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Optimal Image Representation for Linear Canonical Transform Multiplexing

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Abstract—Digital images are widely used in computer applications. To store or transmit the uncompressed images requires considerable storage capacity and transmission bandwidth. Image compression is a means to perform transmission or storage of visual data in the most economical way. This paper explains about how images can be encoded to be transmitted in a multiplexing time-frequency domain channel. Multiplexing involves packing signals together whose representations are compact in the working domain. In order to optimize transmission resources each 4×4 pixel block of the image is transformed by a suitable polynomial approximation, into a minimal number of coefficients. Less than 4×4 coefficients in one block spares a significant amount of transmitted information, but some information is lost. Different approximations for image transformation have been evaluated as polynomial representation (Vandermonde matrix), least squares + gradient descent, 1-D Chebyshev polynomials, 2-D Chebyshev polynomials or singular value decomposition (SVD). Results have been compared in terms of nominal compression rate (NCR), compression ratio (CR) and peak signal-to-noise ratio (PSNR) in order to minimize the error function defined as the difference between the original pixel gray levels and the approximated polynomial output. Polynomial coefficients have been later encoded and handled for generating chirps in a target rate of about two chirps per 4×4 pixel block and then submitted to a transmission multiplexing operation in the time-frequency domain.

Keywords—Chirp signals, Image multiplexing, Image transformation, Linear canonical transform, Polynomial approximation.

I. Introduction

DIGITAL representation of images requires a very large number of bits and it is always highly important to represent the information contained in the image with fewer numbers of bits. Image compression is a technique to reduce the redundancies in data representation in order to decrease data storage requirements and hence reduces the communication costs [1]–[4]. Reducing the storage requirement is equivalent to increasing the capacity of the storage medium and hence communication bandwidth. Basically, image is represented as a combination of information and redundancy. In order to correctly interpret the purpose of image information, the image data must be preserved permanently in its original form. For lossless image compression, it is always required to preserve information absolutely and it needs as more coefficients as pixels for the polynomial approximation. Otherwise, for lossy compression, that some amount of information may be lost if the number of coefficients in the polynomial approximation are reduced [4], [5]. In this paper, different approximations for image transformation have been evaluated as polynomial representation (Vandermonde matrix) [5], least squares + gradient descent [6], 1-D Chebyshev polynomials $T(x)$ (notation T is taken from the older French spellings of Chebyshev as Tchebichef) [7], 2-D Chebyshev polynomials [8], [9] or truncated singular value decomposition (SVD) [10], [11] for lossy/lossless image compression. In all polynomial approximation techniques, each 4×4 pixel block of the image is transformed into a set of polynomial coefficients. By using an 8-coefficient polynomial approximation, 50% of the representation is spared, but some information is lost. These 8 polynomial coefficients can be encoded and handled for generating chirps [12] in a rate of two chirps per block . The generated chirp functions can be rotated and shifted by using offset linear canonical transform (OLCT) [13]–[16] for multiplexing operation in the time-frequency domain [17].

The rest of the paper is organized as below: Section II gives the brief review of the proposed polynomial approximation techniques for image representation. Section III explains the experimental results for comparative analysis of image compression by using different polynomial approximation techniques or SVD. Generation of chirp functions from the polynomial coefficients are explained in Section IV and finally conclusion and future scope is given in Section V.

II. Image Compression

Image compression is an application of data compression that encodes the original image with fewer bits without degrading the quality of the image to an unacceptable level. A gray image as stored in binary code by 8 bits per pixel is able to take account of 0 to 255 gray levels. The target of image compression consists in handling the structure of the image in order to reduce the bulk of data for storage purposes or to transmit data in an efficient form. Image compression plays an important role in applications like tele-video conferencing, remote sensing and medical imaging [18], [19]. Image compression may be classified as lossy or lossless compression. In lossless compression, all original data can be recovered when the file is uncompressed i.e. every single bit of data that was originally in the file remains after the file is uncompressed. On the other hand, lossy compression reduces a file by permanently eliminating certain information, especially redundant information by reducing the

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correlation between the pixels [20], [21]. Three main steps have to be considered for image compression: transformation, quantization and coding [22], [23].

The transformation step consists of finding some invertible transform T for decorrelating the image pixels as much as possible, by removing redundancy. The results of transformation T is a set of coefficients. If such transformation is exactly invertible then the transformation is lossless, otherwise it is a lossy transformation. In this paper, seven different transformations have been applied for image tiling in blocks of 4×4 pixels. In the first phase, a method to obtain an exact or simplified least square representation of an image by a set of polynomial coefficients has been applied [5]. The idea of reducing a matrix into a diagonal representation is an ill-posed mathematical problem, because in the sense of Newton polynomial interpolation "given a finite set of data points, there is only one polynomial, of least possible degree, that passes through all of them", and this polynomial could be variable-separable or not. To meet this challenge, a new algorithm piece-wise least squares-gradient descent polynomial approximation of images with reduced number of coefficients has been developed. By the combination of least squares and gradient descent, it became possible to develop an algorithm for approximating an image tile of 4×4 block $g(x, y)$ to a polynomial with 8 coefficients, 4 for variable x and 4 for variable y in a separate variable context. It has been observed that this algorithm gives better results with large images at the cost of longer time of calculations. The reason is due to the better behavior of polynomials with smoother image variations as details change more slowly in larger images.

To further improve the quality the recovered image, Chebyshev polynomial with spiral scanning method is proposed. Chebyshev polynomials are among the most popular orthogonal polynomials that are used to approximate a set of data and are useful in such contexts as numerical analysis and circuit design [8], [9]. Orthogonal polynomials can be used to make the polynomial coefficients uncorrelated, to minimize the error of approximation, and to minimize the sensitivity of calculations to round off error [24].

The main advantage of Chebyshev polynomials is that the error is equally distributed in the interval of application, while in the general polynomial approximation the error is unevenly distributed. The results obtained by spiral scanning method are found to be superior than other scanning methods such as row scanning, column scanning and diagonal scanning etc. [24], because in this scanning method the sequence is a next-pixel arrangement with a close pixel to pixel correlation that abides well with a 1-D smooth polynomial fitting. To control the desired amount of quality, the Chebyshev algorithm is transformed into an adaptive algorithm that is by establishing an admissible error e and then approximating by polynomials of $1, 2, 3...m$ degree, until we get an squared error or quality factor $Q < e$ to stop the algorithm, then in a frame of 16 coefficients we will have 16-m zero coefficients and then the length of the encoded coefficients will be reduced significantly. In this paper, for all adaptive algorithms, the square error is considered as 0.1 per 4×4 image block and PSNR can be further improved by considering smaller error at the cost of less compression ratio. When square error is zero then PSNR is infinite and worse values of PSNR are obtained by increasing admitted error.

Let us suppose that image $G(x, y)$ is tessellated by a set of squared boxes of 4×4 pixels and we seek approximate each block $g(x, y)$ by a polynomial such that $g(x, y) \approx P(x, y)$, where $P(x, y)$ is the unknown polynomial and is to be obtained by the least squares approximation method. The error function Q is given by

$$Q = \sum_{i=1}^{4} \sum_{j=1}^{4} \Big(g(x_i, y_j) - P(x_i, y_j) \Big)^2 \tag{1}$$

The problem with all 1-D polynomial methods applied to images is in the encoding step, because in 1-D x powers varies from 0 to 15 and in 2-D x and y powers only varies from 0 to 3. In 1-D polynomial, the coefficients are going to be ranged in a big interval and this is difficult and slow to be coded in a binary low rate bit/pixel. Contrary wise, 2-D polynomials with small polynomial degree are more suitable for high-compression and fast calculation performances. Hence 2-D Chebyshev and 2-D adaptive Chebyshev algorithms have been developed for image compression. In 2-D, the degree of polynomial is the sum of maximum power of polynomial in x direction and maximum power of polynomial in y direction and the number of coefficients are obtained by the product of maximum power of polynomial in x direction plus one and maximum power of polynomial in y direction plus one. Approximation of 2-D Chebyshev polynomials is achieved with two separable 1-D Chebyshev polynomials $T(x)$ and $T(y)$ that are discrete and orthogonal [8]. This massively reduces the complexity, as the same principles from 1-D Chebyshev polynomial has been applied, i.e. intervals of x and y are still between $[-1, 1]$. Mathematically, the 2-D Chebyshev polynomial is expressed as [25]

$$\tilde{f}(x, y) = \sum_{m=0}^{M} \sum_{n=0}^{N} a_{m,n} T_m(x) T_n(y) \tag{2}$$

where $a_{m,n}$ are the 2-D Chebyshev polynomial coefficients. Chebyshev approximation (sometimes referred to as Chebyshev moments) have found applications in image analysis, pattern recognition, image segmentation, image reconstruction and rendering [26]–[30]. Finally, another technique, in which a $m \times n$ matrix can be factored into the product of an orthogonal matrix times a diagonal matrix times another orthogonal matrix has been applied for image compression known as singular value decomposition (SVD). For a $m \times n$ image, the truncated SVD originates $k(m + n + 1)$ coefficients. The compressed image will reduce the storage space requirement to $k(m + n + 1)$ bytes as against the storage space requirement of $m \times n$ bytes of the original uncompressed image. The compression is achieved if the storage space required by the compressed image is less than that required by the original image i.e., $(m \times n) > k(m + n + 1)$. Value of k represents the number of eigen values used in the reconstruction of the compressed image. Smaller the value of k, the more is the compression ratio (i.e. less storage space is required) but image quality

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deteriorate. When $k << m$, then the number of coefficients are $< (m \times n)$, but for $k = m$ (0 error), there are more coefficients than the pixels and the transformation is not suitable at all [10], [11].

The outcome of the transformation step is a set of coefficients, that are real numbers, and they may be adjusted to a least high-precision floating point range. These values require a high number of bits to be stored. The aim of the quantization step is replacing these real numbers with an approximation that requires fewer bits to be stored. Then, quantization is a lossy process and exact values of coefficients cannot be recovered. So, some error hast to be admitted in this step.

The coding step takes advantage about that the most of the coefficients will be close to zero and then set to zero in the quantization step. The outcome of the transformation step plus the quantization step will be a sequence of numbers with a great amount of zeros. Such configuration can be successfully compressed to a binary sequence of $0s$ and $1s$. In this paper, the arithmetic encoding and decoding technique has been applied. Arithmetic coding is a common algorithm used in both lossless and lossy data compression algorithms. Arithmetic encoders are better suited for adaptive models than Huffman coding, but they can be challenging to implement [31]–[35].

III. Experimental Results

For testing the proposed compression algorithms, experiments have been done to compare the performance of the different polynomial image approximation techniques using a fixed block of size of 4×4 and truncated SVD. These are applied on a number of well-known standard bitmap images, all images of 256 gray levels (8 bits/pixel) of size 256×256. The peak signal-to-noise ratio (PSNR) in dB, nominal compression rate (NCR) i.e. entropy and compression ratio (CR) in bits/pixel are adopted as objective fidelity measures between the original image and the decoded image. The NCR is measured with respect to the initial storage requirement (ISR) of the original 256 gray scale bitmap image i.e. 8 bits/pixel.

$$\text{Nominal Compression Rate(Entropy)} = -\sum_{k=1}^{q} p_k \log(p_k) \text{ bits/pixel} \quad (3)$$

where q is the number of quantization levels and

$$p_k = \frac{\text{Number of polynomial coefficients in the kth bin}}{\text{Total number of polynomial coefficents}} \quad (4)$$

For entropy calculation of 8 bit images, the number of quantization levels will be equal to $q = 256$. In case of Vandermonde technique, for proper recovery of the image at the decoder side, the number of quantization levels q should be 4096 or more. Hence Vandermonde technique is not suitable for compression purposes, because q has to be a huge number. By increasing q, better PSNR is obtained at the cost of worse CR. The reason is that if the compression will be more i.e. small number q of levels, more information will be lost. When the degree of polynomial is high, generally > 8 then for 1-D Chebyshev polynomial, 256 quantization levels for arithmetic compression will result in a low PSNR because the quantization does not meet the sampling theorem, hence these polynomials have to be discarded. NCR is not the compression of the image but a compression possibility that will be real if coefficients are efficiently encoded. This parameter is important to determine the efficiency of the encoding step and also the information reduction from the image ISR 8 bits/pixel. Final image compression is obtained after encoding the polynomial coefficients such as:

$$\text{Final Compression} = \frac{\text{Length(B2)}}{M \times N} \text{ bits/pixel} \quad (5)$$

where B2 indicates the binary code for polynomial coefficients which are obtained after quantizing and encoding the polynomial coefficients and $M \times N$ indicates the image size.The CR may be defined as

$$\text{Compression Ratio} = \frac{\text{Initial Storage Requirement}}{\text{Final Compression}} \quad (6)$$

The unity value of CR indicates that there is no compression. More will be the value of CR, better will be the image compression at the cost of degraded image quality. The various test results are shown in Tables I and II. In order to test the lossless compression system efficiency for an image block of size 4×4, the proposed algorithms results in infinite (∞) PSNR with the selection of 16 polynomial coefficients. However, for lossy compression the number of polynomial coefficients are < 16 for an image block of size 4×4. To evaluate the performance of the proposed image compression techniques, a set of seven 8 bit gray scale images are put under test. The numerical values of different performance metrics of various test images are shown in Tables I and II. Figs. 1 and 2 show the outcome of different transformations for different 8-bit bitmap gray scale 256×256 size test images. It has been observed from Tables I and II that as the degree of polynomial increases then the compression results in low PSNR for 1-D Chebyshev polynomial. Due to small degree of polynomial, 2-D Chebyshev polynomials ends up with high PSNR as compare to 1-D Chebyshev and gradient-descent polynomial techniques and found more suitable for image compression.

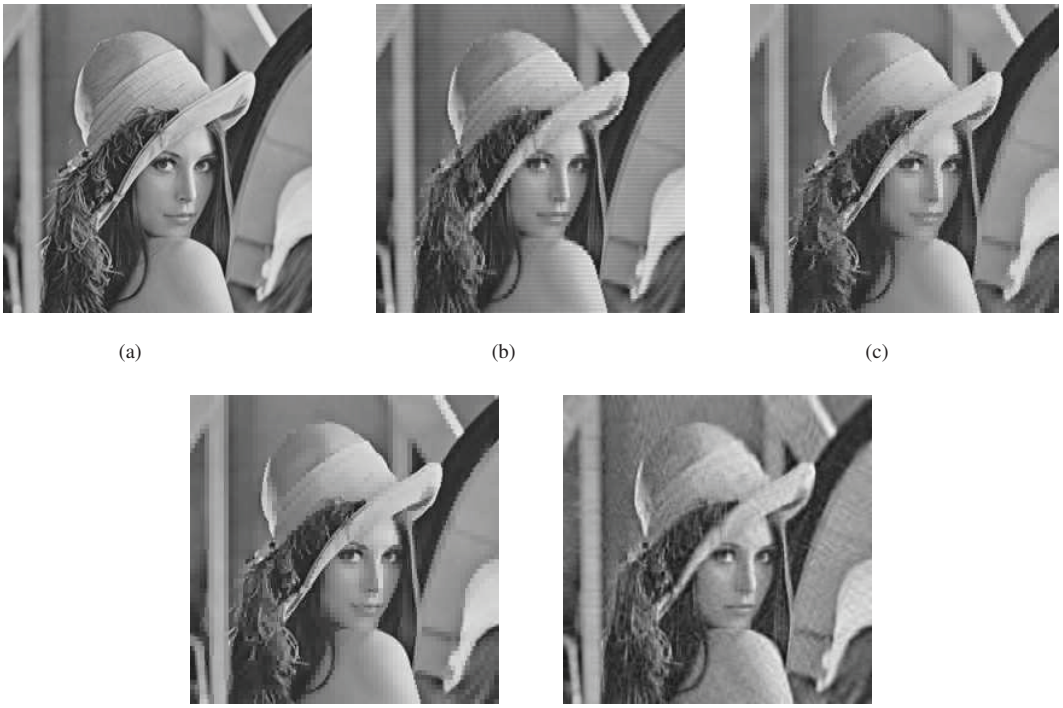
IV. Generation of Chirp Functions from the Polynomial Coefficients

A chirp is a signal in which the frequency increases ('up-chirp') or decreases ('down-chirp') with time. These are ubiquitous in nature as these can be observed in animal communication and echolocation, geophysics, astro-physics, acoustics, or biology. These are also extensively used in manmade systems, such as radar and sonar [36].

Chirps are considered as transient observations that may take account for many non-stationary deterministic signals. The time-frequency plane is a natural representation space for chirps [12]. When the instantaneous frequency is a linear function, then the input signal is referred as a linear chirp function [12]. A certain family of linear chirp functions may be defined as:

$$f(t) = Ae^{-\left(p(t+q)^2 + jrt^2\right)} \quad (7)$$

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(a) (b) (c)

(d) (e)

Fig. 1 Decoded Image from different transformations (a) Original image (b) Gradient-descent, PSNR=27.7228 dB (c) 1-D adaptive Chebyshev with error = 0.1, PSNR=28.3595 dB (d) 2-D adaptive Chebyshev with error = 0.1, PSNR=28.739 dB (e) SVD with $k = 32$, PSNR=27.6712 dB



(a) (b) (c)

Fig. 2 Decoded image from Chebyshev transformations with 8 coefficients (a) Original image (b) 1-D Chebyshev, PSNR=31.76 dB (c) 2-D Chebyshev, PSNR=32.7720 dB

where $A, p, q, r \in R$ and $A > 0, p > 0, p << abs(r)$. This last requirement is introduced to determine appropriate chirp shape in the time-frequency domain. From (7), it has been observed that four elements A, p, q, r are required to generate one chirp function successfully. It has been observed from Tables I and II that the desired value of PSNR [31], [37] i.e. $\approx$ 30 dB is achieved when the numbers of polynomial coefficients are equal to 8. Hence an image block of size 4×4 with 8-coefficient polynomial approximation can be transformed into a set of chirp functions.

The time-frequency response of the chirp function can be shown with the help of the Wigner-Ville Distribution (WVD). For a signal $s(t)$, with analytic associate $x(t)$, the Wigner-Ville Distribution, $W_x(t,u)$ is defined as [38]:

$$W_x(t,u) = \int_{-\infty}^{\infty} x\left(t+\frac{\tau}{2}\right) x^*\left(t-\frac{\tau}{2}\right) e^{-ju\tau} \mathrm{d}\tau \qquad (8)$$

Finally, the chirps generated in each 4×4 are submitted to a transmission operation in the time-frequency domain for image multiplexing.

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TABLE I
PERFORMANCE METRICS OF SVD AND CHEBYSHEV POLYNOMIALS

Bitmap Gray Scale Images 256×256	Truncated SVD				2-D Chebyshev						1-D Chebyshev (Spiral)					
	k	Number of Coefficients for Complete Image	PSNR (dB)	Compression Ratio	Number of Coefficients for 4×4 Image Block	Degree of Polynomial	PSNR (dB)	NCR (bits/pixel)	Compression (bits/pixel)	Compression Ratio	Number of Coefficients for 4×4 Image Block	Degree of Polynomial	PSNR (dB)	NCR (bits/pixel)	Compression (bits/pixel)	Compression Ratio
Barbara	16	8208	25.4418	7.9844	4	1+1=2	27.577	5.529	1.5912	5.028	4	3	27.094	5.277	1.5296	5.23
	32	16416	27.9373	3.992	9	2+2=4	34.728	5.076	3.3316	2.401	8	7	31.76	5.03	2.9436	2.718
	64	32832	32.1365	1.996	12	3+2=5	38.499	4.925	4.3146	1.854	12	11	35.828	4.902	4.2832	1.868
	128	65664	40.6691	0.998	16	3+3=6	∞	4.719	5.5431	1.443	16	15	∞	4.1	4.6248	1.73
Boat	16	8208	24.8743	7.9844	4	1+1=2	25.341	5.11	1.4728	5.432	4	3	25.44	5.07	1.4492	5.52
	32	16416	27.7365	3.992	9	2+2=4	34.352	4.71	3.0989	2.582	8	7	30.826	4.829	2.8075	2.85
	64	32832	32.6388	1.996	12	3+2=5	37.92	4.765	4.2337	1.89	12	11	35.753	4.704	4.0842	1.959
	128	65664	42.2097	0.998	16	3+3=6	∞	4.848	5.668	1.411	16	15	∞	3.202	3.6428	2.196
Cameraman	16	8208	24.0202	7.9844	4	1+1=2	25.795	4.513	1.3317	6.008	4	3	24.95	4.394	1.2763	6.268
	32	16416	26.9669	3.992	9	2+2=4	34.775	4.052	2.7292	2.931	8	7	30.742	4.209	2.5029	3.196
	64	32832	32.246	1.996	12	3+2=5	39.116	4.084	3.6651	2.183	12	11	36.012	3.843	3.3916	2.359
	128	65664	42.5539	0.998	16	3+3=6	∞	4.078	4.9606	1.613	16	15	∞	2.453	2.8835	2.774
Lena	16	8208	24.6385	7.9844	4	1+1=2	27.917	5.238	1.522	5.256	4	3	26.876	5.055	1.4671	5.453
	32	16416	27.6712	3.992	9	2+2=4	35.462	4.595	3.0565	2.617	8	7	32.409	4.786	2.8176	2.839
	64	32832	32.934	1.996	12	3+2=5	41.058	4.382	3.9111	2.046	12	11	37.656	4.306	3.8071	2.101
	128	65664	42.5151	0.998	16	3+3=6	∞	4.552	5.4624	1.465	16	15	∞	3.485	3.9713	2.015
House	16	8208	26.9382	7.9844	4	1+1=2	27.58	3.919	1.0716	7.465	4	3	28.154	4.116	1.1171	7.161
	32	16416	33.0276	3.992	9	2+2=4	40.488	3.52	2.1828	3.665	8	7	36.111	3.821	2.0603	3.883
	64	32832	39.733	1.996	12	3+2=5	44.345	3.396	2.8348	2.822	12	11	41.499	3.978	3.2164	2.487
	128	65664	49.5905	0.998	16	3+3=6	∞	3.399	3.6909	2.618	16	15	∞	3.504	3.6943	2.166
Goldhill	16	8208	26.749	7.9844	4	1+1=2	27.437	5.289	1.5396	5.196	4	3	27.503	5.292	1.5362	5.208
	32	16416	28.5626	3.992	9	2+2=4	35.019	4.883	3.2443	2.466	8	7	32.804	5.035	2.9498	2.712
	64	32832	32.4379	1.996	12	3+2=5	38.156	4.896	4.358	1.836	12	11	37.494	4.598	4.068	1.967
	128	65664	39.9712	0.998	16	3+3=6	∞	5.184	6.0496	1.322	16	15	∞	4.272	4.8195	1.66
Pirate	16	8208	24.1064	7.9844	4	1+1=2	26.264	5.648	1.6288	4.912	4	3	26.148	5.437	1.5701	5.095
	32	16416	27.53	3.992	9	2+2=4	33.96	5.237	3.4319	2.331	8	7	31.529	5.3	3.0821	2.596
	64	32832	32.4454	1.996	12	3+2=5	37.985	5.158	4.552	1.758	12	11	36.282	4.534	4.0046	1.998
	128	65664	41.4567	0.998	16	3+3=6	∞	5.29	6.1572	1.299	16	15	∞	4.349	4.8841	1.638

TABLE II
EXPERIMENTAL RESULTS OF VANDERMONDE, GRADIENT DESCENT AND ADAPTIVE CHEBYSHEV TRANSFORMATIONS

Transform	Number of Coefficients	Degree of Polynomial	Performance Metrics	Bitmap Gray Scale Images of Size 256×256						
				Barbara	Boat	Cameraman	Lena	House	Goldhill	Pirate
1-D Adaptive Chebyshev	Adaptive	Adaptive	PSNR (dB)	27.5373	27.6707	29.3206	28.3595	30.4403	27.5315	27.2266
		0.1 Admitted Error	NCR (bits/pixel)	1.6573	1.4319	0.0112	0.9405	2.0677	1.7699	1.2673
			Compression (bits/pixel)	0.9399	0.8393	0.0113	0.5629	0.8338	0.9803	0.7618
			CR	8.5113	9.5322	706.587	14.2126	9.5944	8.1608	10.5009
2-D Adaptive Chebyshev	Adaptive	Adaptive	PSNR (dB)	27.84	28.0027	29.6157	28.739	31.1426	27.5919	27.4119
		0.1 Admitted Error	NCR (bits/pixel)	1.0928	1.6595	1.1889	1.4446	1.5084	1.3314	1.2057
			Compression (bits/pixel)	0.7612	0.9309	0.8331	0.7844	0.6046	0.7166	0.8551
			CR	10.5104	8.5935	9.6032	10.1988	13.2312	11.1631	9.3556
Vandermonde	16 for Image Block of size 4×4	16	PSNR (dB)	∞	∞	∞	∞	∞	∞	∞
			NCR (bits/pixel)	2.4132	2.7638	2.0059	2.4565	3.2072	3.3093	2.8638
			Compression (bits/pixel)	2.6663	3.1701	2.2883	2.7115	3.4935	3.76	3.1625
			CR	3.0004	2.5236	3.4961	2.9503	2.29	2.1276	2.5296
Gradient Descent	$4 + 4$ for Image Block of size 4×4	3+3	PSNR (dB)	23.6264	25.8832	26.5208	27.7228	28.3438	28.2192	27.1453
			NCR (bits/pixel)	4.5036	3.3578	2.5994	3.0186	4.0074	3.5058	3.7874
			Compression (bits/pixel)	2.6834	1.9615	1.5815	1.7814	2.2293	2.1483	2.1967
			CR	2.9813	4.0784	5.0585	4.491	3.5885	3.724	3.6418

V. CONCLUSION

In this paper, different transformations techniques have been tested for image transformation. It has been observed that Vandermonde technique is not suitable for compression purposes because the number of quantization levels q has to be a huge number. The adaptive Chebyshev algorithms will run until we reach the desired squared error and it results in lossless compression when the squared error is set to zero. In another way, the transformation technique is transformed to lossless compression technique when the number of coefficients for 4×4 image block are equal to 16. From the experimental results, it has been found that the 2-D Chebyshev polynomial performs better in terms of high PSNR. As a future scope, this transformation technique with 8-coefficients could be transformed into two chirps if no encoding is applied to the coefficients, but this will originate an ill-posed problem in the time-frequency domain. This is the reason to introduce a compression step that simplifies the time-frequency operation against an increased number of chirp functions. Finally, we conclude that number of coefficients cannot reduce substantially without experiencing a considerable loss of information and consequently we will be probably limited to keep compression ratio equal to 1 or close to 1 or accept an error that can be fixed by design. Further work will consists in generating suitable chirp signals from the compressed binary code, able to be multiplexed for transmission purposes in the time-frequency domain.

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