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An Approximation Method for Exact Boundary Controllability of Euler-Bernoulli System

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***Abstract*—**The aim of this work is to study the numerical implementation of the Hilbert Uniqueness Method for the exact boundary controllability of Euler-Bernoulli beam equation. This study may be difficult. This will depend on the problem under consideration (geometry, control and dimension) and the numerical method used. Knowledge of the asymptotic behaviour of the control governing the system at time T may be useful for its calculation. This idea will be developed in this study. We have characterized as a first step, the solution by a minimization principle and proposed secondly a method for its resolution to approximate the control steering the considered system to rest at time T.

***Index Terms*—**Boundary control, exact controllability, finite difference methods, functional optimization.

I. Introduction

THE problem of exact controllability is one of the most important analysis of distributed systems (i.e systems whose state is given by solving a partial differential equation). A conventional method of solving this problem was proposed by [15]. Others followed, like HUM (Hilbert Uniqueness Method), developed by [11]-[13], by treating the problem particularly in the context of Euler-Bernoulli beam equation with action on the Dirichlet boundary. This method is to solve this equation:

$$\Lambda\{\varphi^0, \varphi^1\} = \{y^1, -y^0\} \tag{1}$$

where y^0 and y^1 are the initial conditions of the system and Λ an isomorphism between E and it's dual E'.

The resolution of (1) may be difficult. This will depend on the problem under consideration (geometry, control type and dimension) and the numerical method used. According to [1], knowledge of the asymptotic behaviour of the control governing the system at time T may be useful for its calculation. This idea will be developed in this study. More precisely, we determine explicit formulas for φ^0 and φ^1, and therefore explicitly control. An example is presented to illustrate this approach.

The remainder of this paper is organized as follows: Section II defines the exact Dirichlet boundary controllability problem for the Euler-Bernoulli beam equation. Section III describes the proposed method. In Section IV, explicit formulas are presented to explicitly resolve the problem considered. In Section V, an implementation of Hilbert Uniqueness Method is presented. In Section VI, experimental results are presented. Section VII concludes the paper.

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II. Formulation of the Problem

Let T denote a given positive number and let $y^0(x)$ and $y^1(x)$ denote given functions defined on Ω=]0,1[. Let Σ=$\{0,1\}$x]0,T[, Q=]0,1[x]0,T[and $(y^0, y^1) \in L^2(\Omega)\text{x}H^{-2}(\Omega)$ [18].

The Exact Dirichlet boundary controllability problem for the Euler-Bernoulli beam equation is: Find a control function v defined on Σ such that y satisfies:

$$\begin{cases} y_{tt} + y_{xxxx} = 0 & \text{in Q} \\ y(x,0) = y^0(x) \ , \frac{\partial y}{\partial t}(x,0) = y^1(x) & \text{in } \Omega \\ y(x,T) = 0 \ , \frac{\partial y}{\partial t}(x,T) = 0 & \text{in } \Omega \\ y(0,t) = 0 \ , y(1,t) = 0 & \text{t}\in[0,\text{T}] \\ \frac{\partial y}{\partial x}(0,t) = 0 \ \frac{\partial y}{\partial x}(1,t) = v(t) & \text{t}\in[0,\text{T}] \end{cases} \tag{2}$$

It is well known that state y and control function v such that (2) is satisfied exist provided T positive [12], [13], [19].

Motivated by numerical methods presented in [2], [5], [7]-[9] and particularly in [10], we adapt the latter to explicitly solve the problem of exact boundary of Euler-Bernoulli beam equation when the control is the Dirichlet type. For this purpose, we characterized in a first step, the solution of (1) by a minimization principle, which operates in a second step, to determine explicitly $\{\varphi^0, \varphi^1\}$, and therefore the control explicit v* such as y satisfies (2).

III. Proposed Method

A. Characterization of the Solution

The problem (1) may be written in the following form:

$$\begin{cases} Find\ \varphi \in\ E\ \ such\ \ as \\ < \Lambda\varphi, \hat{\varphi} > = < \{y^1, -y^0\}, \hat{\varphi} > \forall\ \hat{\varphi} \in \text{E} \end{cases} \tag{3}$$

where $\varphi = \{\varphi^0, \varphi^1\}$; $\hat{\varphi} = \{\hat{\varphi^0}, \hat{\varphi^1}\ \}$, E=$H_0^2(\Omega)\text{x}L^2(\Omega)$, $E' = H^{-2}(\Omega)\text{x}L^2(\Omega)$ and $< ., . >$ denotes the duality product between E' and E.

A bilinear functional $< \Lambda., . >$ is continuous, symmetric and coercive. These properties, according to [16], are used to characterize the solution of (3) by the principle of following minimization:

B. Principle

Principle 1: Any Solution $\varphi \in E$ of (3) sends the minimum of the functional:

$$J(\varphi) = \frac{1}{2} < \Lambda\varphi, \varphi > - < \{y^1, -y^0\}, \varphi > \tag{4}$$

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and reciprocally.

Proof: (3) is a special case of the following variational problem:

$$\begin{cases} Find\ u \in\ \ V\ \ such\ \ as \\ a(u,v) = L(v); \qquad\qquad \forall\ v \in \mathrm{V} \end{cases} \tag{5}$$

where in (5)
(i) V is a real Hilbert space equipped with the scalar product (.,.), the corresponding norm is $\| \ . \ \|$
(ii) a: V x V $\longrightarrow \mathbb{R}$ is bilinear, continuous, symmetric and coercive.
(iii)L: V $\longrightarrow \mathbb{R}$ is bilinear continuous.
With this assumptions, and after [16], (5) has a unique solution in achieving the minimum in V of the functional J(v)=$\frac{1}{2}a(v,v) - L(v); \forall v \in$ V and reciprocally.
If the principle is satisfied, simply take in [16];
V=E; a(.,.)=$< \Lambda., . >\ \ and\ L : \overset{\wedge}{\varphi} \longrightarrow < \{y^1, -y^0\}, \overset{\wedge}{\varphi} >$ ■

C. Resolution Minimization Problem

Let the functional (4):
J($\{\varphi^0, \varphi^1\}$)=$\frac{1}{2} < \Lambda\{\varphi^0, \varphi^1\}, \{\varphi^0, \varphi^1\} > -(y^1, \varphi^0)+(y^0, \varphi^1)$
According to [12] and [13], we have:

$$\frac{1}{2} < \Lambda\{\varphi^0, \varphi^1\}, \{\varphi^0, \varphi^1\} >= \frac{1}{2}\int_0^T \left[\frac{\partial^2 \varphi(1,t)}{\partial x^2}\right]^2 dt \tag{6}$$

So:

$$J(\{\varphi^0, \varphi^1\}) = \frac{1}{2}\int_0^T \left[\frac{\partial^2 \varphi(1,t)}{\partial x^2}\right]^2 dt - \int_\Omega \left[\varphi^0 y^1 - \varphi^1 y^0\right] dx \tag{7}$$

Solving (1) is then equivalent to solving the minimization problem:

$$\underset{\{\varphi^0,\varphi^1\}\in E}{Inf}\ J\left(\{\varphi^0, \varphi^1\}\right) \tag{8}$$

Let $\{\varphi_T^0, \varphi_T^1\}$ the solution of (8). We are going to transform (7) by introducing the T factor as:

$$T.J(\{\varphi^0, \varphi^1\}) = \frac{T}{2}\int_0^T \left[\frac{\partial^2 \varphi(1,t)}{\partial x^2}\right]^2 dt - \int_\Omega \left[y^1 T\varphi^0 - y^0 T\varphi^1\right] dx \tag{9}$$

Let:
$\rho = T.\varphi$
ρ is then the solution of the system:

$$\begin{cases} \frac{\partial^2 \rho}{\partial t^2} + \frac{\partial^4 \rho}{\partial x^4} = 0 & \text{in Q} \\ \rho(x,0) = \rho^0 \ \ and \ \ \frac{\partial \rho}{\partial t}(x,0) = \rho^1 & \text{in } \Omega \\ \rho = \frac{\partial \rho}{\partial x} = 0 & \text{on } \Sigma \end{cases} \tag{10}$$

$$T.J(\{\varphi^0, \varphi^1\}) = \frac{1}{2T}\int_0^T \left[\frac{\partial^2 \rho(1,t)}{\partial x^2}\right]^2 dt - \int_\Omega \left[y^1\rho^0 - y^0\rho^1\right] dx = J\left(\{\rho^0, \rho^1\}\right) \tag{11}$$

The problem (8) becomes:

$$Inf\ J\left(\{\rho^0, \rho^1\}\right) \tag{12}$$

Let $\{\rho_T^0, \rho_T^1\}$ the solution of (12). One have:
$\rho_T^0 = T\varphi_T^0$ and $\rho_T^1 = T\varphi_T^1$
According [1], if we consider:

$$\rho^0 = \lim_{T\to+\infty} \rho_T^0 \tag{13}$$

$$\rho^1 = \lim_{T\to+\infty} \rho_T^1 \tag{14}$$

it is possible to find explicitly (ρ^0, ρ^1). This method will lead to numerical approximations very useful for computations. In fact, it will be possible to make computations of φ_T^0 and φ_T^1 by using:

$$\varphi_T^0 = \frac{1}{T}\rho^0 \tag{15}$$

$$\varphi_T^1 = \frac{1}{T}\rho^1 \tag{16}$$

IV. Explicit forms

Let us consider the system (2).
Introduce eigenfunctions(ψ_j(x)).

$$\begin{cases} \frac{d^4\psi_j(x)}{dx^4} = \lambda_j^2\psi_j(x) & \text{in } \Omega \\ \psi_j = 0; & \text{on } \{0,1\} \\ \frac{d\psi_j}{dx} = 0; & \text{on } \{0,1\} \end{cases}$$

Let us suppose the eigenvalues multiplicity is 1. The eigenvalues of $\frac{d^4}{dx^4}$ are λ_j^2 where $\lambda_j = \mu_j^2$ and

$$cosh(\mu_j).cos(\mu_j) = 1; j = 1, 2, \cdots \tag{17}$$

The eigenfuctions [3] are:

$$\begin{aligned} \psi_j(x) = &\left(sin(\mu_j) - sinh(\mu_j)\right).cos(\mu_j x) \\ &+ \left(cosh(\mu_j) - cos(\mu_j)\right).sin(\mu_j x) \\ &+ \left(sinh(\mu_j) - sin(\mu_j)\right).cosh(\mu_j x) \\ &+ \left(cos(\mu_j) - cosh(\mu_j)\right).sinh(\mu_j x) \end{aligned} \tag{18}$$

Denote by $\omega_j(x)$ the orthonormal eigenfunctions of $\frac{d^4}{dx^4}$ with homogeneous Dirichlet condition. Consider (11) and look for: $\lim_{T\to+\infty}\ J\left(\{\rho^0, \rho^1\}\right)$
Let:

$$y^0 = \sum_{j=1}^{\infty} y_j^0\omega_j \ and\ y^1 = \sum_{j=1}^{\infty} y_j^1\omega_j \tag{19}$$

with $y_j^0 = (y^0, \omega_j)$ and $y_j^1 = (y^1, \omega_j)$.
Then:

$$\int_\Omega y^0\rho^1 dx = \sum_j (y^0, \omega_j)(\rho^1, \omega_j) \tag{20}$$

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$$\int_\Omega y^1\rho^0 dx = \sum_j (y^1,\omega_j)(\rho^0,\omega_j) \tag{21}$$

By the same, we have:

$$\rho(x,t) = \sum_j \rho_j(t)\omega_j(x) \tag{22}$$

where

$$\rho_j(t) = (\rho^0,\omega_j)cos(\lambda_j t) + \frac{1}{\lambda_j}(\rho^1,\omega_j)sin(\lambda_j t) \tag{23}$$

So:

$$\begin{aligned}&\frac{1}{2T}\int_0^T \left[\frac{\partial^2\rho(1,t)}{\partial x^2}\right]^2 dt\\ &= \frac{1}{2T}\int_0^T \left[\sum_{j,l}\rho_j(t).\rho_l(t)\frac{d^2\omega_j(1)}{dx^2}.\frac{d^2\omega_l(1)}{dx^2}\right] dt \qquad (24)\\ &= \frac{1}{2}\sum_{j,l}\frac{d^2\omega_j(1)}{dx^2}.\frac{d^2\omega_l(1)}{dx^2}\left[\frac{1}{T}\int_0^T \rho_j(t).\rho_l(t)dt\right]\end{aligned}$$

We obtain in developing:

$$\begin{aligned}&\frac{1}{T}\int_0^T \rho_j(t).\rho_l(t)dt\\ &= \frac{1}{T}\int_0^T \left\{(\rho^0,\omega_j)cos(\lambda_j t) + (\rho^1,\omega_j)\frac{sin(\lambda_j t}{\lambda_j}\right\} \qquad (25)\\ &\qquad \left\{(\rho^0,\omega_l)cos(\lambda_l t) + (\rho^1,\omega_l)\frac{sin(\lambda_l t}{\lambda_l}\right\} dt\end{aligned}$$

$$= \frac{1}{T}\int_0^T \left[(\rho^0,\omega_j)(\rho^0,\omega_l)cos(\lambda_j t)cos(\lambda_l t)\right] dt \tag{26}$$

$$+\frac{1}{T}\int_0^T \left[(\rho^0,\omega_j)(\rho^1,\omega_l)cos(\lambda_j t)\frac{sin(\lambda_l t}{\lambda_l}\right] dt \tag{27}$$

$$+\frac{1}{T}\int_0^T \left[(\rho^1,\omega_j)(\rho^0,\omega_l)cos(\lambda_l t)\frac{sin(\lambda_j t}{\lambda_j}\right] dt \tag{28}$$

$$+\frac{1}{T}\int_0^T \left[(\rho^1,\omega_j)(\rho^1,\omega_l)\frac{sin(\lambda_j t).sin(\lambda_l t)}{\lambda_j.\lambda_l}\right] dt \tag{29}$$

and a quite calculation gives, for$j \neq l$:

$$\begin{aligned}&\frac{1}{T}\int_0^T [sin(\lambda_j t).cos(\lambda_l t)]\, dt\\ &\leqslant \frac{1}{T}\left[\frac{1}{\mid \lambda_j+\lambda_l \mid} + \frac{1}{\mid \lambda_j-\lambda_l \mid}\right]\end{aligned} \tag{30}$$

$$\begin{aligned}&\frac{1}{T}\int_0^T [cos(\lambda_j t).cos(\lambda_l t)]\, dt\\ &\leqslant \frac{1}{2T}\left[\frac{1}{\mid \lambda_j+\lambda_l \mid} + \frac{1}{\mid \lambda_j-\lambda_l \mid}\right]\end{aligned} \tag{31}$$

$$\begin{aligned}&\frac{1}{T}\int_0^T [cos(\lambda_j t).sin(\lambda_l t)]\, dt\\ &\leqslant \frac{1}{T}\left[\frac{1}{\mid \lambda_j+\lambda_l \mid} + \frac{1}{\mid \lambda_l-\lambda_j \mid}\right]\end{aligned} \tag{32}$$

$$\begin{aligned}&\frac{1}{T}\int_0^T [sin(\lambda_j t).sin(\lambda_l t)]\, dt\\ &\leqslant \frac{1}{2T}\left[\frac{1}{\mid \lambda_j-\lambda_l \mid} + \frac{1}{\mid \lambda_j+\lambda_l \mid}\right]\end{aligned} \tag{33}$$

Then for $j \neq l$:

$$\frac{1}{T}\int_0^T \rho_j(t).\rho_l(t)dt \overset{T\to\infty}{\longrightarrow} 0 \tag{34}$$

and for $j = l, we\ obtain$:

$$\frac{1}{2} - \frac{1}{4\lambda_j T} \leqslant \frac{1}{T}\int_0^T sin^2(\lambda_j t)dt \leqslant \frac{1}{2} + \frac{1}{4\lambda_j T} \tag{35}$$

$$\frac{1}{2} - \frac{1}{4\lambda_j T} \leqslant \frac{1}{T}\int_0^T cos^2(\lambda_j t)dt \leqslant \frac{1}{2} + \frac{1}{4\lambda_j T} \tag{36}$$

and

$$\frac{1}{T}\int_0^T sin^2(\lambda_j t)dt \overset{T\to\infty}{\longrightarrow} \frac{1}{2} \tag{37}$$

$$\frac{1}{T}\int_0^T cos^2(\lambda_j t)dt \overset{T\to\infty}{\longrightarrow} \frac{1}{2} \tag{38}$$

$$\frac{1}{T}\int_0^T sin(\lambda_j t).cos(\lambda_l t)dt \overset{T\to\infty}{\longrightarrow} 0 \tag{39}$$

So

$$\begin{aligned}&\frac{1}{T}\int_0^T \rho_j(t).\rho_l(t)dt \overset{T\to\infty}{\longrightarrow}\\ &\left[\frac{1}{2}(\rho^0,\omega_j)^2 + \frac{1}{2\lambda_j^2}(\rho^1,\omega_j)^2\right]\end{aligned} \tag{40}$$

Finally:

$$\begin{aligned}&\frac{1}{T}\int_0^T \rho_j(t).\rho_l(t)dt \overset{T\to\infty}{\longrightarrow}\\ &\delta_j l\left[\frac{1}{2}(\rho^0,\omega_j)^2 + \frac{1}{2\lambda_j^2}(\rho^1,\omega_j)^2\right]\end{aligned} \tag{41}$$

where $\delta_j l = 1\ for\ j = l\ and\ \delta_j l = 0\ for\ j \neq l$
and then:

$$\begin{aligned}&\frac{1}{2T}\int_0^T \left[\frac{\partial^2\varphi(1,t)}{\partial x^2}\right]^2 dt \overset{T\to\infty}{\longrightarrow}\\ &\frac{1}{4}\sum_j \left[(\rho^0,\omega_j)^2 + \frac{1}{\lambda_j^2}(\rho^1,\omega_j)^2\right]\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2\end{aligned} \tag{42}$$

The problem (12) is then transformed to the minimization with respect to $\rho^0\ and\ \rho^1$ of:

$$\begin{aligned}&\frac{1}{4}\sum_j \left[(\rho^0,\omega_j)^2 + \frac{1}{\lambda_j^2}(\rho^1,\omega_j)^2\right]\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2\\ &\qquad - \int_\Omega (y^1\rho^0 - y^0\rho^1)dx\\ &= \sum_j \left[\frac{1}{4}(\rho^0,\omega_j)^2\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2 - y_j^1(\rho^0,\omega_j)\right]\\ &+\sum_j \left[\frac{1}{4\lambda_j^2}(\rho^1,\omega_j)^2\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2 + y_j^0(\rho^1,\omega_j)\right]\end{aligned} \tag{43}$$

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The first term of which does not depend on ρ^1 and the second does not depend on ρ^0. The minimization of (43) leads then to minimize:

$$\frac{1}{4}(\rho^0,\omega_j)^2\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2 - y_j^1(\rho^0,\omega_j) \quad (44)$$
$$with\ respect\ to\ \rho^0$$

and

$$\frac{1}{4\lambda_j^2}(\rho^1,\omega_j)^2\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2 + y_j^0(\rho^1,\omega_j) \quad (45)$$
$$with\ respect\ to\ \rho^1$$

The minimum is given by:

$$\frac{1}{2}(\rho^0,\omega_j)\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2 - y_j^1 = 0 \quad (46)$$

$$\frac{1}{2\lambda_j^2}(\rho^1,\omega_j)\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2 + y_j^0 = 0 \quad (47)$$

Finally, when $T \longrightarrow \infty$, we obtain:

$$\rho^0 = \sum_{j=1}^{\infty}\frac{2.(y^1,\omega_j).\omega_j}{\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2} \quad (48)$$

$$\rho^1 = -\sum_{j=1}^{\infty}\frac{2\lambda_j^2(y^0,\omega_j).\omega_j}{\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2} \quad (49)$$

Then we deduce to approximated one:

$$\varphi_T^0 = \frac{2}{T}\sum_{j=1}^{m}\frac{(y^1,\omega_j).\omega_j}{\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2} \quad (50)$$

$$\varphi_T^1 = \frac{-2}{T}\sum_{j=1}^{m}\frac{\lambda_j^2(y^0,\omega_j).\omega_j}{\left[\frac{d^2\omega_j(1)}{dx^2}\right]^2} \quad (51)$$

(50) and (51) can be used directly for computations.

V. Implementation

A. Algorithmic Aspect

Let us consider once again the system (2). According a method HUM of J.L LIONS, the control v* such as satisfied (2) is v*= $\frac{\partial^2\varphi(1,t)}{\partial x^2}$. This computation necessitates the computation of φ, solution of the system:

$$\begin{cases} \frac{\partial^2\varphi(x,t)}{\partial t^2} + \frac{\partial^4\varphi(x,t)}{\partial x^4} = 0 & \text{in Q} \\ \varphi(x,0) = \varphi_T^0 \ \ and \ \ \frac{\partial\varphi(x,0)}{\partial t} = \varphi_T^1 & \text{in } \Omega \\ \varphi = \frac{\partial\varphi}{\partial x} = 0 & \text{on } \Sigma \end{cases} \quad (52)$$

where φ_T^0 and φ_T^1 are the initial conditions given in (50) and (51). To judge the efficiency of the results, we consider a final state error:

$$\|\xi\|^2 = \|y(.,T)\|^2_{L^2(\Omega)} + \|\frac{\partial y(.,T)}{\partial t}\|^2_{L^2(\Omega)} \quad (53)$$

The following general schema is used for the numerical implementation:

Algorithm 1:

Step 1. Initial data $y^0 and\ y^1$.
Step 2. Choice of m.
Step 3. Computation of φ_T^0 and φ_T^1 using formulas (50)-(51).
Step 4. Integration of the system (52) and computation $\|v*\|^2$.
Step 5. Integration of the system (2) using the control v*.
Step 6. Computation of the final error $\|\xi\|^2$.
Return to Step 2.

The numerical method for integration of systems (2) and (52) is based on a symmetric finite difference schema [4]-[17].

As the solutions of these systems are functions of the independent variables x and t, we subdivide the x-t plane into sets of equal rectangles of sides dx=h, dt=k, by equally spaced grid lines, defined by $x_j = j.h, j\ integer$ and equally spaced grid lines, defined by $t_n = n.k, n\ integer$.
The (x_j, t_n) are called grid points, mesh points, or nodes [6]-[14].

A Von Neumann's stability condition of this explicitly schema is:
$V.N.S = \frac{k^2}{h^4} \leq \frac{1}{4}$.

We used 50 discretizations points in space, and in time we used an explicit symmetric finite difference schema with the V.N.S number equal to 0,25.

VI. Example and Discussion

We choose:
$y^0(x) = C.x^2$; $y^1(x) = 2.y^0(x)$; m=4; T=0,5.
C is a coefficient chosen by numerical considerations (so that y^0 and y^1 have reasonable magnitude).

Fig. 1 gives the numerical solution of the discretization system corresponding to (52). Fig. 2 gives the form of approximate control v* steering the system (2) to rest at time T. Fig. 3 gives the numerical solution of the discretization system corresponding to (2). Fig. 4 gives the variation of the cost function. Fig. 5 shows that the final state error is close to zero. This allows us to say that the explicit control v* steering the system(2) to rest at time T.

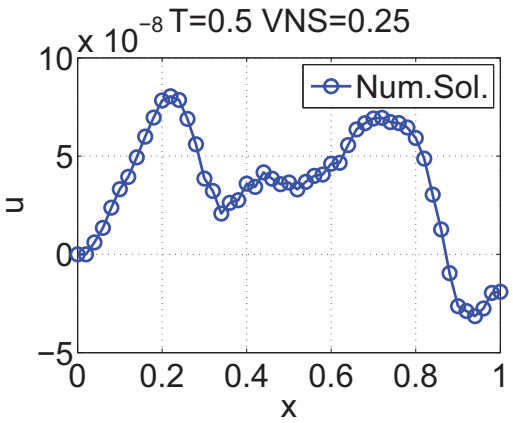


Fig. 1: Numerical solution of system (52)

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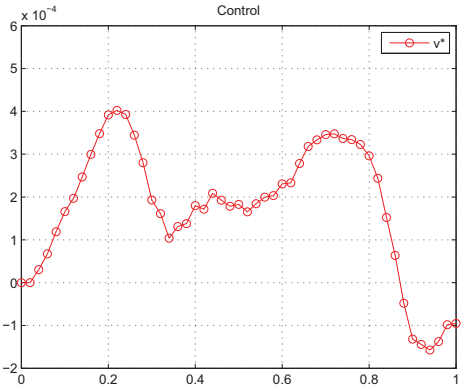


Fig. 2: Approximate control

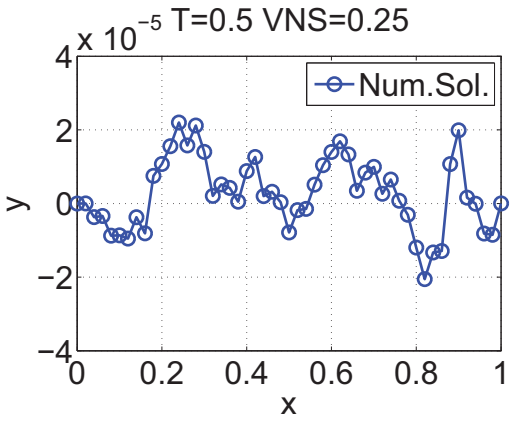


Fig. 3: Numerical solution of system (2)

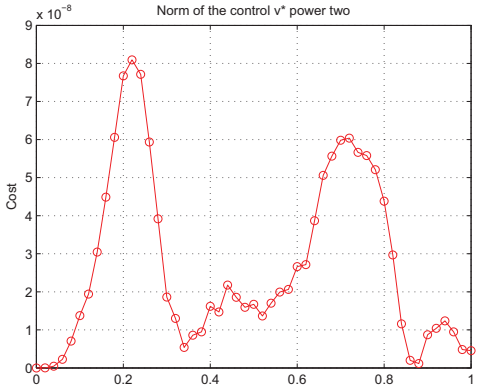


Fig. 4: Variation of the cost function

VII. Conclusion

Hilbert uniqueness method is implemented for the exact boundary controllability of the Euler-Bernoulli beam equation. The results found reflect the effectiveness of approximations methods. However, we think that we can improve the calcul of the final error by using metaheuristics in future and studying the case of dimension two for the same system.

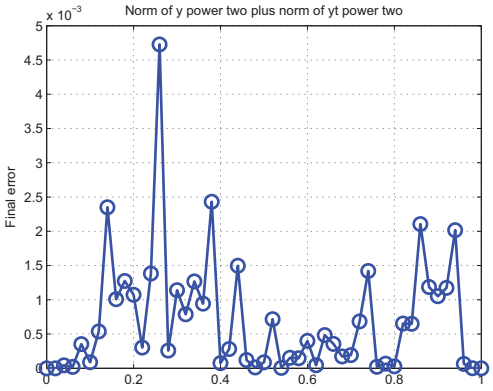


Fig. 5: Variation of the final error

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