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Propagation of Cos-Gaussian Beam in Photorefractive Crystal

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Abstract**—**A physical model for guiding the wave in photorefractive media is studied. Propagation of cos-Gaussian beam as the special cases of sinusoidal-Gaussian beams in photorefractive crystal is simulated numerically by the Crank-Nicolson method in one dimension. Results show that the beam profile deforms as the energy transfers from the center to the tails under propagation. This simulation approach is of significant interest for application in optical telecommunication. The results are presented graphically and discussed.

Keywords**—**Beam propagation, cos-Gaussian beam, Numerical simulation, Photorefractive crystal.

I. Introduction

THE self-focusing of beams of electromagnetic waves in nonlinear optical media has been an interest topic in theoretical and experimental works. In this field, a large number of theoretical investigations on self-focusing have been paid for Gaussian beams. It is well known that the Hermite–sinusoidal-Gaussian beams have been introduced as the exact solutions of the paraxial wave equation [1]. The cos-Gaussian and cosh-Gaussian beams are also regarded as the special cases of sinusoidal-Gaussian beams or Hermite-sinusoidal-Gaussian beams [2], [3]. The cos-Gaussian beam and its propagation characteristics have good potential applications such as optical telecommunication and improved pump lasers with flat top beam shape for more efficient optical lasers and amplifiers [3]. The dynamic properties of cos-Gaussian beams in the presence of Kerr nonlinearity are investigated analytically and numerically, using the nonlinear Schrodinger equation (NLS) [4]. On the other hand, optical spatial solitons that do not require waveguides is one of the interesting topics in modern optics. Spatial solitons are self-trapped optical beams that propagate and interact with each other as real particles. Soliton generation is the result of strong nonlinear interaction between the light and propagating medium. In fact, the medium has a nonlinear response that acts as a positive lens, and compensates the diffraction exactly. In a special case, photorefractive materials exhibit a good potential for many applications in optics and photonics, because of the ability of stable guiding of the wave in one and two dimensions and supporting the soliton generation at low optical power [5]. Additionally, photorefractive solitons have the wavelength dependence of material response. This property allows us to write the waveguides with low power beams to guide high power beams at other wavelengths in which the material is less sensitive. So, the photorefractive spatial solitons can be used as wave-guiding elements for optical telecommunication. One of this category is reported experimentally in 2009 [6], by using two written beams propagate under different angles overlap on the front face of the crystal in order to generate Y-coupler. In this regard, we show that this configuration can also be achieved by choosing a cos-Gaussian beam as incident beam under suitable set of material and optical parameters to generate soliton-like waveguide.

In this paper, the field distribution of cos–Gaussian beam, as a family of Gaussian beams, is introduced and simulation of the beam propagation in strontium barium niobate (SBN) photorefractive crystal is performed numerically. Self-focusing properties of cos-Gaussian laser beams in this medium are also investigated. Numerical simulation shows how the shapes of the beams varying during propagation and write the Y-coupler waveguide in the crystal.

The paper is organized as follows. In Section II, we present the theory. Simulation results and conclusion will be also dealt with in Sections III and IV, respectively.

II. Theory

In this section, we consider how the photorefractive nonlinearity modulates the refractive index of the medium and make a suitable condition for stable propagation of the beam in the crystal.

Let us consider an optical beam which propagates in SBN photorefractive crystal along the z axis, and emits to diffract only along the x direction of the crystal with its optical axis oriented along x coordinate. Moreover, let us assume that the optical beam is linearly polarized along x axis, which is coincident with the c axis of the crystal. The external electric field is applied in the same direction ($\vec{E} = E_0\hat{i}$) perpendicular to the propagation direction. As an optical beam propagates in a photorefractive crystal, the distribution of photo-excited charges induces a space-charge electric field $\hat{E}_{sc}$ that screens out the externally applied electric field which is necessary for modulates the refractive index of the medium via the Pokels linear electro-optic effect [7]:

$$\hat{n}^2 = n_0^2 - n_0^4 r_{33} E_{sc} \qquad (1)$$

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where $\hat{n}$ is the refractive index tensor induced by the propagation beam and n_0 is unperturbed index of refraction. Here r_{33} is effective linear electro optic coefficient. This change in the refractive index creates a lens like media which is necessary to converge the beam in front of the diffraction. On the other hand, the amplitude of the electric field $\vec{E}$ of the optical beam in terms of a slowly varying envelope $\vec{E} = A(x,z)\exp(ikn_0 z)\hat{i}$ satisfies the Helmholtz equation [8]:

$$2ikn_0\partial_z A + \partial_{xx}A + k^2(\hat{n}^2 - n_0^2)A = 0 \quad (2)$$

where $k = \frac{2\pi}{\lambda}$ is the propagation constant, and λ is the free space wavelength of the light wave employed that the medium sensitive to it.

Because typical propagation distances are of the order of millimeters and typical beam diameters are of the order of micrometers, it is more convenient to introduce transverse and longitudinal scaling length. The transverse scaling lengths x_0 is typically of the order of beam diameter, and the propagating scaling length is accordingly determined by the diffraction length $L_D = kn_0 x_0^2$, which is usually as long as the crystal length. Under these conditions and with respect to nonlinear electro-optic equation, the propagation equation is transformed into a dimensionless form:

$$\partial_z A - \frac{i}{2}\partial_{xx}A = -\frac{i}{2}\gamma E_{sc}A \quad (3)$$

where $\gamma = k^2 n_0^4 x_0^2 r_{33}$ is the photorefractive coupling constant.

The nonlinear response of the photorefractive medium which is described by the space-charge field can be obtained from the band-transport model of [9]. Under condition of strong applied external electric field E_0 and low temperature (theoretically assumed zero), the terms associated with the diffusion process can be neglected and the space-charge field with a good approximation is given by:

$$E_{sc} = \frac{1}{1+I}E_0 \quad (4)$$

where I is the power intensity of the optical beam which is normalized to dark intensity I_d, and assumed the optical beam intensity equal to zero at infinity. This assumption is corresponding to generation of bright optical beams.

It is instructive to study Gaussian beam families that do not match with the soliton beam but exhibit stable propagation under a suitable condition for soliton generation. Here, cos-Gaussian beam is introduced as a special case. For investigating the propagation of one-dimensional cos-Gaussian beam in photorefractive media, we assume the optical distribution of the field at the $z = 0$ plane which is expressed as:

$$A(x,0) = \cos(\frac{bx}{w_0})\exp(-\frac{x^2}{w_0^2}) \quad (5)$$

where b is a control parameter and w_0 is the spot size of the Gaussian beam. Fig. 1 shows the typical intensity distribution of this field. This profile shows a particular function as a Gaussian beam modulated with a cos function.

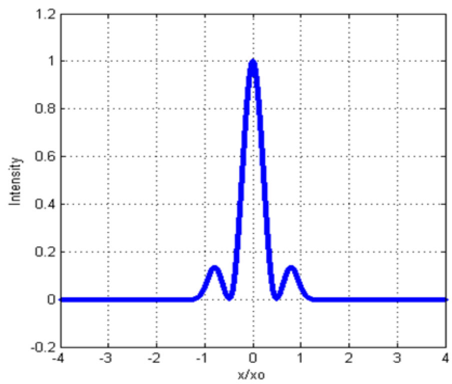


Fig. 1 Intensity of cos-Gaussian field with $b = 3$ and $w_0 = 12\mu m$

III. Numerical Method

The Crank-Nicolson numerical method is used to simulate beam propagation [8]. In this case we seek approximate $A_{m,k}$ to the original function $A(x,z)$ at a set of point x_k and z_t in the $x-z$ plane, where $x_k = x_0 + kh$ and $z_t = z_0 + mt$ for arbitrary mesh constants $0 \le k \le r$ and $m \ge 0$ with the initial points (x_0, z_0). The approximate derivatives in the propagation direction and the transverse Laplacian in this scheme are as:

$$\partial_z A = \frac{1}{t}\{A_{m+1} - A_m\}_k \quad (6)$$

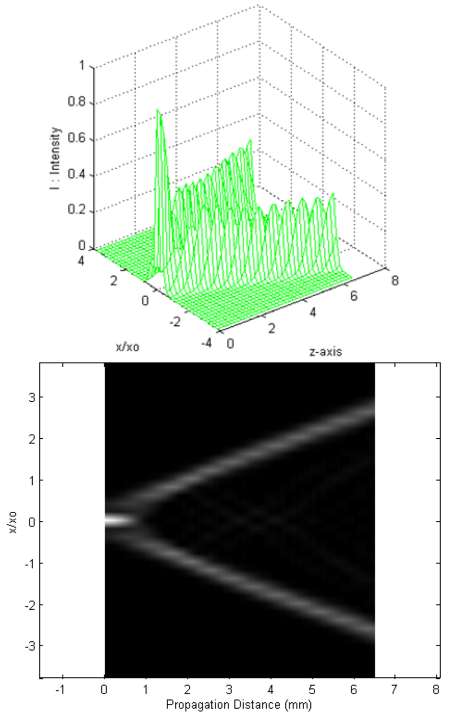
$$\partial_{xx}A = \frac{1}{2h^2}\{A_{k-1} - 2A_k + A_{k+1}\}_{m+1} + \frac{1}{2h^2}\{A_{k-1} - 2A_k + A_{k+1}\}_m \quad (7)$$

where the local truncation error is $O(h^2 + t^2)$.

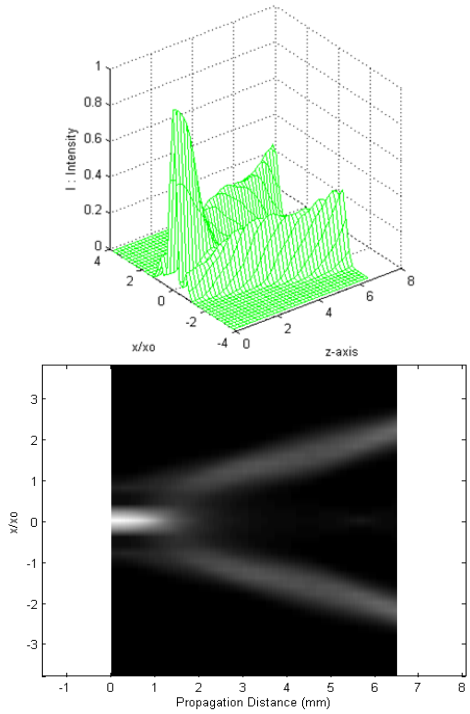
The numerical solution of the problem will be facilitated by the use of the matrix formulation and matrix iterative analysis. The iteration is controlled by the relative infinity norm:

$$R_\infty = \frac{\|A_{new} - A_{old}\|_\infty}{\|A_{new}\|_\infty} \quad (8)$$

and stops when it gets smaller than a tolerance level of $10^{-4} - 10^{-5}$.



(a)

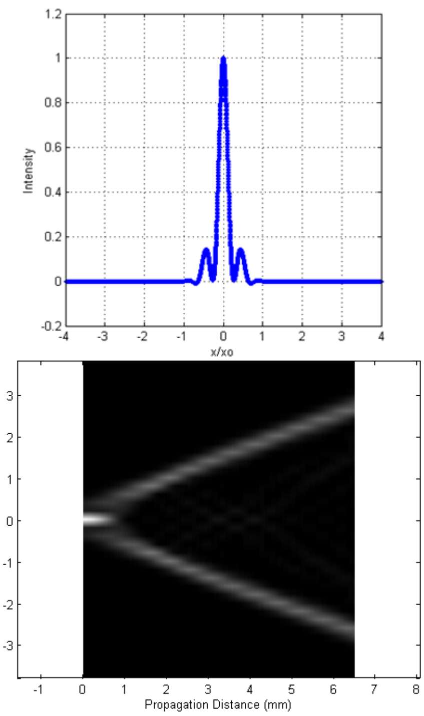


(b)

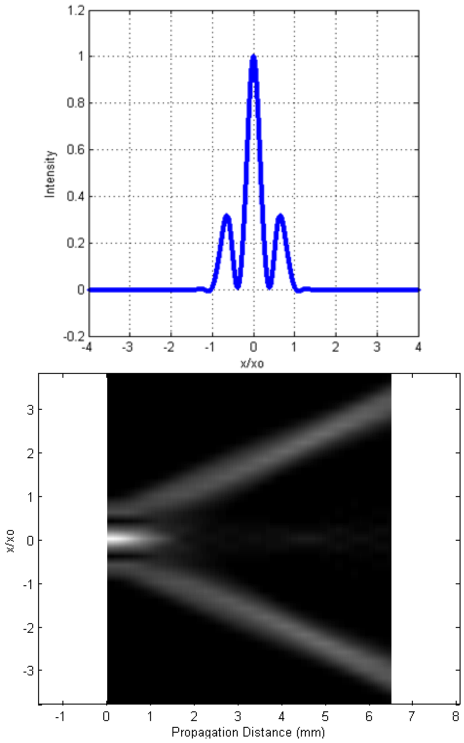
Fig. 2 Propagation of the cos-Gaussian beam in photorefractive crystal; intensity distribution and their contour for $b = 3$ and $w_0 = 12\mu m$: (a) $I_d = 10^{-2}$, (b) $I_d = 10^{-3}$

IV. Simulation

The Numerical method is performed to simulate cos-Gaussian beam propagation for the case of a SBN crystal with $n_0 = 2.35$, $r_{33} = 180$ Pm/V and 6.5 mm length. Applied external field is chosen as $E_0 = 4357$ V/cm, and the optical wavelength is $\lambda = 488$ nm. The incident distribution of the beam is chosen as the Cos-Gaussian beam which is shown in Fig. 1. Fig. 2 shows the propagation of the cos-Gaussian beam with $b = 3$ and $w_0 = 12\mu m$ in photorefractive SBN crystal for different dark intensity $I_d = 10^{-2}$ and $I_d = 10^{-3}$ in Figs. 2 (a) and (b) respectively. The dark intensity can be controlled experimentally by controlling the illumination of the crystal by a wide beam derived from a white light source. From Fig. 2 it is found that the beam width is broadening by decreasing the dark intensity. However, the cos-Gaussian beam profile is not a soliton solution of the wave equation as the set of parameters is approximately chosen which is necessary for soliton generation. This guarantees the stable propagation of the beam. It is interesting that the central part of the beam collapses and the energy transfers to the tails component and deforms the field distribution during propagation. As it is seen in Fig. 2, the Y-coupler is written in the crystal and can be used for guiding the other beam that does not induce the photorefractive nonlinearity. This configuration can be obtained by dividing an incoming light into two output beams or two write beams incident on the crystal under different angles with respect to x axis. These beams interact with each other during propagation and write two solitonic waveguide channels as Y-coupler [6], [10]. In Fig. 3, we demonstrate how the parameter b permits us to control the divergence of the beam. In fact, by increasing the b parameter the tails of the beam are more affected in energy transfer.



(a)



(b)

Fig. 3 Intensity profile and propagation of the cos-Gaussian beam in photorefractive crystal for $I_d = 10^{-3}$ and $w_0 = 12\mu m$: (a) $b = 3$, (b) $b = 4$

V. Conclusion

In this paper, we simulate the propagation of the cos-Gaussian beam as the special cases of Sinusoidal-Gaussian beams in photorefractive SBN crystal. The Crank-Nicolson numerical method is performed to simulate this propagation. By choosing the solitonic parameters and considering the controllable parameters, stable propagation of the beam is introduced. The beam diameter can be controlled by controlling the applied external electric field and the dark intensity, while the beam divergence can be controlled by controlling the parameter b of the cosine part function of the beam. Results show that the beam distribution reshapes and the energy transfers to the tails of the beam during propagation. This structure can be suitable for some applications such as waveguiding elements for optical telecommunication in particular Y-coupler as studied in [6]. The Y-coupler written in this case can be used for guiding the other beams that do not have any nonlinear sensitivity in the photorefractive crystal.

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