decompression cycle of the colloidal spoke, length of the water column varies from the initial length ($l_{w,i} = \beta S$) to final length ($l_{w,f} = (\beta-1)S$), where S is the piston stroke and β is a water surplus coefficient. On the other hand, the piston displacement x varies between 0 and the piston stroke. Consequently, the colloidal spoke spring constant can be rewritten as:

$$k_{sp} = \frac{A_p E_w}{S} \frac{\overline{E}}{\overline{x} + \overline{E}(\beta - \overline{x})} \tag{26}$$

or in dimensionless form as:

$$\overline{k_{sp}} = \frac{k_{sp} S}{A_p E_w} = \frac{\overline{E}}{\overline{x} + \overline{E}(\beta - \overline{x})} \tag{27}$$

where $\overline{E} = E_{si}/E_w \cong 0.01$ is the dimensionless elastic modulus and $\overline{x} = x/S \in [0;1]$ is the dimensionless piston displacement.

Next, Fig. 11 illustrates the variation of the dimensionless spring constant of the colloidal spoke versus the dimensionless piston displacement, calculated for various values of the water

World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:10, No:1, 2016

surplus coefficient $\beta = 1/2$ (i.e., deficit of water), $\beta = 1$ (i.e., initial quantity of water outside the silica gel particles exactly equals the quantity of water able to penetrate the nanopores), $\beta = 2$, and 4 (i.e., initial quantity of water outside the silica gel particles largely exceeds the quantity of water able to penetrate the nanopores). As expected, the dimensionless spring constant of the colloidal spoke monotonically decreases versus the dimensionless piston displacement. Colloidal spoke becomes stiffer as the water surplus coefficient increases, but this kind of influence becomes significant only for relative small piston displacements ($\bar{x} = x/S \in [0;0.2]$). Note that initial stiffness of the colloidal spoke, which in fact equals the stiffness of the water spring, can be adjusted as desired by properly selecting the water surplus coefficient.
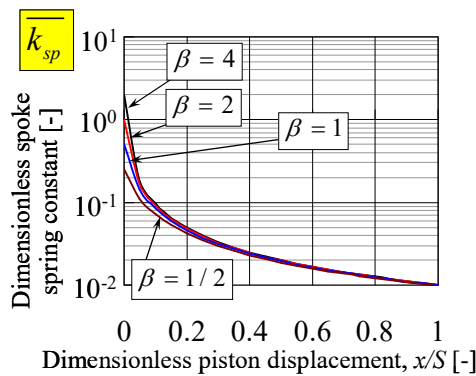

Fig. 11 Variation of the dimensionless spring constant of the colloidal spoke versus the dimensionless piston displacement, calculated for various values of the water surplus coefficient

In conclusion, by mixing silica gel particles in water, reduction of the bulk modulus of elasticity of the resulting colloidal solution leads to an important decrease of the spoke stiffness. Thus, the colloidal spoke buffer can be designed to be sufficiently flexible to accommodate practical applications, such as a wheel system with a built-in suspension structure.

Beneficial stiffness reduction of the colloidal spoke is accompanied by an advantageous dissipative effect, produced during the cyclical penetration/exudation of water in/from the liquid-repellent nanopores. As a result, colloidal spoke displays a dual function, of absorber and compression spring [19]. In such circumstances, comfortable vehicles can be achieved even in the absence of the classical absorber (see the simplified suspension proposed in Fig. 13, where the classical oil absorber is omitted), or further, in the absence of the whole classical suspension (see the simplified system proposed in Fig. 14, where both the helical spring and absorber are omitted, vehicle body being rigidly connected to the tire's wheel system).

V. RIDE COMFORT EVALUATION

As already stressed in the introductive section, although the classical wheels are able to provide for a buffering function, their elastic and dissipative properties are insufficient to ensure the desired level of ride comfort for the usual vehicles. Therefore, a suspension is customarily inserted between the

wheel and the vehicle body (Fig. 12). Such suspension is consisted of a dashpot (e.g., hydro-pneumatic damper, etc.), with a damping coefficient c_{OD} of about 10 times larger than the damping coefficient c_t of the tire, connected in parallel with a spring (e.g., compression helical spring, air spring, etc.), with a spring constant k_{CS} of about 10 times lower than the spring constant k_t of the tire [1], [2]. On Fig. 12, M_w and M_b are the wheel mass and body mass, respectively, x_0 represents the displacement excitation produced by the ground roughness, x_1 is the wheel vibration, and x_2 is the body vibration.
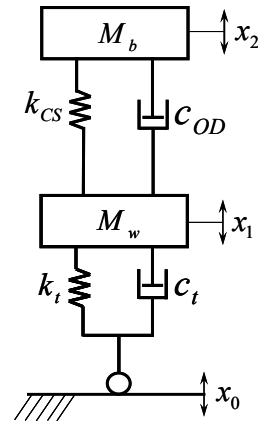

Fig. 12 Vibration model of a quarter vehicle equipped with classical suspension

Ride comfort of a vehicle can be roughly estimated from the transmissibility X_2/X_0 of vibration from the rough road to the vehicle body, where X_2 and X_0 are amplitudes of vibration for the response (x_2) and the excitation (x_0), respectively. Thus, in the case of classical suspension shown in Fig. 12, the transmissibility of vibration can be calculated as:

$$\frac{X_2}{X_0} = \frac{k}{m}\sqrt{\frac{[1-g(\Omega)\frac{c}{k}]^2 + g(\Omega)(1+\frac{c}{k})^2}{[f(\Omega)-g(\Omega)\frac{c}{m}]^2 + g(\Omega)[\frac{k+c}{m}-\Omega^2(1+c\frac{1+m}{m})]^2}}$$

(28)

where the functions $f(\Omega)$ and $g(\Omega)$ are given by:

$$f(\Omega) = \Omega^4 - \Omega^2(1+k\frac{1+m}{m}) + \frac{k}{m} \quad ; \quad g(\Omega) = 4\zeta_w^2\Omega^2 \qquad (29)$$

Here $\Omega = \omega/\omega_w$ is the dimensionless circular frequency, ω is the circular excitation frequency of the road, $\omega_w = \sqrt{k_t/M_w}$ is the natural circular frequency of the wheel, $m = M_b/M_w$ is the dimensionless mass, $\zeta_w = 0.5c_t/\sqrt{k_tM_w}$ is the damping ratio of the wheel, $k = k_{CS}/k_t$ is the dimensionless stiffness, and $c = c_{OD}/c_t$ is the dimensionless damping.

As already stressed in the end of the previous section, since

World Academy of Science, Engineering and Technology
International Journal of Mechanical and Mechatronics Engineering
Vol:10, No:1, 2016

the colloidal spoke buffer can be designed to be sufficiently flexible to accommodate practical wheel systems, and since the colloidal spoke displays a dual function, of absorber and spring, comfortable vehicles can be achieved even if the traditional suspension is partially simplified, or totally omitted. Thus, Fig. 13 illustrates a partially simplified suspension, where the classical oil absorber is omitted. In this case, the transmissibility of vibration can be calculated as:

$$\frac{X_2}{X_0} = \frac{k}{m} \sqrt{\frac{1 + g(\Omega)}{[f(\Omega)]^2 + g(\Omega)(\frac{k}{m} - \Omega^2)^2}} \qquad (30)$$
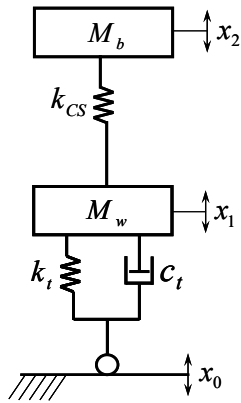

Fig. 13 Vibration model of a quarter vehicle equipped with a partially simplified suspension, where the classical oil absorber is omitted

Further, by providing adequate elastic (spring constant k_t) and dissipative (damping coefficient c_t) characteristics to the proposed colloidal spoke buffer, the wheel and suspension system can be redesigned at a larger scale (Fig. 14), by omitting both the helical compression spring and the oil absorber. In such circumstances, the vehicle body is rigidly connected to the tire's wheel system, and the transmissibility of vibration can be calculated as:

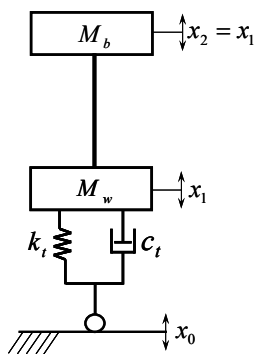$$\frac{X_2}{X_0} = \sqrt{\frac{1 + g(\Omega)}{[1 - (1 + m)\Omega^2]^2 + g(\Omega)}} \qquad (31)$$

Fig. 14 Vibration model of a quarter vehicle with the wheel and suspension system redesigned at a larger scale, by omitting both the classical helical compression spring and the oil absorber

A. Case of the Classical Pneumatic Tire

Fig. 15 shows variation of the transmissibility of vibration from the rough road to the vehicle body X_2 / X_0 versus the dimensionless frequency, for a vehicle equipped with classical pneumatic tires and three types of suspensions: 1) *classical suspension* ($\zeta_w = 0.017$, $m = 10$, $k = 0.1$, and $c = 10$); 2) partially simplified suspension system with *omitted absorber* ($\zeta_w = 0.017$, $m = 10$, $k = 0.1$, and $c = 0$); and 3) omitted suspension system, where both the *helical compression spring* and the *oil absorber* are *omitted* ($\zeta_w = 0.017$, $m = 10$, $k = 0$, and $c = 0$).
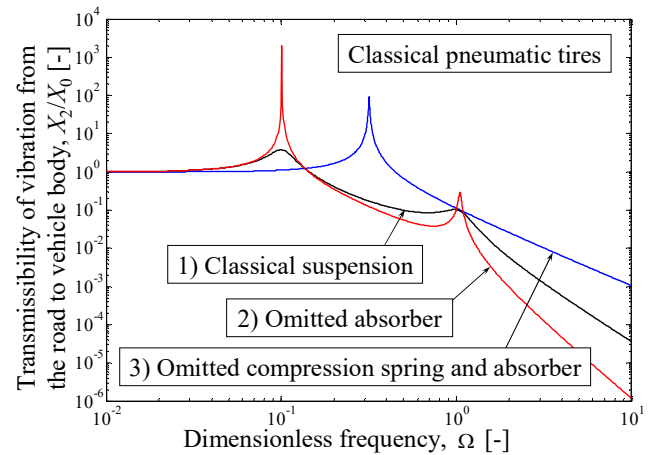

Fig. 15 Variation of the transmissibility of vibration from the rough road to the vehicle body versus the dimensionless frequency, for a vehicle equipped with classical pneumatic tires and three types of suspensions: 1) classical suspension; 2) proposed system with omitted absorber; 3) proposed system with omitted spring and absorber

As expected, Fig. 15 shows that by simply removing the oil absorber, without redesigning the whole suspension-wheel system, the transmissibility of vibration from the rough road to the vehicle body worsens. Thus, the two peaks corresponding to the critical frequency of the body and wheel become higher. A little bit unexpected is the fact that removing the entire suspension and creating a rigid link between the wheel and the car body might be a better solution of structural simplification, since the resonant peak decreases.

B. Proposed Redesign of the Wheel-Suspension System, by employing Non-Pneumatic Tires with Colloidal Spoke Buffer

Fig. 16 shows variation of the transmissibility of vibration from the rough road to the vehicle body X_2 / X_0 versus the dimensionless frequency, for a vehicle equipped with three types of wheel-suspension systems: 1) *classical suspension* and *classical pneumatic tires* ($\zeta_w = 0.017$, $m = 10$, $k = 0.1$, and $c = 10$); 2) *non-pneumatic tires* with *colloidal spoke buffer* and the *proposed suspension* with *omitted absorber*; and 3) *non-pneumatic tires* with *colloidal spoke buffer* and the *proposed suspension* with *omitted spring* and *absorber*.

World Academy of Science, Engineering and Technology
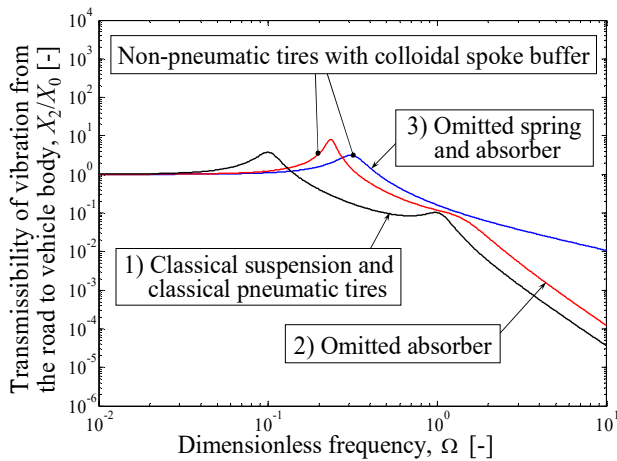International Journal of Mechanical and Mechatronics Engineering
Vol:10, No:1, 2016

Fig. 16 Variation of the transmissibility of vibration from the rough road to the vehicle body versus the dimensionless frequency, for a vehicle equipped with three types of wheel-suspension systems: 1) classical suspension and pneumatic tires; 2) non-pneumatic tires with colloidal spoke buffer and omitted absorber; 3) non-pneumatic tires with colloidal spoke buffer and omitted spring and absorber

Fig. 16 shows that similar transmissibility of vibration from the rough road to the vehicle body, as in the case of the classical suspension-wheel system, can be obtained for non-pneumatic tire with colloidal spoke buffer. Proposed suspension structural simplification (only absorber omission, or omission of both the spring and absorber) should be performed together with an adequate redesign of the whole wheel-suspension system. The redesign technique can be summarized as follows:

1) Damping coefficient of the non-pneumatic tire with colloidal spoke buffer should be increased from the low value corresponding to the pneumatic tire (e.g., $c_t = 81$N·s/m [1]) to the equivalent value of the classical wheel-suspension system (e.g., $c_t + c_{OD} = 81 + 701 = 811$N·s/m [1]).

2) Spring constant of the non-pneumatic tire with colloidal spoke buffer should be decreased from high value corresponding to the pneumatic tire (e.g., $k_t = 215$N/mm [1]) to the equivalent value of the classical wheel-suspension system, which is closed to the helical compression spring (e.g., $k_{CS} = 21.6$N/mm [1]).

In this way, the damping ratio ζ_w of the wheel increases from is the 0.017 to 0.531 and the dimensionless stiffness k increases from 0.1 to 1. Accordingly, the numerical values of the parameters used to achieve the results shown in Fig. 16, were taken as follows:

For the case 2) of *non-pneumatic tires* with *colloidal spoke buffer* and the *proposed suspension* with *omitted absorber*: $\zeta_w = 0.531$, $m = 10$, $k = 1$, and $c = 0$; and

For the case 3) of *non-pneumatic tires* with *colloidal spoke buffer* and the *proposed suspension* with *omitted spring* and *absorber*: $\zeta_w = 0.531$, $m = 10$, $k = 0$, and $c = 0$.

VI. CONCLUSIONS

In this work, by replacing the traditional solid spokes with colloidal spokes, a vehicle wheel with a built-in suspension structure was proposed. From the analytical and numerical analysis of the proposed wheel-suspension system, the following conclusions can be drawn:

1) In order to increase the lifespan of the tire, to reduce the fuel consumption of the vehicle, and to improve the ride comfort of the vehicle, it is generally preferable to design the hub and rim as rigid, but the spoke region as elastic.

2) Previous design conditions cannot be achieved by the standard pneumatic tires, since the elastic buffer (pressurized tread tube) is placed at the outer part of the tire, and besides, the central spoke region is rigid; however, design conditions can be easily satisfied by non-pneumatic (airless) tires with radially deployed colloidal spokes.

3) Since the tread spring constant of a non-pneumatic tire is considerably larger than the spring constant of the proposed colloidal spokes, the equivalent spring constant of the tire equals the spring constant of the colloidal spoke buffer.

4) Under the assumption that the spoke region is rigid, the necessary but sufficient number of spokes was found to be 3. Such case of study is relevant for the liquid spokes using ideal incompressible liquids, and it is also relevant for the standard pneumatic tires, where the spoke region is rigid.

5) Under the assumption that the spoke region is elastic, the necessary but sufficient number of spokes was found to be 5. In such circumstances, the stiffness of the spoke buffer equals the stiffness of the colloidal spoke. Additionally, stiffness of the spoke buffer when the contact of the tire with the ground is exactly on the spoke equals the stiffness of the spoke buffer when the contact is exactly in the middle between two spokes.

6) Since the domain of variation for the stroke of the proposed colloidal spokes is easily achievable from a practical standpoint, one concluded that colloidal spokes can be used to construct efficient spoke buffers for non-pneumatic tires.

7) By mixing silica gel particles in water, reduction of the bulk modulus of elasticity of the resulting colloidal solution leads to an important decrease of the spoke stiffness. Thus, the colloidal spoke buffer can be designed to be sufficiently flexible to accommodate practical applications, such as a wheel system with a built-in suspension structure.

8) Beneficial stiffness reduction of the colloidal spoke is accompanied by an advantageous dissipative effect, produced during the cyclical penetration/exudation of water in/from the liquid-repellent nanopores. As a result, colloidal spoke displays a dual function, of absorber and compression spring. In such circumstances, comfortable vehicles can be achieved even in the absence of the classical absorber (see the simplified suspension proposed, where classical oil absorber is omitted), or further, in the absence of the whole classical suspension (see the simplified system proposed, where both the helical spring

and absorber are omitted, vehicle body being rigidly connected to the tire's wheel system).

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