World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:9, No:6, 2015

Playing Games with Genetic Algorithms: Application on Price-QoS Competition in Telecommunications Market

M'hamed Outanoute, Mohamed Baslam, Belaid Bouikhalene

Open Science Index, Mathematical and Computational Sciences Vol:9, No:6, 2015 publications.waset.org/10002944/pdf

Abstract—The customers use the best compromise criterion between price and quality of service (QoS) to select or change their Service Provider (SP). The SPs share the same market and are competing to attract more customers to gain more profit. Due to the divergence of SPs interests, we believe that this situation is a non-cooperative game of price and QoS. The game converges to an equilibrium position known Nash Equilibrium (NE). In this work, we formulate a game theoretic framework for the dynamical behaviors of SPs. We use Genetic Algorithms (GAs) to find the price and QoS strategies that maximize the profit for each SP and illustrate the corresponding strategy in NE. In order to quantify how this NE point is performant, we perform a detailed analysis of the price of anarchy induced by the NE solution. Finally, we provide an extensive numerical study to point out the importance of considering price and QoS as a joint decision parameter.

Keywords—Pricing, QoS, Market share game, Genetic algorithms, Nash equilibrium, Learning, Price of anarchy.

I. Introduction

RECENTLY, game theory has been widely used to analyze the selfish behavior of customers and service providers in telecommunications systems. Several studies have shown that the selfish behavior of customers leads to a typical prisoner's dilemma situation that causes a network collapse. In the literature, a single decision action (e.g. the price) is commonly used for computing an equilibrium. However, it is necessary to include more than one parameter in the model to take into account the quality of service. The competition in terms of price and QoS between SPs entails the formation of non-cooperative games. We consider a game of several SPs, in which each player tries to maximize its own revenue. The whole system of SPs would have no incentive to deviate from the Nash equilibrium point. In this work, we present a model to calculate a bi-criteria Nash equilibrium (here, service price and quality of service) for several SPs. Then we will analyze the interactions between different SPs who wont attract more clients and maximize their respective profits. Our model is mainly inspired from [6], where the authors have constructed a Markov model that derive the behavior of customers depending on the strategic actions of the SPs, to study a non-cooperative game for pricing problem considering QoS as an extra decision parameter. We base our study on the concepts of demand for services of a given SP. This demand given by linear function that depends on the vectors of prices and QoSs, is a commonly used function in research related to competitive network and equilibrium models [9][5], to calculate the reputation of an SP in the market.

Rationality is the most fundamental assumption in game theory's works; every player looks to maximize its own utility [27]. In this context, the players know all the information about the game, i.e. there is a complete information. So, we consider that all players are said to be rational and intelligent, i.e. every player acts in such a way as to maximize his or her expected payoff or utility as economists would say and can deduce what his or her opponent will do when acting rationally. In fact, humans use a propositional calculus in reasoning, the propositional calculus concerns truth functions of propositions, which are logical truths (statements that are true in virtue of their form). For this reason, the assumption of rational behaviour of players in telecommunications systems is more justified, as the players are usually devices programmed to operate in certain ways. However, there are previous studies that have shown that humans do not always act rationally [10].

A. Related Works

Applying game theory in telecommunications problems is an active research area, in which game-theoretic models have been developed and studied in the last decades, [6], [9], [19], [2], [1], [7], [22]. These models are interested to pricing issue, they proposed non-cooperative game formulations to analyze behaviours of players that selfishly decide their strategies to maximize their respective profits. Other works consider the criteria of price as an implicit parameter, which is determined as a function of the degree of saturation on the network. Typically in these approaches, the price is a shadow price. For more details on those approaches see, [17], [18], [29]. Nonetheless, the price of anarchy has been studied in a large and diverse number of games, e.g., in areas like wireless ad-hoc networks [8], [15], routing and congestion [4], [23], network creation [3], or facility location [26]. In our model, we do not take into account network topology, but rather the effective service proposed by each SP as a single entity. In other words, the price and QoS proposed by an SP will not depend on the source or destination, distance, etc. that

M'hamed Outanoute is PhD candidate on Computer Science, Faculty of Sciences and Techniques, Sultan Moulay Slimane University, Beni Mellal, Morocco (e-mail: mhamed.outanoute@gmail.com).

Mohamed Baslam is with the Department of Computer Science, Faculty of Sciences and Techniques, Sultan Moulay Slimane University, Beni Mellal, Morocco (e-mail: baslam.med@gmail.com).

Belaid Bouikhalene is with the Department of Mathematics and Computer Science, Polydisciplinary Faculty, Sultan Moulay Slimane University, Beni Mellal, Morocco (e-mail: bbouikhalene@yahoo.fr).

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underlies the request of each user. After we have proved existence of Nash equilibrium, we propose a genetic algorithm that allows learning the Nash equilibrium of price and QoS strategies decided by SPs.

II. Problem Modeling

In this work, we formulate the interaction among service providers (SPs) as a non-cooperative game. Each SP chooses the Quality of Service to guarantee (it depends on the amount of requested bandwidth) and the corresponding price.

We consider a system with N service providers. Let p_i and q_i be, respectively, the tariff/pricing policy and the QoS guaranteed by SP-i. Now, each customer seeks to subscribe to the operator which allows him to meet a QoS sufficient to satisfy his/her needs, at suitable price. We considers that behaviors of customers has been handled by a simple function so called demand functions, see equation (1). This later depends on the price and QoS strategies of all SPs. From a tagged SP's point of view, the question is to set the best pricing strategy and the best QoS (amount of bandwidth to request from the network owner). SPs are supposed to know the effect of their policy on the customers subscription policy. Whereas from customers point of view, the question is to find the SP that has the best price-QoS tradeoff conditions.

A. Demand Model

For simplicity, we consider that the demand function D_i for services of the tagged SP-i is linear with respect to the set price p_i and the promised QoS q_i [2]. This demand function depends also on prices $\mathbf{p}_{-i}$ and QoS $\mathbf{q}_{-i}$ set by the competitors. Namely, the demand function of SP-i depends on $\mathbf{p} = [p_1, .., p_N]$ and $\mathbf{q} = [q_1, .., q_N]$. Eventually, D_i is decreasing w.r.t. p_i and increasing w.r.t. $p_j, \quad j \neq i$. Whereas it is increasing w.r.t q_i and decreasing w.r.t. $q_j, \quad j \neq i$. Then, the demand functions w.r.t services of SP-i can be written as follows:

$$D_i(\mathbf{p}, \mathbf{q}) = D_i^0 - \alpha_i^i p_i + \beta_i^i q_i + \sum_{j, j \neq i} \left[\alpha_i^j p_j - \beta_i^j q_j \right], \quad \forall i \in \{1, .., N\}. \quad (1)$$

where D_i^0 is a positive constant used to insure non-negative demands over the feasible region. While α_i^j and β_i^j are positive constants representing respectively the sensitivity of service provider i to price and QoS of service provider j.

B. Utility Model

The total revenue of SP-i is $D_i(\mathbf{p}, \mathbf{q})p_i$. We assume that we have a single network owner, this latter charges each SP-i a cost ϑ_i per unit of requested bandwidth. In order to insure the customers loyalty, the amount of bandwidth μ_i required by SP-i should depend on $D_i(.)$ and on the QoS q_i it wishes to offer to its customers. Therefore, the net profit of SP-i is simply the difference between the total revenue and the fee paid to the network owner:

$$U_i(\mathbf{p}, \mathbf{q}) = D_i(\mathbf{p}, \mathbf{q})p_i - F_i(q_i, D_i), \quad \forall i \in \{1, .., N\}.$$

where $F_i(q_i, D_i)$ is the fee paid by SP-i (investment of SP-i):

$$F_i = \vartheta_i \mu_i (q_i, D_i)$$

where μ_i is the amount of bandwidth required by SP-i, such that ϑ_i is a cost per unit of requested bandwidth. We assume that the QoS corresponds to the expected delay, also we consider the Kleinrock delay which is a common delay used in Networking Games, so:

$$q_i = \frac{1}{\sqrt{Delay_i}} = \sqrt{\mu_i - D_i}$$

that mean that:

$$\mu_i = q_i^2 + D_i$$

While, the utility function of the SP-i is given by the following formula:

$$U_i(\mathbf{p}, \mathbf{q}) = D_i(\mathbf{p}, \mathbf{q}) (p_i - \vartheta_i) - \vartheta_i q_i^2, \quad \forall i \in \{1, .., N\}. \quad (2)$$

III. Non-cooperative Game Formulation

Let $\mathbf{G} = [\mathcal{N}, \{P_i, Q_i\}, \{U_i(.)\}]$ denote the non-cooperative price and QoS game (NPQG), where $\mathcal{N} = \{1, .., N\}$ is the index set identifying the SPs, P_i is the price strategy set of SP-i, Q_i is the QoS strategy set of SP-i, and $U_i(.)$ is the utility function. Each SP-i selects a price $p_i \in P_i$ and a QoS measure $q_i \in Q_i$. Let the price vector $\mathbf{p} = (p_1, .., p_N)^T \in P^N = P_1 \times P_2 \times ... \times P_N$, QoS vector $\mathbf{q} = (q_1, .., q_N)^T \in Q^N = Q_1 \times Q_2 \times ... \times Q_N$ (where T represents the transpose operator). The utility of SP-i when it decides the strategy price p_i to allocate the QoS q_i is given in (2). We assume that the strategy spaces P_i and Q_i of each SP are compact and convex sets with maximum and minimum constraints, For any given user i we consider strategy spaces the closed intervals $Pi = [p_min, p_max]$ and $Qi = [q_min, q_max]$. In order to maximize their utilities, each SP-i decides a price p_i and QoS q_i. Formally, the NPQG problem can be expressed as:

$$\max_{p_i \in P_i, q_i \in Q_i} U_i(\mathbf{p}, \mathbf{q}), \quad \forall i \in \mathcal{N}. \quad (3)$$

A. Nash Equilibrium

Taking rationality of service providers into account, the Nash equilibrium concept is the natural concept solution of the NPQG game. We first will investigate the Nash equilibrium solution for the induced game as defined in the previous section. We show that Nash equilibrium solution exists and is unique by using the theory of concave games [24]. We recall that a non-cooperative game $\mathbf{G}$ is called concave if all players' utility functions are strictly concave with respect to their corresponding strategies [24].

According to [24], a Nash equilibrium exists in a concave game if the joint strategy space is compact and convex, and

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the utility function that any given player seeks to maximize is concave in its own strategy and continuous at every point in the product strategy space. Formally, if the weighted sum of the utility functions with nonnegative weights:

$$\varphi = \sum_{i=1} x_i U_i, \quad x_i > 0 \quad \forall i. \tag{4}$$

is diagonally strictly concave, this implies that the Nash equilibrium point is unique. The notion of diagonal strict concavity means that an individual user has more control over its utility function than the other users have on it, and is proven using the pseudo-gradient of the weighted sum of utility functions [24].

Fixed-Price Game: Considering some fixed price policy, a Nash equilibrium in QoS is formally defined as:

Definition 1: A QoS vector $\mathbf{q}^* = (q_1^*, .., q_N^*)$ is a Nash equilibrium of the NPQG : $\mathbf{G} = [\mathcal{N}, \{P_i, Q_i\}, \{U_i(.)\}]$ if, for every $i \in \mathcal{N}$, $U_i(q_i^*, \mathbf{q}_{-i}^*) \geq U_i(q_i', \mathbf{q}_{-i}^*)$ for all $q_i' \in Q_i$.

Theorem 1: A Nash equilibrium in terms of QoS for game $\mathbf{G} = [\mathcal{N}, \{P_i, Q_i\}, \{U_i(.)\}]$ exists and is unique.

Proof: To prove existence, we note that each SP's strategy space Q_i is defined by all QoSs in the closed interval bounded by the minimum and maximum QoSs. Thus, the joint strategy space Q is a nonempty, convex, and compact subset of the Euclidean space $\mathbb{R}^N$. In addition, the utility functions are concave with respect to QoSs as can be seen from the second derivative test:

$$\frac{\partial^2 U_i(\mathbf{p}, \mathbf{q})}{\partial q_i^2} = -2\vartheta_i < 0, \quad \forall i \in \mathcal{N}, \tag{5}$$

which ensures existence of a Nash equilibrium.

In order to prove uniqueness, we follow, [24], and define the weighted sum of user utility functions.

$$\varphi(\mathbf{q}, \mathbf{x}) = \sum_{i=1}^{N} x_i U_i(q_i, \mathbf{q}_{-i}), \tag{6}$$

The pseudo-gradient of (6) is given by:

$$g(\mathbf{q}, \mathbf{x}) = \left[x_1 \nabla U_1(q_1, \mathbf{q}_{-1}), ..., \quad x_N \nabla U_N(q_N, \mathbf{q}_{-N})\right]^T \tag{7}$$

The Jacobian matrix $\mathbf{J}$ of the pseudo-gradient (w.r.t. $\mathbf{q}$) is written

$$\mathbf{J} = \begin{pmatrix} x_1 \frac{\partial^2 U_1}{\partial q_1^2} & x_1 \frac{\partial^2 U_1}{\partial q_1 \partial q_2} & \cdots & x_1 \frac{\partial^2 U_1}{\partial q_1 \partial q_N} \\ x_2 \frac{\partial^2 U_2}{\partial q_2 \partial q_1} & x_2 \frac{\partial^2 U_2}{\partial q_2^2} & \cdots & x_2 \frac{\partial^2 U_2}{\partial q_2 \partial q_N} \\ \vdots & \vdots & \ddots & \vdots \\ x_N \frac{\partial^2 U_N}{\partial q_N \partial q_1} & x_N \frac{\partial^2 U_N}{\partial q_N \partial q_2} & \cdots & x_N \frac{\partial^2 U_N}{\partial q_N^2} \end{pmatrix}$$
$$= \begin{pmatrix} -2x_1\vartheta_1 & 0 & \cdots & 0 \\ 0 & -2x_2\vartheta_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & -2x_N\vartheta_N \end{pmatrix}.$$

Thus, $\mathbf{J}$ is a diagonal matrix with negative diagonal elements. This implies that $\mathbf{J}$ is negative definite. Henceforth $[\mathbf{J} + \mathbf{J}^T]$ is also negative definite, and according to Theorem (6) in [24], the weighted sum of the utility functions $\varphi(\mathbf{q}, \mathbf{x})$ is diagonally strictly concave. Thus the fixed-price Nash equilibrium point

$$q_i^* = arg \max_{q_i \in Q_i} U_i(q_i, \mathbf{q}_{-i}^*), \quad \forall i \in \mathcal{N}. \tag{8}$$

is unique.

■

Fixed-QoS Game: When fixing the QoS, a Nash equilibrium in terms of price is formally defined as:

Definition 2: A price vector $\mathbf{p}^* = (p_1^*, .., p_N^*)$ is a Nash equilibrium of the NPQG : $\mathbf{G} = [\mathcal{N}, \{P_i, Q_i\}, \{U_i(.)\}]$ if, for every $i \in \mathcal{N}$, $U_i(p_i^*, \mathbf{p}_{-i}^*) \geq U_i(p_i', \mathbf{p}_{-i}^*)$ for all $p_i' \in P_i$.

Theorem 2: A Nash equilibrium in terms of price for the game $\mathbf{G} = [\mathcal{N}, \{P_i, Q_i\}, \{U_i(.)\}]$ exists and is unique.

Proof: To prove existence, we note that each SP's strategy space P_i is defined by all prices in the closed interval bounded by the minimum and maximum prices. Thus, the joint strategy space P is a nonempty, convex, and compact subset of the Euclidean space $\mathbb{R}^N$. In addition, the utility functions are concave with respect to prices as can be seen from the second derivative test:

$$\frac{\partial^2 U_i(\mathbf{p}, \mathbf{q})}{\partial p_i^2} = -2\alpha_i^i < 0, \quad \forall i \in \mathcal{N}, \tag{9}$$

which ensures existence of a Nash equilibrium.

To prove uniqueness, we define now the weighted sum of user utility functions

$$\phi(\mathbf{p}, \mathbf{x}) = \sum_{i=1}^{N} x_i U_i(p_i, \mathbf{p}_{-i}), \tag{10}$$

the pseudo-gradient of this later is given by

$$g(\mathbf{p}, \mathbf{x}) = \left[x_1 \nabla U_1(p_1, \mathbf{p}_{-1}), ..., \quad x_N \nabla U_N(p_N, \mathbf{p}_{-N})\right]^T. \tag{11}$$

In order to show that $\phi(\mathbf{p}, \mathbf{x})$ is diagonally strictly concave in this case we use the following lemma proved in [12].

Lemma 1: If each $U_i(\mathbf{p})$ is a strictly concave function in p_i, each $U_i(\mathbf{p})$ is convex in p_{-i} and there is some $\mathbf{x} > 0$ such that $\phi(\mathbf{p}, \mathbf{x})$ is concave in $\mathbf{p}$, then $[\mathbf{J}(\mathbf{p}, \mathbf{x}) + \mathbf{J}^T(\mathbf{p}, \mathbf{x})]$ is negative definite, where $\mathbf{J}(\mathbf{p}, \mathbf{x})$ is the Jacobian of $g(\mathbf{p}, \mathbf{x})$.

From (9), we know that $U_i(\mathbf{p})$ is strictly concave in p_i. Further

$$\frac{\partial^2 U_i}{\partial p_j^2} = 0, \quad \forall i \neq j,$$

which implies that $U_i(\mathbf{p})$ is convex in p_{-i} as well. Also, we have that

$$\begin{aligned} \frac{\partial^2 \phi(\mathbf{p}, \mathbf{x})}{\partial p_i^2} &= x_i \frac{\partial^2 U_i(p_i, \mathbf{p}_{-i})}{\partial p_i^2} + \sum_{j \neq i}^{N} x_j \frac{\partial^2 U_j(p_i, \mathbf{p}_{-i})}{\partial p_i^2} \\ &= -2x_i\alpha_i^i < 0, \quad \forall i, \end{aligned}$$

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then $\phi(\mathbf{p},\mathbf{x})$ is concave in p_i and from *Lemma* 1 we have that $[\mathbf{J}(\mathbf{p},\mathbf{x})+\mathbf{J}^T(\mathbf{p},\mathbf{x})]$ is negative definite. Thus the weighted sum of utility functions $\phi(\mathbf{p},\mathbf{x})$ is diagonally strictly concave. The fixed-QoS Nash equilibrium point is then unique and is given by

$$p_i^* = arg \max_{p_i \in P_i} U_i(p_i, \mathbf{p}^*_{-i}), \quad \forall i \in \mathscr{N}. \tag{12}$$

■

B. Joint Price and QoS Game

The utility functions $U_i(\mathbf{p},\mathbf{q}), \quad \forall i \in \mathscr{N}$, are concave respectively w.r.t. q_i and p_i. So, for all, $i \in \mathscr{N}$, the QoS and price conditions which maximizes the utility given in (2) are respectively:

$$\begin{cases} \frac{\partial U_i(\mathbf{p},\mathbf{q})}{\partial q_i} &= 0 \\ \frac{\partial U_i(\mathbf{p},\mathbf{q})}{\partial p_i} &= 0 \end{cases} \tag{13}$$

Thus, the computation of Nash Equilibrium can be performed by solving latter system.

C. Social Welfare and Price of Anarchy

The concept of social welfare [20] or total surplus [25] is defined as the sum of the utilities of all agents in the systems (i.e. Providers). It is well known in game theory that agent selfishness, such as in a Nash equilibrium, does not lead in general to a socially efficient situation. As a measure of the loss of efficiency due to the divergence of user interests, we use the Price of Anarchy (PoA) [23], this latter is a measure of the loss of efficiency due to actors' selfishness. This loss has been defined in [23] as the worst-case ratio comparing the global efficiency measure (that has to be chosen) at an outcome of the noncooperative game played among actors, to the optimal value of that efficiency measure. A PoA close to 1 indicates that the equilibrium is approximately socially optimal, and thus the consequences of selfish behavior are relatively benign. The term Price of Anarchy was first used by Koutsoupias and Papadimitriou [23] but the idea of measuring inefficiency of equilibrium is older. The concept in its current form was designed to be the analogue of the "approximation ratio" in Approximation Algorithms or the "competitive ratio" in Online Algorithms. As in [14], we measure the loss of efficiency due to actors' selfishness as the quotient between the social welfare obtained at the Nash equilibrium and the maximum value of the social welfare:

$$PoA = \frac{\min_{\mathbf{p},\mathbf{q}} W_{NE}(\mathbf{p},\mathbf{q})}{\max_{\mathbf{p},\mathbf{q}} W(\mathbf{p},\mathbf{q})} \tag{14}$$

where $W(\mathbf{p},\mathbf{q}) = \sum_{i=1}^{N} U_i(\mathbf{p},\mathbf{q})$ is a welfare function and $W_{NE}(\mathbf{p},\mathbf{q}) = \sum_{i=1}^{N} U_i(\mathbf{p}^*,\mathbf{q}^*)$ is a sum of utilities of all actors at Nash Equilibrium.

IV. Genetic Algorithm Learning

Genetic Algorithms (GAs), developed by Holland [16] and his student Goldberg [11], are based on the mechanics of natural evolution and natural genetics. GAs differ from usual inversion algorithms because they do not require a starting value. The GAs use a survival-of-the-fittest scheme with a random organized search to find the best solution to a problem. Solve an optimization problem is find the optimum of a function from a finite number of choices, often very large. The practical applications are numerous, whether in the field of industrial production, transport or economics - wherever there is need to minimize or maximize digital functions in systems simultaneously operate a large number of parameters. Algorithm (1) represents the genetic algorithm used for learning the problem studied in this work.

Algorithm 1 Genetic Algorithm learning for Nash Equilibrium on Price-QoS Competition.

1- Initialize price and QoS vectors **p** and **q** randomly;
2- Repeat:
- For each service provider $i \in \mathscr{N}$ (Maximizing his utility)
  a) Create an initial population;
  b) Repeat:
     i) Selection;
     ii) Crossover;
     iii) Mutation.
  c) Verify whether end condition is met.

3- Until stabilization of prices and service qualities.

In the following, we describe each step mentioned in Algorithm (1).

A. Coding of Individuals and Initial Population

Coding or chromosomal representation is a way to encode the solutions to a problem or individuals. Choosing an encoding must take into account the complexity of the coding/decoding process that can slow the calculations and considerably influence the convergence of the GA. Historically, the encoding used by the GAs was in the form of bit strings containing all the information needed to describe a point in the state space. However, this type of coding is not always good [28]. GAs using real vectors avoid these problems by keeping the variables of the problem in the coding of the population element without going through the intermediate binary encoding [28]. The structure of the problem is stored in the coding. Our utility function is described in (2). For the problem of Nash equilibrium, we try to find the pair (p_i, q_i) that maximizes the utility function of a given SP-i, knowing a priori the other couples $(p_j, q_j), j \neq i$ for his competitors SPs. Each individual is represented by a pair (p, q), such as $p \in [\underline{p}, \overline{p}]$ and $q \in [\underline{q}, \overline{q}]$. In order to build an initial population, we randomly generate a number N_p of individuals belonging to the state space $[\underline{p}, \overline{p}] \times [\underline{q}, \overline{q}]$.

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B. Selection

This operation is based on the principle of adaptation of each individual in a population to its environment, according to the theory of natural selection introduced by Charles Darwin. Thus, only the fittest individuals to meet certain criteria will be selected to survive and reproduce. The evaluation function of adequacy of individuals evaluates each chromosome of the population, and only those whose quality is sufficient will be kept from one generation to another. Other individuals will take the place of the worst. The selection of each individual k belonging to the population depends on its probability PS_k called probability of selection. In the case of a maximization problem (finding a maximum of the adaptation function, which is the case in this paper), PS_k is given by:

$$PS_k = \frac{U_i\left(p_k, q_k\right)}{\sum_{j=1}^{Np} U_i(p_j, q_j)} \tag{15}$$

C. Crossover

The crossing is to combine any two individuals (called parents) to come out two other individuals (called children). The crossover operator does not act on all the pairs of individuals of the population. An individual has a chance to participate in a cross, denoted Pc (often caught between 0.8 and 1). At this level, we use an arithmetical crossover [13]. Let the two chromosomes of the current population be $C1$ and $C2$. We obtain both descendants of $C1$ and $C2$ by:

$$\begin{cases} C'1 = r * C1 + (1-r) * C2 \\ C'2 = (1-r) * C1 + r * C2 \end{cases} \tag{16}$$

where r is the random number generated between 0 to 1.

D. Mutation

A mutation is a random modification of a parameter (gene) used to ensure variability in the evolutionary process. A probability of mutation is defined, Pm, usually chosen to be between 0.0001 and 0.1. Should mutation occur, a non-uniform mutation [21] is performed. One of the parameters is modified as follows, after a flip from an unbiased coin:

$$C_k = \begin{cases} C_k + \delta(UB - C_k) \\ C_k - \delta(C_k - LB) \end{cases} \tag{17}$$

where UB is the upper bound of the parameter being mutated and LB is the lower bound. The delta function is defined as:

$$\delta(y) = y[r(1 - t/T)^B] \tag{18}$$

where r is a random number between 0 and 1, t is the current generation, T is the maximum generation, and B is a parameter that determines the degree of dependence on the actual generation (usually between 1 and 5). From (18) it can be seen that the amplitude of the mutation decreases as the number of generations increases. This kind of mutation is called non-uniform.

V. Numerical Results and Discussions

To clarify and show how to take advantage from our theoretical study, we suggest to study numerically the market share game while considering the genetic algorithm described in the previous section and expressions of demand as well as utility functions of SPs. Hence, we consider a system with three SPs seeking to maximize their respective revenues. Table I represents the system parameter values considered in this numerical study.

TABLE I
System Parameters Used for Numerical Examples

$\alpha_1^1 = \alpha_2^2 = \alpha_3^3$	$\alpha_j^i, \quad i \neq j$	$\beta_1^1 = \beta_2^2 = \beta_3^3$	$\beta_j^i, \quad i \neq j$
0.7	0.15	0.7	0.15
D_0^1	D_0^2	D_0^3	$\vartheta_1 = \vartheta_2 = \vartheta_3$
350	300	250	20
$\underline{p_1} = \underline{p_2} = \underline{p_3}$	$\overline{p_1}, \overline{p_2}, \overline{p_3}$	$\underline{q_1} = \underline{q_2} = \underline{q_3}$	$\overline{q_1}, \overline{q_2}, \overline{q_3}$
100	500	0	10

Figs. 1 and 2 present respectively curves of the convergence to Nash Equilibrium Price and to Nash Equilibrium QoS. It is clear that the genetic algorithm converges to the unique Nash equilibrium price and QoS. We also remark that the speed of convergence is relatively high (around 7 rounds are enough to converge to the joint price and QoS equilibrium).

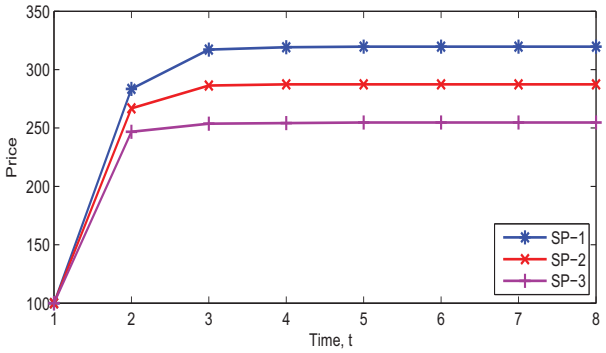


Fig. 1 Price game: Convergence to the Price Nash equilibrium

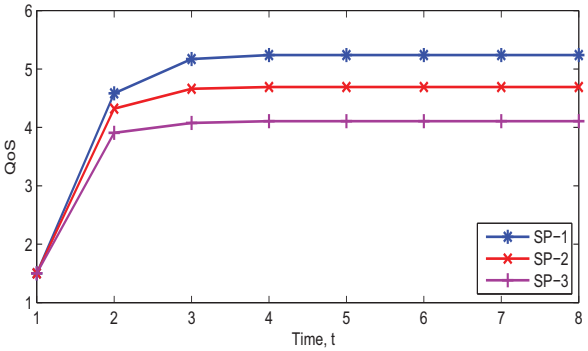


Fig. 2 QoS game: Convergence to the Price Nash equilibrium

Next we plot in Figs. 3 and 4, respectively, the interplay of bandwidth cost ($\vartheta_i, \quad i \in \{1,2,3\}$ on the price and QoS

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at Nash equilibrium, for both SPs that we consider in this example. On one hand, we note that the equilibrium price for both SPs is increasing with respect to the bandwidth cost. On the other hand, we note that the equilibrium QoS for all SPs is decreasing with the bandwidth cost. When the cost of bandwidth decided by the network owner is cheaper, the SPs invest for more bandwidth, so as to offer better QoS and an attractive price.

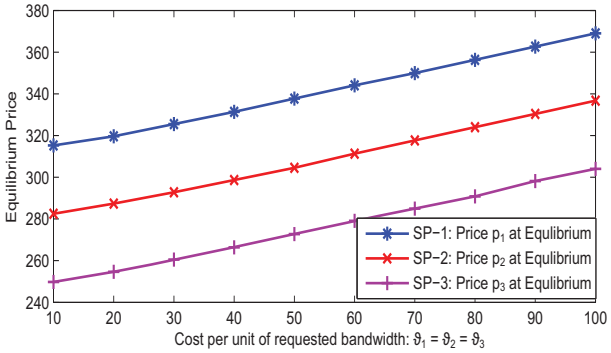


Fig. 3 Equilibrium Prices w.r.t cost per unit of requested bandwidth ϑ_i

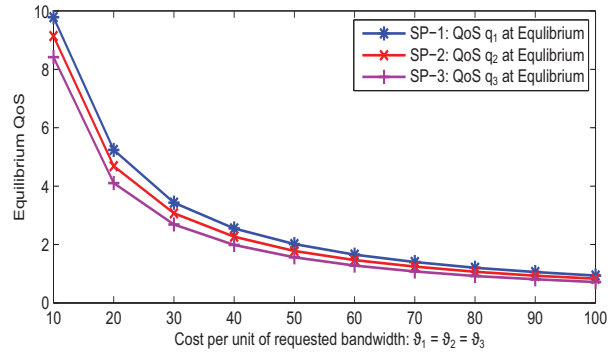


Fig. 4 Equilibrium QoSs w.r.t cost per unit of requested bandwidth ϑ_i

In the following, we discuss the impact of the system parameters on the system efficiency in terms of Price of anarchy:

Influence of ϑ_i (cost per unit of requested bandwidth): Fig. 5 shows the PoA variation curve as a function of the providers' bandwidth cost ϑ_i. Without loss of generality, we assume that $\vartheta_1 = \vartheta_2 = \vartheta_3$. A special feature is that the Nash equilibrium performs well and the loss of efficiency is only around 8%. This result indicates that the Nash equilibrium of this game is fair and socially efficient. Henceforth, selfish players would not need the help of a third-part regulator (who recommends the players the best strategy profile to achieve their respective best outcomes) to get attracted by the optimum social welfare. However, the network owner can use the value of the bandwidth cost to control the selfishness/aggressiveness of the service providers, which will improve the whole network performance.

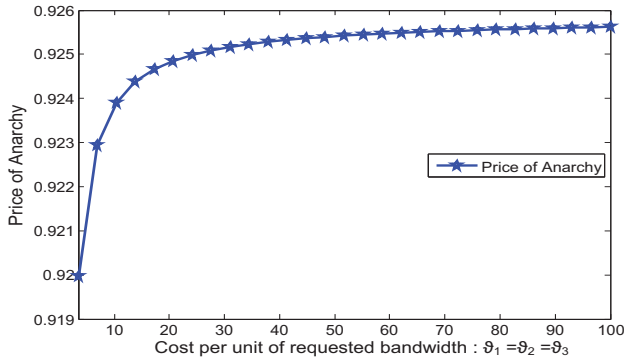


Fig. 5 Price of Anarchy as a function of cost per unit of requested bandwidth ϑ_i

Influence of α (Sensitivity of SP-i to his price p_i): Fig. 6 plots the variation curve of price of anarchy with respect to α which represents the sensitivity of SP-i to his price p_i. In that figure, we first notice that the price of anarchy increases when α increases, the fact that the price of anarchy increases with α finds the simple intuition that increasing the sensitivity of SPs to their prices gives more and more freedom to SPs for optimizing the Nash equilibrium. On the other hand, when $\alpha = \alpha_1^1 = \alpha_2^2 = \alpha_3^3 = 1$, in the other word, when the sensitivity of an SP to the price of its competitors is zero ($\alpha_1^2 = \alpha_1^3 = \alpha_2^1 = \alpha_2^3 = \alpha_3^1 = \alpha_3^2 = 0$), price of anarchy converges to 1 and so the equilibrium is approximately socially optimal.

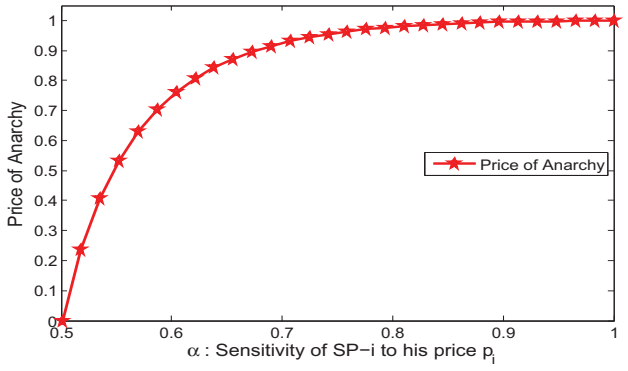


Fig. 6 Price of Anarchy as a function of $\alpha = \alpha_1^1 = \alpha_2^2 = \alpha_3^3$ (Sensitivity of SP-i to his price p_i)

Influence of β (Sensitivity of SP-i to his QoS q_i): Fig. 7 illustrates variations of PoA as a function of, β, which is the sensitivity of SPs to their respective own QoS. We first notice that the loss of efficiency is around 8%. Moreover the curve of PoA is concave, this latter mean that there are some, $\beta^* < 1$, which optimizes the equilibrium, ($\beta^* = \beta_1^1 = \beta_2^2 = \beta_3^3 = 0.76, \quad PoA^* = 0.925$). Surprisingly, the price of anarchy varies slightly (variation of almost 0.001). To explain this behaviour, Figs. 8-10 depict, respectively, the curves of equilibrium Price, equilibrium QoS and equilibrium Utility of each SP-i. We find that the induced variation of the price is

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much higher compared to that of QoS, and subsequently, β (Sensitivity of SPs to their QoS) has a smaller impact on the system.

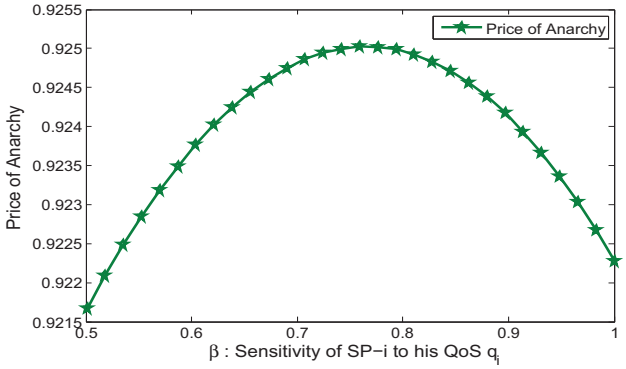


Fig. 7 Price of Anarchy as a function of $\beta = \beta_1^1 = \beta_2^2 = \beta_3^3$ (Sensitivity of SP-i to his QoS q_i)

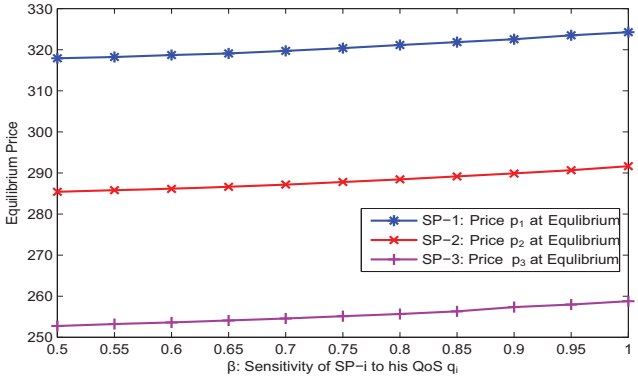


Fig. 8 Equilibrium Price of SP-i as a function of $\beta = \beta_1^1 = \beta_2^2 = \beta_3^3$ (Sensitivity of SP-i to his QoS q_i)

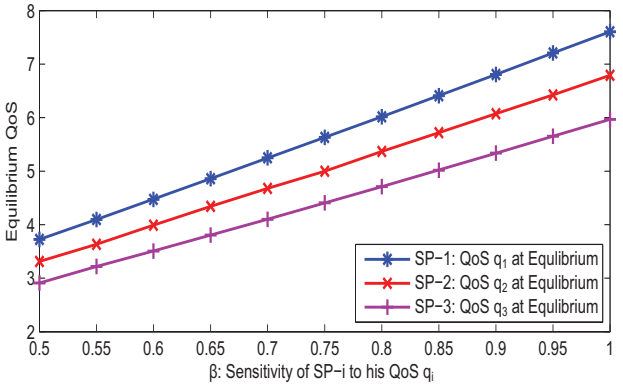


Fig. 9 Equilibrium QoS of SP-i as a function of $\beta = \beta_1^1 = \beta_2^2 = \beta_3^3$ (Sensitivity of SP-i to his QoS q_i)

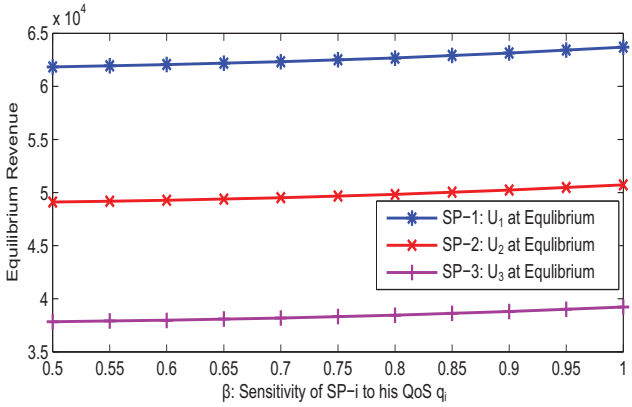


Fig. 10 Equilibrium Revenue of SP-i as a function of $\beta = \beta_1^1 = \beta_2^2 = \beta_3^3$ (Sensitivity of SP-i to his QoS q_i)

VI. Conclusion

In this work, we gave a modeling competition between different SPs who share the same telecommunications market. For this, we have defined the function of demand for service of each SP based on prices and service qualities guaranteed by all SPs. We also defined the function of the utility measuring the profit made by each SP and formulated this problem as a non-cooperative game converging to a Nash equilibrium strategy. We have shown by the results found by implementing a genetic algorithm for learning this equilibrium and clarify the equilibrium strategy of the system. Then, our proposed algorithm finds very fast the equilibrium price and the equilibrium QoS to be chosen by each provider. Our scheme is different from previous approaches since it involves two varying parameters in a simple implementation and low complexity. Yet, we have obtained some insightful results such as the interplay of bandwidth cost. Results found in this work can be further extended to general network considerations, in particular under non-neutrality perspective or non-linear demand.

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