

Spatial Structure of First-Order Voronoi for the Future of Roundabout Cairo since 1867

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Abstract—The Haussmannization plan of Cairo in 1867 formed a regular network of roundabout spaces, though deteriorated at present. The method of identifying the spatial structure of roundabout Cairo for conservation matches the voronoi diagram with the space syntax through their geometrical property of spatial convexity. In this initiative, the primary convex hull of first-order voronoi adopts the integral and control measurements of space syntax on Cairo's roundabout generators. The functional essence of royal palaces optimizes the roundabout structure in terms of spatial measurements and the symbolic voronoi projection of 'Tahrir Roundabout' over the Giza Nile and Pyramids. Some roundabouts of major public and commercial landmarks surround the pole of 'Ezbekia Garden' with a higher control than integral measurements, which filter the new spatial structure from the adjacent traditional town. Nevertheless, the least integral and control measures correspond to the voronoi contents of pollutant workshops and the plateau of old Cairo Citadel with the visual compensation of new royal landmarks on top. Meanwhile, the extended suburbs of infinite voronoi polygons arrange high control generators of chateaux housing in 'garden city' environs. The point pattern of roundabouts determines the geometrical characteristics of voronoi polygons. The measured lengths of voronoi edges alternate between the zoned short range at the new poles of Cairo and the distributed structure of longer range. Nevertheless, the shortest range of generator-vertex geometry concentrates at 'Ezbekia Garden' where the crossways of vast Cairo intersect, which maximizes the variety of choice at different spatial resolutions. However, the symbolic 'Hippodrome' which is the largest public landmark forms exclusive geometrical measurements, while structuring a most integrative roundabout to parallel the royal syntax. Overview of the symbolic convex hull of voronoi with space syntax interconnects Parisian Cairo with the spatial chronology of scattered monuments to conceive one universal Cairo structure. Accordingly, the approached methodology of 'voronoi-syntax' prospects the future conservation of roundabout Cairo at the inferred city-level concept.

Keywords—Roundabout Cairo, first-order Voronoi, space syntax, spatial structure.

I. INTRODUCTION

THE 19th century westernization addresses a cross-cultural urban heritage. The French expedition in 1798 influenced the cultural change of Egypt, with the prominent plan in 1867 of Haussmann for the new European Quarter in Cairo on his Parisian precedence of roundabouts joining radial boulevards. The plan composed a regular network of 15-roundabouts, which contrasted with the adjacent irregular traditional town. The building stock of neo-classical styles used to function for commercial, residential, and cultural activities. Haussmann's foundation plan became the modern downtown of Cairo, while

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influencing the latter expansions up to total 153-roundabouts of more boulevards towards satellite suburbs. However, the sudden nationalization policies of Egypt in 1952 had caused cultural change of terminated roundabout development with rundown condition all over the European Quarter at present Cairo e.g. [3]. Meanwhile, [8] and [9] document the cultural values of special architecture with the socioeconomic structure of downtown Cairo for conservation. Besides, the Egyptian urban planning authority has collaborated with Cairo University and Cairo Governorate on the project of conserving downtown Cairo, which details the special merits of the historical quarter for visual enhancements [12]. In another front, this study qualifies the urban heritage of downtown Cairo from the conceptual structure of space. The topic has gained a memorial dimension through Haussmann's core roundabout of 'Tahrir' that focused the nationwide up-rise, with impacts on the future vision of development [11]. The interdisciplinary method of prospective spatial structure attempts to combine the separate morphologies of voronoi diagram and space syntax [6], [7]. Despite the differences in configuration, their common theme of spatial partitioning superimposes a wide range of analytical layers with numerous spatial measurements. Specifically, the geometry of convex space is a common vocabulary among both literatures awaiting observation. However, the convex space by itself is a standalone geometrical property, which has been incorporated in the spatial processes of the two methodologies. According to the property of spatial convexity, the puzzling question of; 'which convex space matches the other spatial structure' is the key issue to think of. Supposing the pivotal spatial convexity between the two methods enables their joint spatial structure of unprecedented hybrid conception. Through this interfacing framework of convex space, the interdisciplinary spatial structure shall have rethinking, reanalyzing and redesigning in universal scopes. In this respect, the following section constructs the convex hull of first-order voronoi for Cairo's roundabout measurements of spatial integration and control. The third section details the functional convex hull of the first-order voronoi diagram, while the further fourth section explores the roundabout geometrical properties of point pattern distribution. The conclusive fifth section correlates the measurements and functions of the convex hull, which defines the 'voronoi-syntax' of roundabout Cairo towards the future strategy of conservation.

II. SPATIAL MEASUREMENTS OF THE CONVEX HULL

The concept of voronoi diagram generates points in the middle of convex spaces to span the Euclidean plane. The

convex boundary line bisects the imaginary line joining between any two adjacent generators. Complete voronoi edges configure the polygonal convex hull of first-order or higher, with the outer cells open to infinity. Generating the first-order voronoi of Cairo inserts Haussmann's total 15-points of roundabouts at the geometric centers of the subdivided convex hull, Fig. 1. The resulted layout examines the syntactic measures of spatial integration and control, with the functional

contents of voronoi polygons. The measure of spatial integration counts the stepping adjacency from each polygon to all others of the layout, with the lower values of relative asymmetry 'RA' indicating the more integration of the global system. Meanwhile, the local system of spatial control counts the adjacency ratio of each polygon in relationship to its neighbors, where the higher value indicates the more control over the surrounding polygons, Table I.

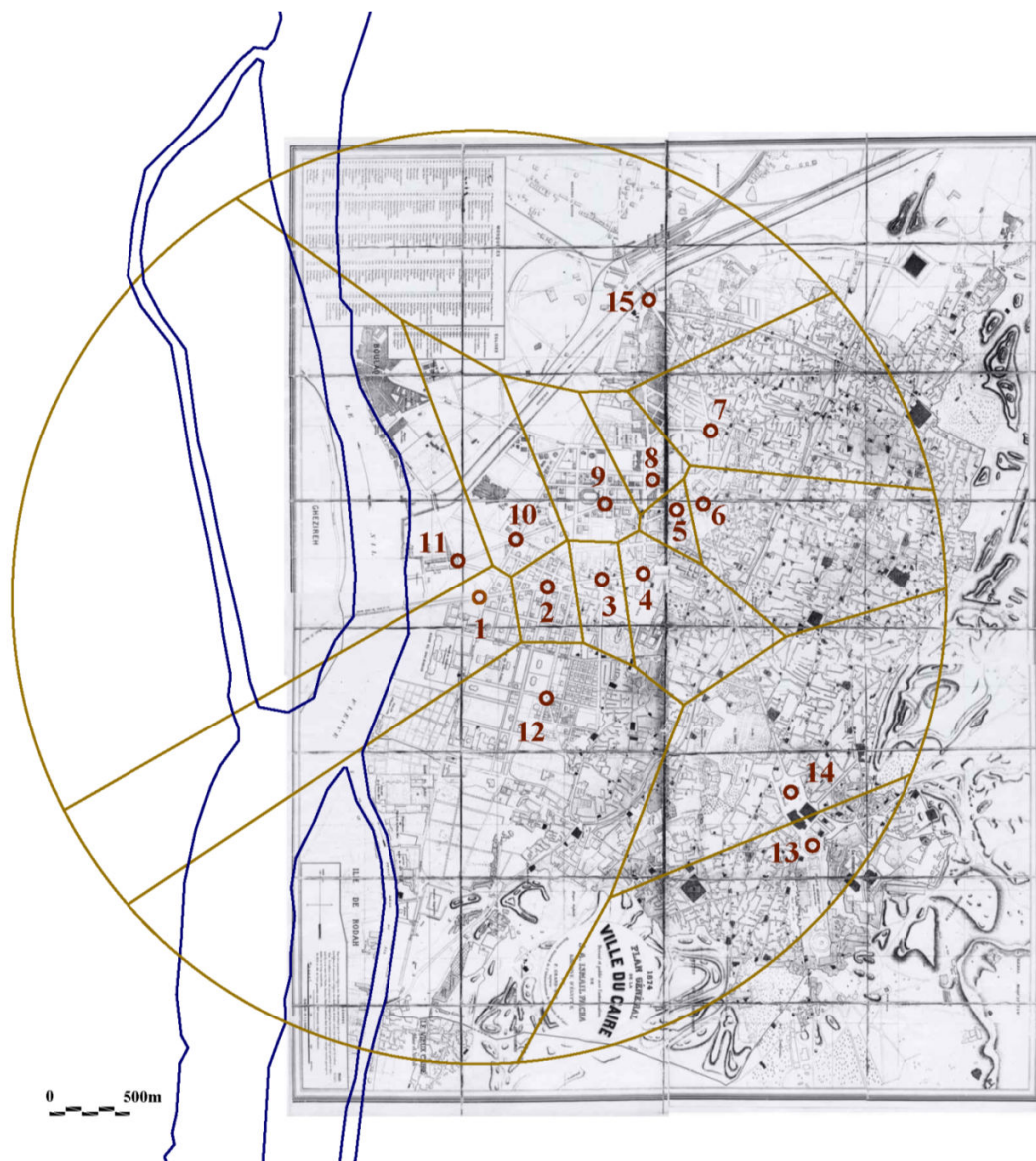


Fig. 1 Overlaid first-order Voronoi on Cairo since 1867

The adjacency matrix of roundabout generators measures an average value of '0.14' relative asymmetry (RA), which indicates a highly integrated overall scheme. Nevertheless, the integral measure of each roundabout observes some classification in range, Fig. 2. Less integrated generators lie on the peripheral convex hull. The tri-pole generators '13, 11 & 7' measure the least integral spaces, while surrounded by polygonal neighbors of an average integral range. The least integrity of 'generator-13' belongs to the elevated monument

of 'Cairo Citadel' with the modern royal complex on top. The closest 'generator-14' of a higher integration fronts the other monument of 'Sultan Hasan Mosque'. In this regard, the twin roundabouts differ according to the accessibility of less integrated high-ground versus the more integral flat topography. The other side of 'genertaor-11' disintegrates the global voronoi system, which logically accounts for the bulk of workshops in this peripheral polygon. The opposite pole of clustered roundabouts at the 'Ezbekia Garden' segregates only

'generator-7' at the border between old and new Cairo. Thus, the spatial logic of the least integral structure determines the filtration process at the interface of different town planning.

The most integrative generators '9 & 4' of less than '0.1' RA values are set halfway along the zigzagging roundabouts between the two opposite poles of 'Ezbekia Garden' and the Nile edge. The symbolic structure of 'generator-4' at 'Abdean Palace' indicates the most integrative royal spanning over the infinite convex hull, with the realized polar connection of triangular boulevards. On the other side, the 'generator-9' of top integrity corresponds to the 'Hippodrome' of largest public facility. Therefore, the integration of the global spatial system determines dual symbols of the public at large with the royal dominance. The average integrity of moderate range divides into two clusters of generators '2, 12, 3, 8 & 10' and '5, 6, 14, 15 & 1' in sequence. Both clusters accumulate at the respective poles of the Nile edge and the 'Ezbekia Garden' in separate. Nevertheless, only 'generator-1' of 'Tahrir' exchange the cluster with 'generator-8' of 'L'Opera' landmark, with their nearest integral measure to the global average of RA values. In this respect, both generators represent the layout integral structure by virtue of the RA measure, with emphasized swap of analogical grouping. Meanwhile, the clustered roundabouts of 'Ezbekia Garden' radiate boulevards in direct links to generators '14, 15 & 4' of respective 'Sultan Hasan Mosque, Cairo Station & Abdean Palace' landmarks to form poles of spatial nodes. Accordingly, the common sharing of 'Ezbekia Garden' in the integral structure of roundabout Cairo adds to its role of filtration from the old town. The renowned French engineer, De Schamps, designed the landscape of 'Ezbekia Garden' on the precedence of the 'Tivoli Gardens' in Paris [1].

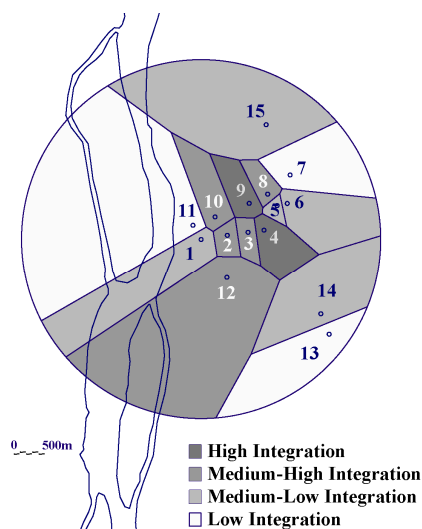


Fig. 2 Integral degree of roundabout Cairo

The spatial measure of control determines the amount of distributiveness with respect to the adjacent generators. The control measure of roundabout Cairo observes high values of the inner polygons and decreases towards the peripheral convex hull, Fig. 3. In spite of this observation, the outer polygon of 'generator-12' qualifies as the most controlling

space of the layout with the highest value of '1.61'. This exceptional generator accommodates a cluster of royal palaces and prestigious chateaux housing in spacious botanical gardens along the Nile promenade, which also forms an adjacency with royal palaces from all sides. Thus, the control measure emphasizes the symbolic royal dominance with polygonal coverage to infinity. The further classification of the control values divides into three ranges of 'over-one, around-one and below-one' measures. The 'over-one' range forms the set of generators '12, 9, 4, 15 & 6' in sequence. The royal 'generator-4' of 'Abdean Palace' extends the symbolic global integrity with a high spatial control, in addition to its extensive adjacency to the three top-tier controlling generators. The similar emphasis of 'generator-9' adds a high control measure to the extreme public integrity of the 'Hippodrome'. The other 'generator-15' of 'Cairo Station' represents a high control point for the public, while the interface of 'generator-6' forms a high control space to maintain the filtration with old Cairo. Thus, the highest range of the control measure extends the logic of spatial structure with multiple symbolic conceptions.

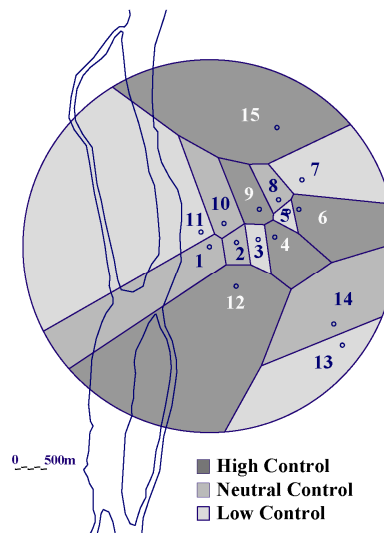


Fig. 3 Control degree of roundabout Cairo

The average control measure forms a set of generators '10, 8, 14, 2 & 1' in sequence, with three of them in spatial adjacency. The concentrated structure of neutral control at the 'Tahrir' pole contrasts with the higher control of the 'Ezbekia Garden' pole. The combined 'Tahrir' with 'generator-8' of 'L'Opera' in the same control range sustains their integral analogy, which extends to the third pole of 'Sultan Hasan Mosque'. Meanwhile, the least control measures of generators '5, 3, 11, 7 & 13' scatter all over the convex hull. Although set apart, the three generators of minimal control correlate with the least integral spaces. Thus, the three voronoi polygons of 'Cairo Citadel, workshops and Ezbekia Garden' exclude the space syntax of integral and control measures in logical structure with the respective topographic, bulky and gardened characteristics. Nevertheless, generators '5 & 3' buffer the high control of 'generator-4', which emphasizes their contrast to highlight the royal structure of 'Abdean Palace'.

TABLE I
 SPATIAL MEASUREMENTS OF ROUNDABOUT CAIRO

Gen.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	RA	Control
1		1	2	2	3	3	3	3	2	1	1	1	2	2	2	0.154	0.9
2	1		1	2	2	3	3	2	1	1	2	1	2	2	2	0.121	1.009
3	2	1		1	2	2	3	2	1	2	3	1	2	2	2	0.132	0.676
4	2	2	1		1	1	2	2	1	2	3	1	2	1	2	0.099	1.259
5	3	2	2	1		1	2	1	1	2	3	2	3	2	2	0.143	0.709
6	3	3	2	1	1		1	1	2	3	3	2	2	1	2	0.143	1.2
7	3	3	3	2	2	1		1	2	2	2	3	3	2	1	0.176	0.6
8	3	2	2	2	1	1	1		1	2	2	3	3	2	1	0.132	1.126
9	2	1	1	1	1	2	2	1		1	2	2	3	2	1	0.088	1.5
10	1	1	2	2	2	3	2	2	1		1	2	3	3	1	0.132	1.126
11	1	2	3	3	3	3	2	2	2	1		2	3	3	1	0.187	0.65
12	1	1	1	1	2	2	3	3	2	2	2		1	1	3	0.121	1.617
13	2	2	2	2	3	2	3	3	3	3	3	1		1	4	0.219	0.417
14	2	2	2	1	2	1	2	2	2	3	3	1	1		3	0.143	1.033
15	2	2	2	2	2	2	1	1	1	1	1	3	4	3		0.143	1.209

III. FUNCTION OF THE CONVEX HULL

The overlay of the first-order voronoi convex hull on the realized plan of Cairo clarifies perceptive issues of functional context, Fig. 4. The unbound projection of the convex hull recognizes the unique context of 'Tahrir' polygon that encloses the plateau of Giza Pyramids, Fig. 5. The same polygon projects over the clearest Nile between the obstructive islands. The distinction of the Pyramids and the Nile in one cognitive polygon of 'Tahrir' symbolizes Egypt in a universal context. In addition, the opposite 'generator-6' of 'Ezbekia Garden' projects the polygonal context up to the heritage zone of the 'Azhar' landmark with a direct link through 'Boulevard Neuve' access. Further observation of 'generator-6' forms an edge in the same direction of 'Boulevard Sultan Hasan' that bisects traditional Cairo up to the roundabouts of 'Sultan Hasan & Cairo Citadel' symbolic monuments. Therefore, the conceptual roundabout structure determines the contextual symbolism of the voronoi convex hull through the cognitive projection over the eternal symbols with all times spatial extents of roundabout Cairo.

The legislative framework had set the policy of real-estate development for the new expansion of Cairo. The early modernization policy had started since 1831 through the law of real-estate, which allowed the expropriation of deteriorated lands and buildings for governmental redevelopment and stated:

"The Royal Decree approves the agent of engineers and supervisors to investigate the condition of lands and buildings in town, such that if any is found to be below standard the owner must redevelop the property or evacuate to be redeveloped as a governmental property." [1, p.14].

The law enforcement transformed the lands and buildings of worst condition into hygienic gardens and streets with upgraded buildings. Meanwhile, the royalty had added incentives to the European settlers in Egypt with the privilege of: *"freedom in acquiring properties and immunity from taxation, arrest and the local judicial system."* [4, p.12]. Moreover, in 1869 the royalty granted lands of the new

Hausmannization plan of Cairo to any national on condition of developing stylish buildings with more gardens added to the existing ones [5]. The ratio of built-up area of the new European urbanism in 1872 had reached 13% representing '1,071' permissions issued for construction [10], with improved public infrastructures such as the established water and gas companies [5]. The whole remaining percentage was designated as gardens along Haussmann's executed roundabouts with radial boulevards [1]. The new boulevards were regulated a 12m minimum width [5], which supposes the conversion from the indigenous 'Dira' unit of measurement of approximately '0.6m' or '20-Dira' [3].

The enforcements of policies were reflected on the detailed resolution of the building characteristics. The public landmark buildings of the 'Post Office, Mixed Tribunal, Fire Brigade and L'Opera' clustered at the joint polygons of generators '5, 6, 7 & 8' where the majority of modern socioeconomic establishments had been structured. However, each building belongs to a certain polygon of the voronoi subdivision. The 'Mixed Tribunal' occupies the polygonal corner of 'generator-6' with the roundabout frontage on 'Ataba Plaza'. This special landmark formed a symbol at the crossways of enlarged Cairo with the vista of judicial power. At the back locates the other 'L'Opera' landmark building of 'generator-8'. Although set back-to-back, the splitting polygonal edge sets 'L'Opera, Hotel Shepherds & Hotel Grand Continental' with a major part of 'Ezbekia Garden' together in one polygon of 'generator-8', which symbolizes the cultural hub of new roundabout Cairo. Similar functions are added to the adjacent polygon of 'generator-7' with added missionary buildings and European consulates against the indigenous town with cultural filtration.

The inner polygon of 'generator-5' concentrates commercial use, which extends to the surrounding polygons of generators '4, 6 & 7'. In addition, this polygon has the two public services of the 'Fire Brigade' and the 'Post Office' at the corner between 'L'Opera' and the 'Mixed Tribunal' landmarks. Their concise location supposes the shortest distance of ideal function to all catchment areas. However, the new banking establishments are shifted inside the polygon of 'generator-4', which adds an economic symbol to the 'Abdean Palace'. Further south, the polygon of 'generator-14' clearly delineates 'Helmeya Palace' with its surrounding gardens at the heart of old Cairo. The adjacency edge of generators '13 & 14' bisects the two similar landmark mosques of the old 'Sultan Hasan' and the new 'Refā' one. The same edge mirrors the 'Helmaya Palace' with the impressive royal complex of 'Gawhara Palace' and 'M. Ali Mosque' on top of the old 'Cairo Citadel'. Thus, generators '13 & 14' enrich the cognitive structure of roundabout Cairo with symbolic revivalism along the edge of voronoi adjacency.

The opposite infinite polygon of 'generator-15' corresponds to the 'Choubra' suburb of 'garden city' concept, while locating the roundabout in front of the 'Cairo Station' landmark. More residential quarters designate the polygons of generators '9, 10, 1, 2 & 12' in the form of detached 'Chateaux' housing with gardens. The community 'Hippodrome' of 'generators-9' provides an easy access for the visitors with spatial integrity. Nevertheless, the royal quarter stretched along the Nile front

in juxtaposed palaces up to the 'Roda Island' suburb. The open polygon of 'Tahrir' roundabout sets 'Dubara Palace' next to the other 'Kasr El Nile Palace' of 'generator-11' apart from 'Beaulac' quarter where the royal workshops, press publishing houses, the first Egyptian Museum, the Water Company and

others further locate. The 'generator-12' adds more palaces with 'Royal School' and the royal hospital of 'Kasr El Aini' towards the 'Roda Island', which defines the royal community. In this regard, the voronoi polygons of roundabout Cairo articulate the social structure at different levels of spatial resolutions.

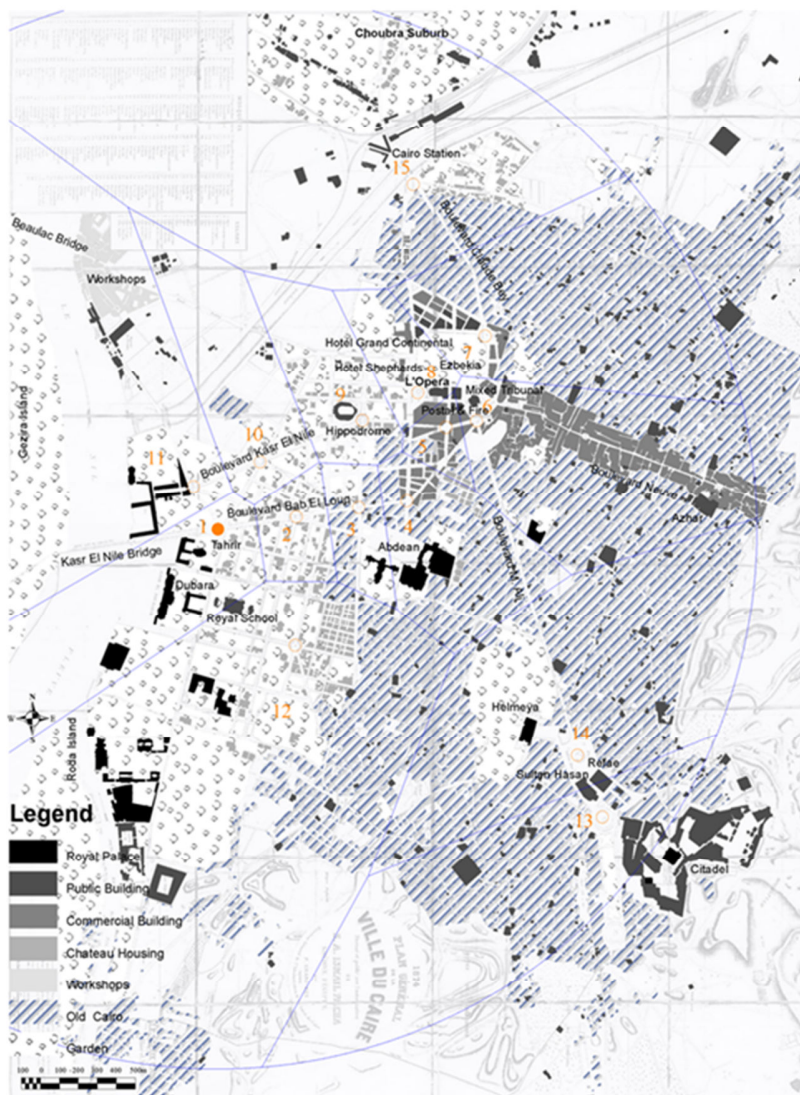


Fig. 4 Functional characteristics of the Voronoi polygons

The architectural characteristics of voronoi polygons develop massive commercial blocks around 'Ezbekia Garden' with Neo-Renaissance façades of 27m (or 45-Dira) maximum height [8]. Nevertheless, the eclectic 'Okelle' commercial-residential block of hybrid structure between the Italian 'Galleria' and the indigenous 'Wekala' spread in polygons '6 & 7' [2], which softens the filtered cognitive structure from traditional to modern Cairo. Meanwhile, the commercial arcades stretched along the two projecting boulevards of 'S. Hasan & Claude Bey' that bisect old Cairo with a maximum 15m (25-Dira) building height [8]. Along the Nile bank, developed classical style palaces with public gardens that had

been planned since 1874 by the French engineer, N. Brocard, to form a riverside promenade [1]. From 'Tahrir' roundabout to 'Ezbekia Garden' developed Baroque style chateaux housing with gardens along the two major boulevards of 'Kasr El Nile' and 'Bab El Louq' of radial roundabout structure. The two boulevards converge at 'Tahrir' roundabout with the 'Kasr El Nile' further bridging towards the Giza suburbs, Fig. 6. Overview of the cognitive structure determines two nodes of the centralized 'Ezbekia Garden' between old and new Cairo, while projecting the 'Tahrir' roundabout towards the legendary Giza Pyramids with the picturesque landscape along the Nile.

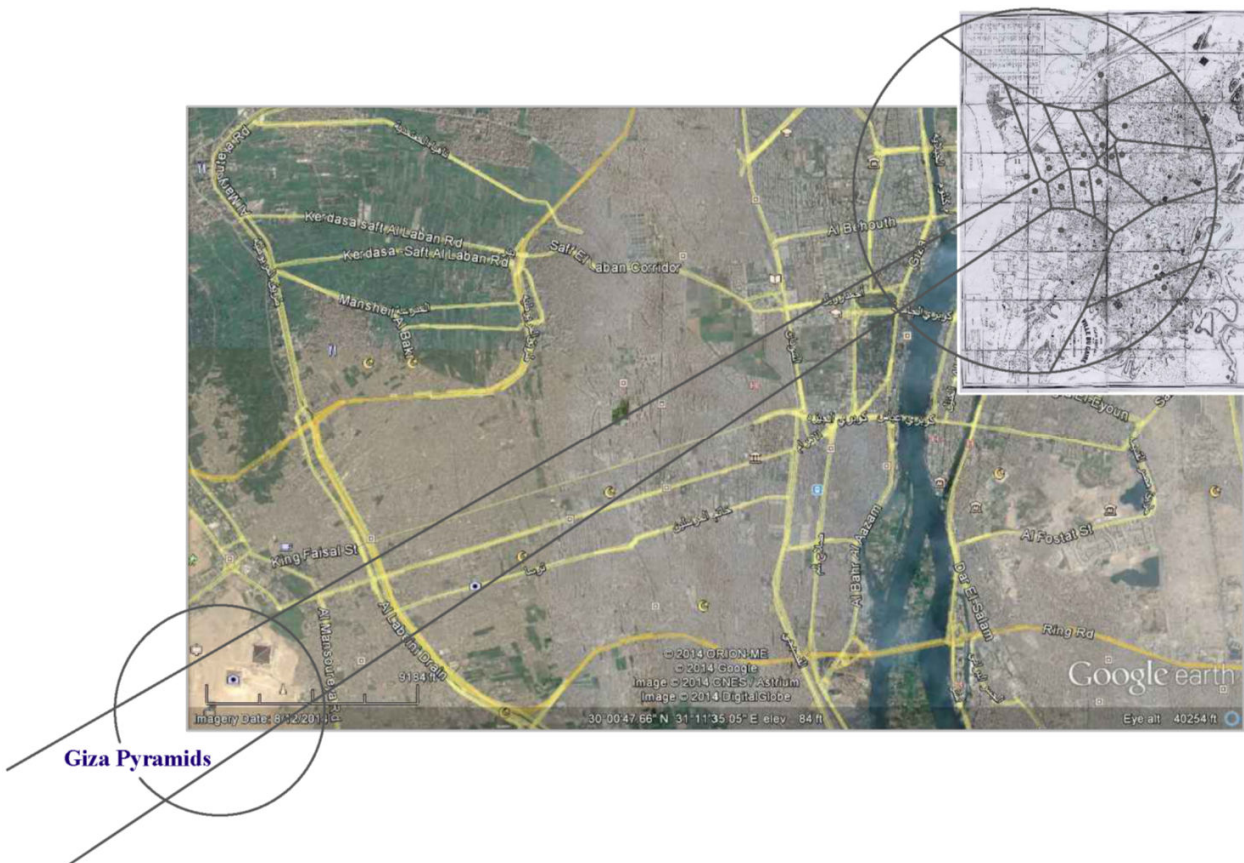


Fig. 5 Offset of first-order Voronoi on the Giza Pyramids

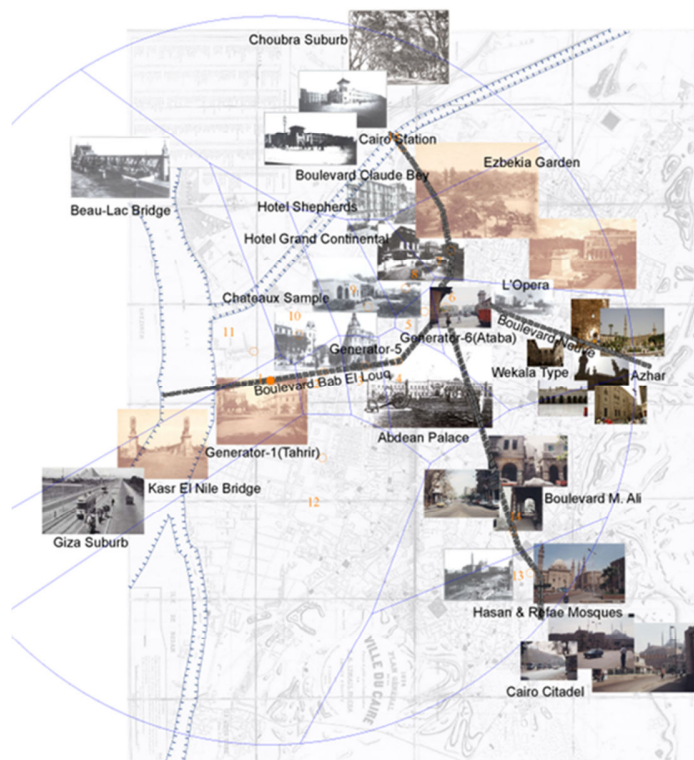


Fig. 6 Visual characteristics of the Voronoi polygon

IV. GEOMETRICS OF THE CONVEX HULL

The geometrical characteristics of the voronoi polygons explore the 'measurable space' of random point process. The few generators, which together with the edge-effect of unbound polygons of half-probability, disqualify the 'Poisson' point distribution of Hausmann's roundabout random process. Alternatively, the random point process observes parametric criteria of cluster distribution, which simulates the measurable space of voronoi polygons. The parametric fiber process of edge length, degree, generator-vertex distance, polygonal angles, in addition to the area and perimeter with their ratio, represent the geometrical analyses of the generated point distribution, Table II and Figs. 7-9. Each parameter is classified into bound, unbound, and overall measures of average values. In this respect, the polygons are compared in groups and in relationship to each other as well.

The lengths of edges are sorted per polygon in an ascending order to identify their structure. In addition, the available finite edges of the unbound polygons are listed in order. The only polygon without any finite edge corresponds to 'generator-13' of elevated topography where the 'Citadel' is located. This completes the exceptional structure of the 'Citadel' through the undefined voronoi edges with the least integral and control measures of space syntax, but maintains the visual overwhelm of high ground. The two equal groups of bound and unbound polygons divide their shortest edges in wide differences, with extreme measurements. The shared edges of adjacency between bound and unbound polygons observe the shortest lengths of either of them, but not both in common. Meanwhile, the order of shortest edges observes bound polygons of shorter lengths more than the unbound polygons. The average length of '100m' of the shortest edges are found in generators '5, 6, 8 & 4', which clusters bound polygons around the 'Ezbekia Garden' pole. The shortest edge of 'Tahrir' measures a length of '94.33m', which clusters with the opposite 'Ezbekia' pole. The longer lengths of the shortest edges spread at the outer convex hull in generators '12, 7, 14 & 11' without adjacency. The second range of shortest edges change strategy with a closer variance among the various lengths. This range forms two clusters of '200-250m' in generators '8, 5 & 3' at the 'Ezbekia Garden' pole, and followed by the longer lengths of '250-300m' in generators '2, 12, 4, 1 & 10' at the opposite cluster of 'Tahrir' pole. The observation of polygons '3 & 4' of 'Abdean Palace' overlap with the two clusters, while 'Tahrir' of '293.21m' length is the most approximate measure to the overall average length of '320.24m'. The third range of edge lengths shifts the location towards the central clustering of polygons '8, 12, 2, 4 & 5' in sequence, with a narrow variance of '250-320m' edges. The longer edges of extended lengths over '500m' re-cluster the polygons into pairs of generators '10-15, 4-5, 3-6 & 2-12' with expanded coverage, except the 'generator-9' of an individual cluster. The pairing phenomenon extends to the next fourth rank, but with a wider range of lengths for generators '9-2, 5-8, 4-3 & 10-12' without any shared edge. Further lengths of edges scatter over the bound polygons without any adjacency with the infinite polygons. Overview of the parametric length of voronoi edges structures

the shortest lengths in clustered zones of 'Ezbekia Garden' and 'Tahrir' poles with shared edges between bound and unbound polygons, while cluster into pairs of longer lengths towards the central zone of bound polygons, but changes to unshared longest edges all over the bound polygons only. Among all of the generators, only 'generator-9' reserves the shortest length in sequence, which enforces the integrative structure of the 'Hippodrome' within a centralized polygon of relatively shortest edges. Meanwhile, the unique structure of 'Tahrir' forms an interactive clustering with the shortest edges of the 'Ezbekia' pole and the longer edges of the 'Tahrir' pole as well.

The second parameter sorts the 'generator-vertex' distance in an ascending order. The shortest distance from each of the roundabout generators to the voronoi vertices observes a clear structure of polygons. The shortest distances subdivides into three ranges of generators '5, 6 & 8', followed by generators '1, 10, 11, 9, 2, 4, 3 & 7', and finally generators '12, 15, 14 & 13' in sequence. The three ranges correspond to the respective poles of the first 'Ezbekia Garden', the second 'Tahrir' and the third 'Citadel' structure. The first two clusters, nevertheless, have a minimal variance of '140-180m' in comparison to the third cluster of '270-900m' wider variance. The shallowest 'Ezbekia Garden' maximizes the variety of three-polygonal choice at the crossways of enlarged Cairo. Nevertheless, the generator of 'Tahrir' has the closest measure to 'Ezbekia' at the border between the two ranges. In addition 'generator-7' cluster with the 'Tahrir' pole instead of its location at the 'Ezbekia' pole. Meanwhile, the northern 'generator-15' of 'Cairo Station' clusters with the southern 'Citadel' pole. Accordingly, the shortest distance of generator-vertex forms an interactive structure between each two opposite poles instead of being clustered in separate zones. The second measure combines 'Tahrir' with 'Ezbekia' in one cluster of generators '5, 8, 1 & 10'. This cluster has the only infinite polygon of 'Tahrir' generator. However, the following range divides between the two poles into generators '9, 6 & 4' of 'Ezbekia' and generators '2 & 3' of 'Tahrir' of all bound polygons. The further range of infinite polygons clusters into generators '12, 7, 15, 14 & 11' with extreme measure over one-kilometer distance. The sequence of this cluster enforces the interactive structure of Cairo poles. Thus, the generator-vertex structure changes from filtered zones of clustering the shortest distance to longer interactive poles of mixed clusters. The third measure generator-vertex of fewer generators subdivides in sequence with the clear clustering of generators '8, 9 & 5' of 'Ezbekia' versus generators '2 & 10' of 'Tahrir', while generators '4, 6, 1 & 3' stretch from the central zone of 'Abdean Palace' towards generators '12, 15 & 14' of opposite poles. Therefore, the third range varies the structure from zoned to interactive clusters in continuation to the previous two ranges. The fourth measure, however, changes in extreme variance below or above the total average of the convex hull with a scattered pattern of distribution. This cluster of dismantled zones observes the increased variance from bound to unbound polygons, which separates through the sequence of generators '5 & 2', '8, 3 & 4', '12 & 6' and '10 & 15' towards the infinite polygons. However, the symbolic measure of

'generator-9' of the 'Hippodrome' continues to form an individual cluster of shortest distances, which adds to its most integrative space syntax. Meanwhile, the first pairs of '5 & 2' generators maintain the analogy of 'Tharir & Ezbekia' structure. The gradual increase of variance observes the pairs of generators '12 & 6' with the latter changing from an unbound polygon at the detailed resolution to a bound

structure at the further zoomed-out level. The further fifth rank dissolves the clustering structure with a large variance of segregated pattern. The sorted sequence maintains 'generator-9' of the shortest structure, and followed by the two generators of '2 & 8' that continue the polar analogy of 'Tahrir & Ezbekia', while the extreme longest measures concern the two generators '12 & 10' of adjacency with the 'Tahrir' pole.

TABLE II
 GEOMETRICAL CHARACTERISTICS OF THE CONVEX HULL

G	Area	Perim.	A/P	E	E1	E2	E3	E4	E5	E6	E7	Av.E	D1	D2
1	∞	∞		4	94.33	293.21	∞	∞				∞	154.48	167.27
2	102195.5	1913	53.42	5	3.98	272.75	293.21	298.81	459			265.55	167.27	226.72
3	108547.66	1487.17	72.99	4	218.3	251.14	459	555.85				371.0725	178.48	226.72
4	316549.48	2288.16	138.34	6	107.56	276.89	317.32	489.52	540.73	555.85		381.3117	178.47	191.36
5	52933.82	1022.08	51.79	4	67.4	252.58	317.32	384.78				255.52	140.27	164.66
6	∞	∞		5	64.34	384.78	489.52	∞	∞			∞	140.27	179.28
7	∞	∞		3	435.09	∞	∞					∞	179.28	411.17
8	129960.25	1574.87	82.52	5	64.34	213.24	252.58	435.09	609.62			314.974	140.27	164.66
9	232158.54	2138.43	108.56	7	3.98	67.4	107.56	218.3	347.65	609.62	783.16	305.3814	164.66	178.48
10	339998.65	2840.4	119.7	5	94.33	298.81	505.07	783.16	1159.02			568.078	154.48	167.27
11	∞	∞		3	1159.02	∞	∞					∞	154.48	1094.24
12	∞	∞		6	251.14	272.75	276.89	911.63	∞	∞		∞	270.13	292.96
13	∞	∞		2	∞	∞						∞	924.15	
14	∞	∞		4	540.73	911.63	∞	∞				∞	609.84	690.13
15	∞	∞		5	213.24	347.65	505.07	∞	∞			∞	411.17	509.17
Bou.	183191.98	1894.87	89.62	5.14	79.98	233.26	321.72	452.22	623.2	582.74	783.16	351.7	160.55	188.55
Inf.	∞	∞		4	393.98	442	423.83	911.63				515.77	355.47	477.74
Aver.	183191.98	1894.87	89.62	4.53	236.98	320.24	352.35	509.64	623.2	582.74	783.16	433.73	264.51	333.14
G	D3	D4	D5	D6	D7	Av.D	A1	A2	A3	A4	A5	A6	A7	Av.A
1	270.13					197.2933	115	118	131					121.3333
2	227.48	270.13	292.96			236.912	81	100	112	114	133			108
3	292.96	413.98				278.035	57	79	100	123				89.75
4	264.73	413.98	609.48	690.13		391.3583	75	108	111	123	136	167		120
5	191.36	264.73				190.255	50	63	113	134				90
6	264.73	690.13				318.6025	110	122	143	143				129.5
7						295.225	75	136						105.5
8	179.28	411.17	509.17			280.91	65	77	114	129	155			108
9	191.36	226.72	227.48	509.17	728.65	318.0743	56	124	129	146	148	149	149	128.71
10	227.48	728.65	1094.24			474.424	41	102	115	141	142			108.2
11						624.36	99	146						122.5
12	413.98	609.84	924.15			502.212	107	146	156	167	172			149.6
13						924.15	141							141
14	924.15					741.3733	47	145	163					118.3333
15	728.65	1094.24				685.8075	156	163	167	173				164.75
Bou.	224.95	389.90	546.66	599.65	728.65	309.99	60.71	93.29	113.43	130	142.8	158	149	106.30
Inf.	520.32	798.07	924.15			615.15	106.25	139.43	152	161	172			131.56
Aver.	348.02	512.35	609.58	599.65	728.65	430.59	85	116.36	129.5	139.3	147.67	158	149	120.35

The parameter of angular measure is sorted in an ascending order to extend the comparison of point pattern distribution. The variance of measured angles in sequence configures the polygonal structure of the convex hull. The sharpest angle of each polygon forms a network with clear difference between the bound acute angles and the unbound obtuse angles. The exceptional measure, however, structures the acute angle of the unbound 'generator-14' next to the most acute angle of 'generator-10' of a bound polygon. Similarly, the unbound 'generator-7' precedes the other 'generator-2' of slightly acute angle. In this regard, the two bound polygons adjacent to

'Tahrir' form a network with the unbound polygons of 'Ezbekia & Citadel' poles to structure an extended tri-pole analogy. Nevertheless, this analogy sustains the obtuse angle of the 'Tahrir' pole to emphasize its infinite structure. The second angular measure maintains the clear filtering of bound-unbound clusters in sequence. The unbound group, however, changes the clustering strategy with several observations. The previous grouping of unbound acute measure with the bound polygons is reversed through the clustering of the bound 'generator-9' with the unbound polygons of obtuse angles, which further enforces the special 'Hippodrome' structure.

Another structural change of the tri-poling analogy adjusts the second angular measure of 'Tahrir' at the border between the bound-unbound clusters. This observation structures the 'Tahrir' pole with two-sided clustering interface, while also introducing the obtuse angles towards the bound polygons. The third angular measure restructures the polygons into bound-unbound clusters, with the bordering of 'Tahrir' and 'generator-9' in close angular measures of the two ranges. The wider group of fourth angle restructures the 'Hippodrome' into the obtuse cluster of unbound infinity. Further angles of few encounters disperse in the bound polygons without order.

this regard, the adjacency structure of 'Tahrir' with 'polygon-2' allows the double-structure of 'Tahrir & Ezbekia' in the area measure, with the central interface of the public landmarks of the 'L'Opera & Hippodrome' in the perimeter measure. In addition, the order of largest areas and perimeters of polygons '10 & 4' reverses the sequence in the area/perimeter measure. Accordingly, the larger 'polygon-10' is more compact than the smaller 'polygon-4'. This inverted measure enforces the stretching of smaller 'polygon-4' towards the 'Citadel & Ezbekia' poles with a high integral correlation, but the largest 'polygon-10' contracts in front of the adjacent 'Tahrir' infinity.

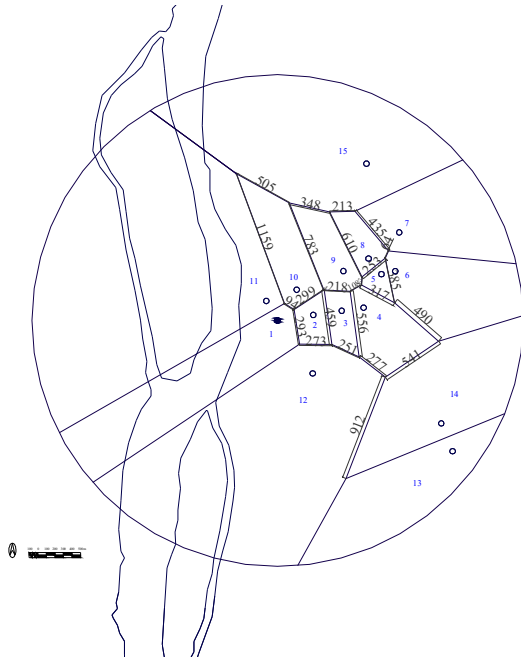


Fig. 7 Voronoi length of edges

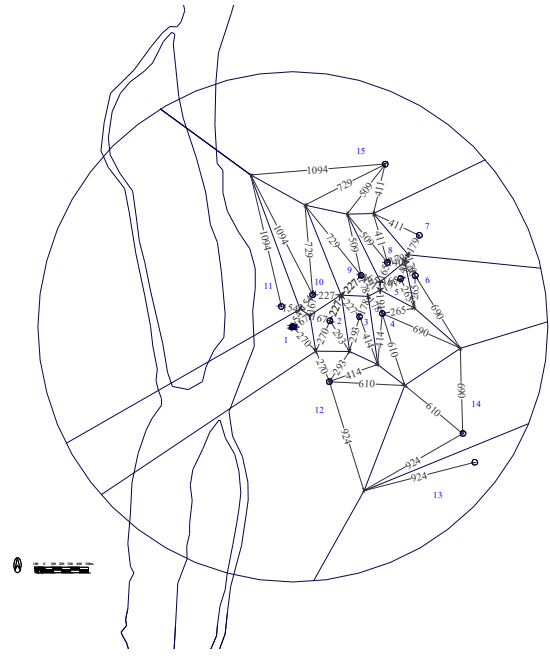


Fig. 8 Voronoi distance of generator-vertex

The parameters of area and perimeter with their ratio complete the polygonal structure. The measure of areas minimizes in 'polygon-5' of 'Ezbekia' and maximizes in 'polygon-10' of the 'Tahrir' pole. However, the adjacency of 'polygon-2' with 'polygon-10' sorts the area measure next to 'polygon-5'. Thus, the dual interface of 'Tahrir' adjacencies forms a contrasting, and also analogous structure of polygonal areas. The extended structure observes 'generator-4' of area measure next to 'polygon-10', while also the area of 'polygon-3' follows the measure of 'polygon-2'. Nevertheless, both polygons '3 & 4' link between the 'Citadel & Ezbekia', which realizes the tri-pole structure through the high integrity of 'Abdean Palace'. Meanwhile, another two successive areas in middle range belong to the special public buildings of 'L'Opera' of 'polygon-8' and the 'Hippodrome' of 'polygon-9'. The former clusters with the near areas in close variance, but the latter continues to cluster in separate with the highest integral structure. The perimeter measure maintains the order of areas, except 'polygon-2' of a shift from the second to the fourth rank. Their ratio, nevertheless, resets the order back as it was in areas. Therefore, the change of 'polygon-2' from the sequence of sorted areas disaffects the 'area/perimeter' ratio. In

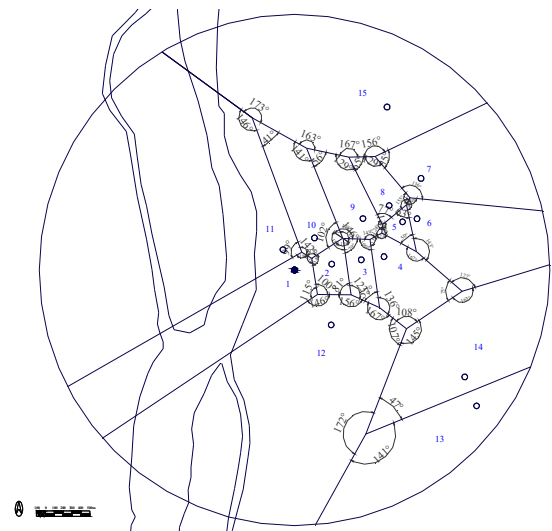


Fig. 9 Voronoi angles

In summary, the correlation of geometrical parameters concentrates the minimal measurements of lengths, distances, angles and areas towards the pole of 'Ezbekia Garden' with

clustered roundabouts. The concise geometry emphasizes the spatial experience of pedestrian character with landmarks from one roundabout to the other of controlled syntax, while filters the various townscapes of Cairo through this central zone of mixed functions. The maximized geometrical measurements, however, shifts to the farthest pole of 'Cairo Citadel' across old Cairo in logic with the less integral syntax. Between these two extremes, the roundabouts of 'Tahrir' pole represent the intermediary structure with analogical modeling of the layout.

V. CONCLUSION

The spatial findings of first-order voronoi, which is assigned to measure the explicit syntax of roundabout Cairo, validate the cognitive structure for the conceptual conservation in the future as follows:

1. The exclusive cognitive structure of 'Tahrir' generator contains the Giza Pyramids with super breadth over the Nile through the unbound polygonal offset, which symbolizes Egypt in a universal context. Meanwhile, the opposite generator of 'L'Opera' contains the socioeconomic hub of 'Ezbekia Garden', with further projection towards the Islamic complex of 'El Azhar' landmark. The average integral and control measurements of 'Tahrir & L'Opera' generators demonstrate the analogical convex hull in axial cross-section from ancient to modern Egypt.
2. The layout of roundabouts is highly integrated through the royal 'Abdean Palace' with the maximal control measure of the other royal palaces along the Nile, thus a dual symbolic syntax of royal dominance. On the contrary, the bulk of royal workshop area measures the least integral and control syntax, in addition to the other royal complex on elevated topography of visual overwhelm instead of space syntax. Thus, the logic of extreme difference between the royal roundabouts verifies the syntactic fit to the voronoi space.
3. The interface of old and new Cairo generates the symbolic high control roundabout of the 'Mixed Tribunal' landmark at the intersecting crossways of enlarged Cairo. However, the adjacent roundabout of neither spatial integration nor control overlooks the landscape of 'Ezbekia Garden' to separate the two towns in compensation. Nevertheless, the spread of residential use generates more spatial control than integration, which secures decentralized garden cities of infinite polygons. However, the 'Hippodrome' of public node justifies the layout of most integral neighborhood. Meanwhile, the commercial use attracts an average spatial integrity, but measures less control for the logic of more distributed bypasses. Therefore, the difference of spatial structuring in correlation to the functional roundabouts further enforces the logic of voronoi with space syntax.
4. The geometrical characteristics of point pattern configure the roundabout distribution. The length of voronoi edges forms two poles of 'Ezbekia Garden & Tahrir'. The longer edges re-cluster into pairs towards the concentric zone, while the longest edges scatter over the bound polygons. The 'generator-vertex' distance changes from the filtered

zones of shortest distances to the longer range at the poles of Cairo with interactive mixed clusters. The longest measure scatters with an extreme variance below or above the average of the convex hull. Meanwhile, the minimal range of angles forms a network of bound polygons adjacent to 'Tahrir' and unbound polygons of 'Ezbekia & Citadel' landmarks to form a tri-pole analogy. The more obtuse angles re-cluster into bound polygons of scattered structure. Nevertheless, the 'Hippodrome & Tahrir' are set in close bordering measures of the two angular ranges. More parameters of area and perimeter measures observe the alternating interface of 'Tahrir' adjacencies to form a contrasting or analogous structure of the convex hull. However, the larger 'polygon-10' is more compact than the smaller 'polygon-4' through the area/perimeter ratio. This stretches the smaller polygon of 'Abdean Palace' towards the 'Citadel & Ezbekia' poles, but the larger cell contracts the interface with the infinite 'Tahrir' structure to emphasize the tri-pole geometry. Thus, the conceptual conservation of roundabout Cairo extends the visual limits of the area with the correlated logic between the spatial measurements, functions, and geometrics of major 'Tahrir' analogy. This prospective conclusion verifies the hybrid methodological strategy of voronoi into space syntax with future developments of universal scopes yet to come.

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