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On the Study of the Electromagnetic Scattering by Large Obstacle Based on the Method of Auxiliary Sources

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Abstract—We consider fast and accurate solutions of scattering problems by large perfectly conducting objects (PEC) formulated by an optimization of the Method of Auxiliary Sources (MAS). We present various techniques used to reduce the total computational cost of the scattering problem. The first technique is based on replacing the object by an array of finite number of small (PEC) object with the same shape. The second solution reduces the problem on considering only the half of the object.These two solutions are compared to results from the reference bibliography.

Keywords—Method of Auxiliary Sources, Scattering, large object, RCS, computational resources.

I. Introduction

THE investigation of scattering problem of an incident field by large objects is a subject of great interest in the study of the electromagnetic phenomena. Thus, the scattering by large object has been an active research topic for many years, due to the complexity of its computation, which needs high computer performance and expensive computational cost. The complexity of such problem leads to the use of efficient numerical methods for computation of the scattered field. Different methods have been used to solve this problem like the finite element method (FEM) [1] in which the geometry is partitioned into smaller sections , the domain decomposition method (DDM) [2] based on the decomposition of structure into many domains, the localized iterative generalized multiple technique (LIGMT) [3] or method of moment (MoM) [4]. However, the solutions proposed by these methods still having significant computation time and memory cost due to the meshing of the structure. So, these methods require more memory space and long computation time. The method of auxiliary sources (MAS) is another alternative to these techniques which has many advantages; being meshless, not needing a complicated discretisation of the domain, being simple to implement and broadly being used to model scattering problems (photonics, metamaterials, arrays, etc.) [5], [6].

MAS is a numerical method suitable for the resolution of electromagnetic scattering problem. The solution is obtained by interchanging the differential equation and boundary conditions. This method is widely applied for scattering problem by small objects (compared with the medium wave length λ) like [9]. The MAS has demonstrated its efficiency on reducing the time-consuming and the complexity of such problem, using a finite number of auxiliary sources chosen to be placed regularly on the auxiliary surface, either filaments for 2-D problems or pairs of elementary dipoles for 3-D problems.

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The convergence of the MAS depends on the choice of the parameters defined in the formulation of this method which are called: Number of auxiliary sources, auxiliary distance and auxiliary surface. It is shown that the choice of the number of auxiliary sources depends on the dimension of the scattering object. These auxiliary sources are represented by coefficients in the linear system of the scattering problem. So, for a large scattering object illuminated by an electromagnetic wave, we require an important number of auxiliary sources, so, the linear system, to be resolved, will have a large matrix size. The inversion of this matrix requires an expansive computation cost.

In the present paper, we propose a solution to simplify the problem by reducing the number of the auxiliary sources. For this, two techniques of solving the problem of scattering by large objects, based upon the method of auxiliary sources (MAS) [7], [8] are presented and discussed. The first idea consists of neglecting the half of the large scattering object in the formulation of the problem. The second one substitutes the large object by a finite number of small objects with the same shape occupying the same volume. The verification of this approximated method of MAS and the validation of the computer written code is made by solving the problem of scattering by an infinite perfectly conducting cylinder with a plane-wave excitation.

The cylindrical structures, more than other structures, offer suitable and efficient model for the study of many practical objects such as trees, human body, antennas and the big building, which can be the case of our present paper. The validity of the present approximation is made by comparison to published results.

II. Method of Auxiliary Sources

Let us consider an infinite cylinder with a surface S, occupied by a perfectly conducting medium. The surface is illuminated by an electromagnetic plane wave U^i (Fig. 1). In this problem, a time factor e^{jwt} has been assumed and suppressed, where $w = 2\pi f$ an f is the frequency of the incident wave. The scattering problem is reduced to the resolution of Helmholtz equation given by:

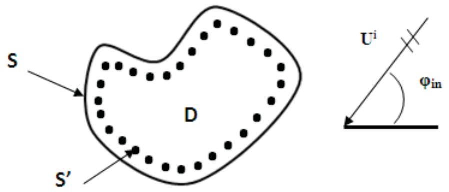

Fig. 1 Geometry of problem

$$\Delta U^s(x,y,z) + k^2 U^s(x,y,z) = 0 \tag{1}$$

Avec k is the wave number in the free space. This equation must satisfy the boundary conditions:

$$W\left\{U^s(x,y,z) + U^i(x,y,z)\right\} = 0, M(x,y,z) \in S \tag{2}$$

where: $U^i(x,y,z)$ is the incident wave; $U^s(x,y,z)$ is the scattered field by the surface S of the structure given in (Fig. 1); W is the boundary condition operator.

The resolution of the scattering problem with the MAS consists on considering an auxiliary surface S', placed in the domain D (Fig. 1)). On this auxiliary surface a finite number of auxiliary sources are placed on the positions $\{r_n\}_1^\infty$. Then, we consider fundamental solution of Helmholtz equation (1):

$$\{U_n(|\vec{r_n} - \vec{r}|)\}_{n=1}^{\infty} \tag{3}$$

This fundamental solution, when projected on surface S, is written as:

$$\{U_n(|\vec{r_n} - \vec{r_s}|)\}_{n=1}^{\infty} = W\{U_n(|\vec{r_n} - \vec{r}|)\}$$

$|\vec{r_n} - \vec{r_s}| = \sqrt{(x_n - x_s)^2 + (y_n - y_s)^2 + (z_n - z_s)^2}$; $M(x_n, y_n, z_n) \in S'$; $M(x_s, y_s, z_s) \in S$

In the case of homogeneous, isotropic structure the fundamental of the Helmholtz equation, for the 2-D problem, U_n is given by this equation:

$$U(|\vec{r_n} - \vec{r}|) = H_0^{(1)}(k(|\vec{r_n} - \vec{r}|)), \tag{4}$$

where H_0^1 are first kinds Hankel function of the order 0.

If the structure is a perfectly conducting one, (2) can be written as:

$$W\left\{\sum_{n=1}^{N} a_n U(|\vec{r_n} - \vec{r}|)\right\}_S = -W\left\{U^i(x,y,z)\right\}_{S'} \tag{5}$$

So, the approximate solution of the boundary outside the domain D is:

$$\tilde{U^s}(x,y,z) = \sum_{n=1}^{N} a_n U_n(|\vec{r_n} - \vec{r}|) \tag{6}$$

The application of the collocation method consists on expressing the boundary condition in a finite number M (r_m^S); $(m = 1, ..., M)$ from the surface S.

For each collocation point M the elementary boundary condition equation is written as:

$$\sum_{n=1}^{N} a_n U_n(|\vec{r_n} - \vec{r_m}|) = -U_m^i; m = 1, ..., M \tag{7}$$

where U_m^i is the incident wave calculated on the collocation point with index m.

Equation (7) gives the following equation:

$$\begin{bmatrix} U_{11} & \cdots & U_{1N} \\ U_{21} & \cdots & U_{2N} \\ \vdots & \ddots & \vdots \\ U_{M1} & \cdots & U_{MN} \end{bmatrix} \times \begin{bmatrix} a_1 \\ \vdots \\ \vdots \\ a_N \end{bmatrix} = - \begin{bmatrix} U_1^i \\ \vdots \\ \vdots \\ U_M^i \end{bmatrix} \tag{8}$$

where $U_{mn} = U_n(|\vec{r_n} - \vec{r_m}|)$.

The resolution of the scattering problem consists on finding the coefficients a_n,$(n = 1, ...n = N)$ by the inversion of the matrix.

III. Formulations and Equations

Assume an infinite z-axis perfectly conducting cylinder (PEC) with arbitrary section, placed in free space. The structure is illuminated by a transverse magnetic (TMz) plane wave with respect to the z-axis. We denote E^{inc} and H^{inc} respectively the electric and the magnetic field, the components of the electromagnetic fields. A Cartesian coordinate system x, y, z is introduced. Under these assumptions, the incident electromagnetic fields are given by [10]:

$$E_z^{inc}(x,y) = E_0 exp\{j(K_0(x\cos\varphi_{inc} + y\sin\varphi_{inc}))\}\hat{z} \tag{9}$$

$$H_z^{inc}(x,y) = -\frac{E_0}{Z_0}(\hat{x}\cos\varphi_{inc} - \hat{y}\sin\varphi_{inc}) \times exp\{j(K_0(x\cos\varphi_{inc} + y\sin\varphi_{inc}))\} \tag{10}$$

where k_0 is the wave number in the free space, $\hat{x}$, $\hat{y}$ and $\hat{z}$ are unit vector respectively in the x, y and z-direction.

According to the MAS fundamental concept [6] for the perfectly conducting cylinder (PEC), a set of auxiliary sources are located inside the scatterer, residing on a fictitious auxiliary surface (a circle of radius a), and surround with circular surface containing M collocation points (CPs).

The two surfaces are separated by a distance d_{aux} named auxiliary distance which will be adjusted in order to find the convergent solution of scattering problem. However for the dielectric cylinder two auxiliary surfaces are placed surrounding the surface of the scatterer. In this case, we suppose that $M = N$.

The boundary condition in the case of perfectly conducting cylinder (PEC) is given by:

$$\hat{n} \times \left(E^{inc} + E^{scat}\right) = 0 \tag{11}$$

where E^{scat} is the total electric scattered field from the cylinder, and E^{inc} is the incident wave.

The solution of this problem is given by the resolution of (8).

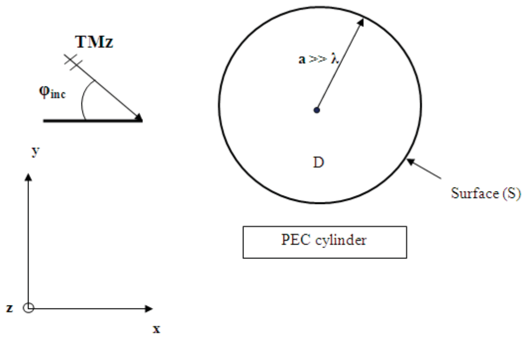

Fig. 2 Large cylinder with $a >> \lambda$

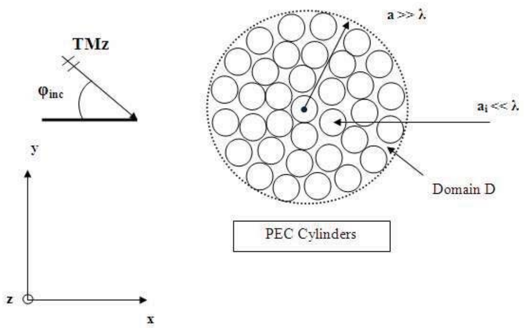

Fig. 3 Array of small cylinders

IV. Optimization of the MAS Solution for Large Cylinders

A. Finite Number of Small Cylinders

The convergence of scattering problem solution depends on the number of auxiliary sources. This number also depends on the width of the scattering object. So, for the large cylinder we need a large number of auxiliary sources.

The scattering of an electromagnetic wave from a PEC cylinder, with a radius very bigger then the wave length λ, has an important number of unknowns in the system of the problem. This requires an important computational cost.

In this section, we introduce an optimization of the method based on substitution of the large cylinder by a finite number of an array of small PEC cylinders. This technique uses the principle of equal volume model.

We consider a PEC cylinder with length and parallel to (Oz) axis. Its radius a is very big compared to the wave length λ. The structure is given by Figs. 2 and 3. The structure is illuminated by a monochromatic plane wave E_{inc} TMz.

We consider an array of Q PEC cylinders. We suppose now that a set of Q perfectly conducting cylinders are regularly distributed in the domain D of the large cylinder PEC as shown in Fig. 2. The formulation of the problem of diffraction by a cylinder radius of perfectly conducting larger then λ by the MAS, is the same compared to the case of a PEC cylinder having a very small radius compared to λ.

The principle of the proposed method is to replace the large cylinder with a set of Q cylinders with very small radius with respect to λ. The problem is reduced to solving the diffraction problem by an array of a finite number of perfectly conducting cylinders.

The results of simulations of the two structures are compared in the following figures. The numerical results (Figs. 4 and 5) obtained by the numerical implementation of the code of MAS are presented to verify the validity and accuracy of the above numerical model. In both examples, all cylinders are perfectly conducting one and illuminated to a plane wave TMZ with a frequency of $300MHz$.

The spatial distribution of scattered energy is characterized by a cross section. Therefore, the comparison of the results is based on values of the scattered power from the object which is characterized by the cross section, for 2D problem, as:

$$SW = \lim_{\rho \to +\infty} \left[2\pi\rho \frac{|E^{sca}|^2}{|E^{inc}|^2} \right] \tag{12}$$

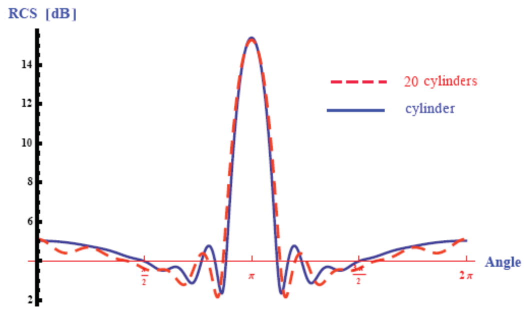

Fig. 4 Radar cross section of a cylinder with radius λ $(-)$ illuminated by TMz with incident angle $\varphi_{inc} = 0$ compared to RCS of 20 cylinders with radius $a_i = 0.1\lambda$

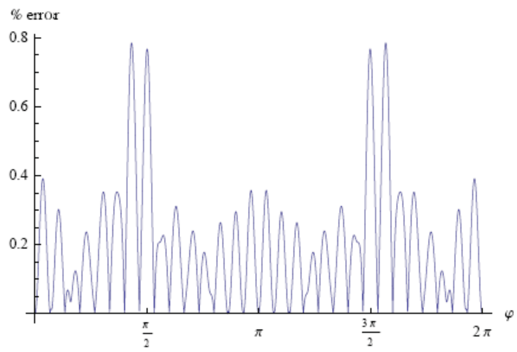

Fig. 5 Error on the boundary of cylinder (radius λ) illuminated by TMz wave (angle $\varphi_{inc} = 0$)

Fig. 4 gives the radar cross section of a circular PEC cylinder with a radius $a = \lambda$ placed in the free space and illuminated by a TMZ plane wave with $\varphi_{inc} = 0$ compared to the radar cross section obtained from an array of 20 homogeneous PEC cylinders. All cylinders have identical radius $a_i = 0.1\lambda$.

The small cylinders ($a_i = 0.1\lambda$) occupy the same volume with the same shape as that of the large cylinder ($a = \lambda$).

The convergence of the solution is checked by the error on the boundary of the large cylinder (Fig. 5).

The solution obtained by the MAS, applied directly to the large cylinder (solid line in Fig. 4), and that obtained by the

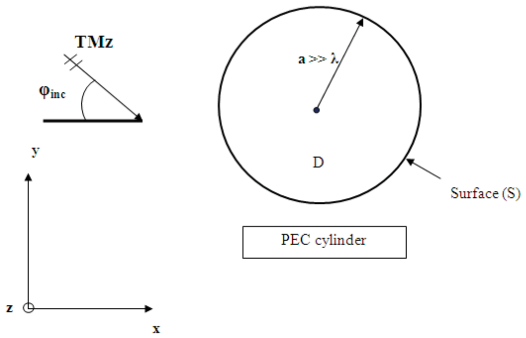

Fig. 6 RCS of a cylinder radius = 9.6λ, $\varphi_{inc} = \pi$ compared to 91 PEC cylinders radius= $a_i = 0.48\lambda$

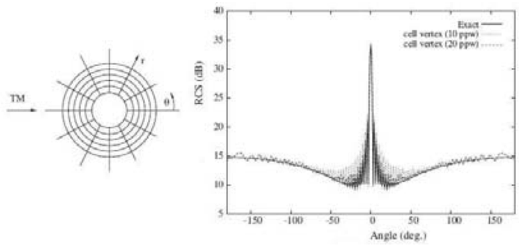

Fig. 7 Normalized RCS of a cylinder $b = 9.6\lambda$ obtained by the Cell-vertex Method based on Method of finite volumes FVTD [11]

MAS applied to the array of small cylinders (dashed line), agree very well, except that there are some differences due to the distance separating these small cylinders.

The second example considers, a hand, a cylinder of radius $a = 9.6\lambda$ illuminated by a TMZ plane wave incidence angle $\varphi_{inc} = \pi$ and also an array of 91 PEC cylinders ($a_i = 0.48\lambda$). The radar cross section of these two structures is given in Fig. 7. The red curve represents the radar cross section of an array of 91 cylinders, while the blue curve represents the radar of the cylinder of radius 9.6λ. This solution is validated by the result obtained in [11].

B. Optimization of the Solution

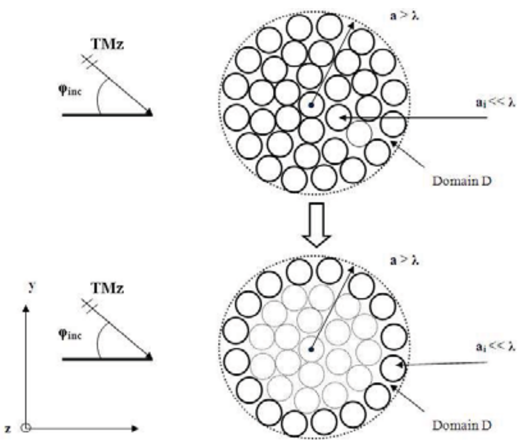

Fig. 8 Optimization of the structure given in Fig. 3

The solution obtained by an array of small cylinders can be optimized by neglecting all the cylinders placed inside the contour given in Fig. 3 when the distance between them is very small compared to the length of wave λ.

The resolution of the scattering problem by an array of cylinders system is reduced to the resolution of given by Fig. 8. For that we consider the example of the 91 cylinders studied in the previous section. We will eliminate the cylinders placed inside the array as shown in Fig. 9.

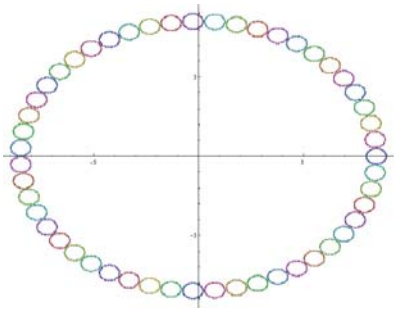

Fig. 9 Optimization of the structure of 91 cylinders by 51 cylinders placed in the boundary

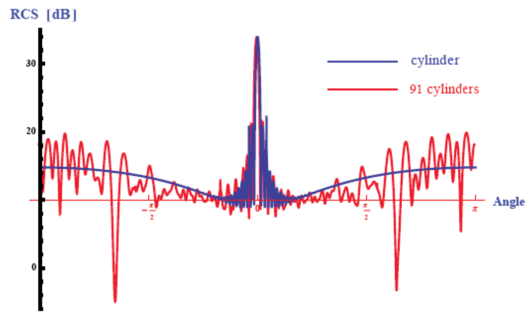

Fig. 10 Radar cross section of a cylinder radius 9.6λ obtained by an array of 51 cylinders illuminated by a plane wave with incidence π

Fig. 10 represents the radar cross section of 51 cylinders placed periodically in the boundary compared to the radar cross section of 91 cylinders. This technique offers an optimization and simplification of the resolution scattering problem by large cylinder because the calculation scattering from the small cylinder does not need an important amount of computational resources.

C. Optimization of the MAS Solution by Minimizing the Surface of Large Cylinder

The electromagnetic scattering fields are obtained when a wave illuminates an obstacle. The incident wave does not affect the surface uniformly in each point of the cylinder. In the case of miniature object, the effect of the incident wave is uniform but when we consider a large cylinder it is not the case. Many experiences are tried that show that the effect of the incident wave is very important on the surface directly exposed to this wave.

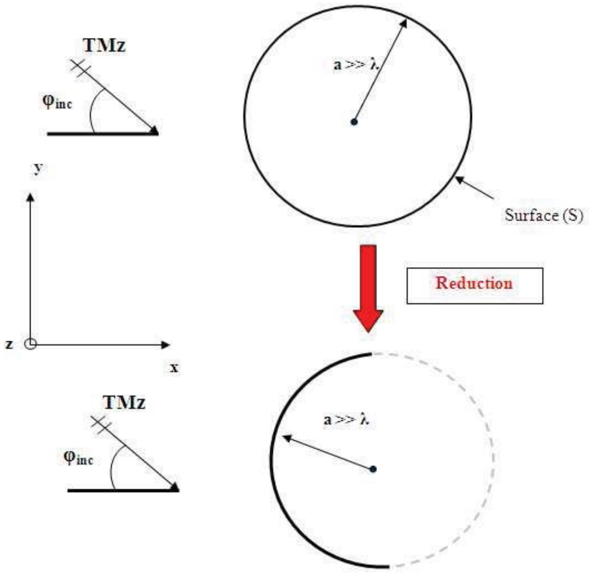

Fig. 11 Surface reduction method

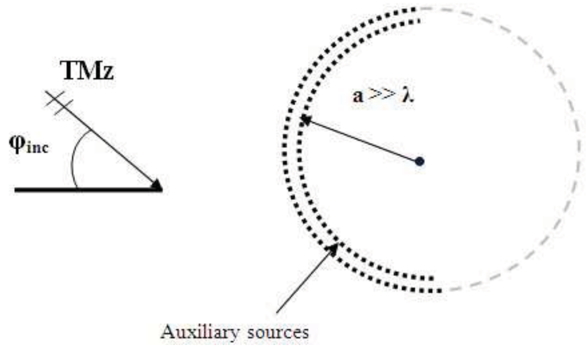

Fig. 12 Bases of auxiliary sources for surface reduction method

The idea in this section is minimize the number of auxiliary sources in the formulation of the scattering problem by large cylinder. For this reason, we propose a technique base on the MAS which consists on applying the standard MAS only on the half of the cylinder's surface which is directly illuminated by the incident plane wave. Indeed, the proposed technique eliminates, in the formulation of the scattering problem, the half of this object when the radius of the object is very higher than the wave length in free space. Fig. 12 shows the idea of this technique.

In the formulation, we consider a PEC cylinder with radius $a >> \lambda$, parallel to (Oz) axis, and illuminated by a TMz plane wave expressed by:

$$E_z^{inc}(x, y) = E_0 exp\{j(k_0(x\cos\varphi_{inc} + y\sin\varphi_{inc}))\}\hat{z} \quad (13)$$

According to the MAS, an auxiliary surface is placed parallel to the physical surface of the cylinder. Fig. 12 shows the repartition of the auxiliary sources.

The boundary conditions applicated of the half of cylinder's surface is written as:

$$\hat{n} \times \left(E^{inc} + E^{scat}\right) = 0 \quad (14)$$

where E^{scat} is the scattered electric field by the half of the surface of the structure.

The scattered field is expressed on a collocation $M(x_m, y_m)$ as:

$$E^{scat}(x_m, y_m) = \sum_{n=1}^{N/2} I_n H_0^{(2)}\left[k_0\sqrt{(x_m - x_n)^2 + (y_m - y_n)^2}\right] \quad (15)$$

The incident field evaluated on the point M is written as:

$$E_m^{inc}(x_m, y_m) = E_0 exp\{j(k_0(x_m\cos\varphi_{inc} + y_m\sin\varphi_{inc}))\}\hat{z} \quad (16)$$

So the boundary conditions can be expressed as:

$$\sum_{n=1}^{N/2} I_n H_0^{(2)}\left[k_0\sqrt{(x_m - x_n)^2 + (y_m - y_n)^2}\right] = -E_0 exp\{j(K_0(x_m\cos\varphi_{inc} + y_m\sin\varphi_{inc}))\} \quad (17)$$

We obtain a linear system with $N/2$ unknowns:

$$\begin{bmatrix} H_0^{(2)}[k_0 r_{11}] & \cdots & H_0^{(2)}[k_0 r_{1(N/2)}] \\ H_0^{(2)}[k_0 r_{21}] & \cdots & H_0^{(2)}[k_0 r_{2(N/2)}] \\ \vdots & \ddots & \vdots \\ H_0^{(2)}[k_0 r_{(M/2)1}] & \cdots & H_0^{(2)}[k_0 r_{(M/2)(N/2)}] \end{bmatrix} \times \begin{bmatrix} I_1 \\ \vdots \\ \vdots \\ I_{N/2} \end{bmatrix} = - \begin{bmatrix} E_1^{inc} \\ \vdots \\ \vdots \\ E_{M/2}^{inc} \end{bmatrix}$$

where E is the incident field vector and I is the unknown current. In order to validate this formulation, we consider a PEC cylinder with radius $a = 9.6\lambda$ illuminated to a plane wave TMz with incident angle π.

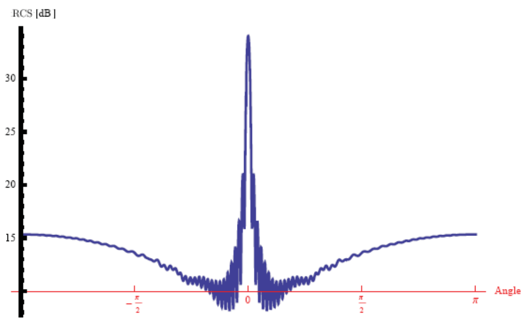

Fig. 13 Normalized RCS of a cylinder $b = 9.6\lambda$ obtained by the MAS with incident angle π

Fig. 13 presents the radar cross section of this cylinder obtained with the MAS applied only to the half of this structure. A good concordance with the result obtained in [11].

The convergence of the solution is obtained with $N/2 = 93$ auxiliary sources. The same solution is represented when we apply the MAS for all the surface of the cylinder with $N = 185$ auxiliary sources. In conclusion, we have reduced the number of auxiliary sources on its half.

V. Conclusion

In this paper, we have developed techniques that simplifies the MAS in the case of the study of scattering problems by large-size objects. Firstly, the structure is substituted by a finite number of small cylinders. Secondly, this array of cylinders is reduced b eliminating the interior one. In the third technique, we propose to apply the MAS method only to the half of the large cylinder in order to solve the scattering problem.

The simplified proposed techniques based on the MAS reduce the computational cost and memory size needed to simulate large-size objects gives a simplified formulation of the standard MAS.

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