

Now we establish the virtual satellite orbital coordinate system: the origin is virtual satellite, and the X-axis points to the flight direction of the virtual satellite in the horizontal plane, and the Z-axis points to the direction of center of the earth and the Y-axis depends upon the right hand system.

When the satellite is running on the near circular orbit before avoid, the dynamic equations of the satellite relative to the virtual satellite can be described by (1).

$$\begin{aligned} \dot{x} &= 2\omega_1 z + a_{cx} \\ \dot{y} &= -\omega_1^2 y + a_{cy} \\ \dot{z} &= -2\omega_1 \dot{x} + 3\omega_1^2 z + a_{cz} \end{aligned} \quad (1)$$

where ω_1 the orbit angular velocity of the virtual satellite; $[a_{cx} \ a_{cy} \ a_{cz}]^T$ the Acceleration of the satellite maneuver.

As we can see from Fig. 1, the satellite maneuvers from the virtual satellite to target-point. Therefore, there are two key points to be addressed in the strategy of satellite avoidance:

- a) The method of avoid maneuver
- b) The best target-point to avoid

There are two maneuver control methods: impulse guidance and continuous thrust guidance. This paper mainly uses the multi-pulse guidance, as noted by the references paper [1]. The acceleration of the satellite maneuver control can be written as:

$$[a_{cx} \ a_{cy} \ a_{cz}]^T = \left[\sum_{i=1}^n v_{ix} \delta(t-t_i) \quad \sum_{i=1}^n v_{iy} \delta(t-t_i) \quad \sum_{i=1}^n v_{iz} \delta(t-t_i) \right]^T \quad (2)$$

Where $\sum_{i=1}^n v_{ix} \delta(t-t_i)$ the velocity pulse of the satellite maneuver; δ Dirac function.

Now we focus on the best target-point to avoid. We'll find these target-points. On these target points, the satellite shares the consistent cycle time and the same semi-major axis with the virtual satellite, which ensures that the properties of the Sun-synchronous orbit remain unchanged. That is to say, the drift of the satellite on the target-point is small enough.

To be in the consistent cycle time with the virtual satellite, the orbital energy of the two orbits should be the same.

$$\frac{1}{2}v^2 - \frac{\mu}{r} = \frac{1}{2}v_0^2 - \frac{\mu}{r_0} \quad (3)$$

and

$$\frac{1}{2}(v^2 - v_0^2) = \frac{\mu}{r} - \frac{\mu}{r_0} \quad (4)$$

where v, v_0, r, r_0 the orbital speed orbit radius of the satellite and virtual satellite; μ gravity constant.

If the satellite is stationary near the X-axis in the virtual satellite orbital coordinate system, then

$$\begin{aligned} \frac{1}{2}(v^2 - v_0^2) &= \frac{1}{2}[(\vec{v}_0 + \vec{\omega} \times \vec{\rho}) \cdot (\vec{v}_0 + \vec{\omega} \times \vec{\rho}) - v_0^2] \\ &\approx -\omega z_0 v_0 + \frac{1}{2} \rho^2 \omega^2 \end{aligned} \quad (5a)$$

$$\begin{aligned} \frac{\mu}{r} - \frac{\mu}{r_0} &= \mu((\vec{r}_0 + \vec{\rho}) \cdot (\vec{r}_0 + \vec{\rho}))^{-1/2} - \frac{\mu}{r_0} \\ &\approx \frac{\mu}{r_0} \left(\frac{z_0}{r_0} - \frac{\rho^2}{2r_0^2} \right) \end{aligned} \quad (5b)$$

The above equations are substituted in (4) and get

$$z_0 = \frac{1}{2} \frac{x_0^2}{r_0} \quad (6)$$

Because of the use of the Taylor expansion, the formula has a certain error. Through simulation, we revised the equation as

$$z_0 = 0.65 \frac{x_0^2}{r_0} \quad (7)$$

Now we can get the target point to avoid in the orbit coordinate system of the virtual satellite:

$$\left(x \quad 0 \quad 0.65 \frac{x^2}{r} \right)$$

The following simulation illustrates the drift of the satellite on the target point in the orbital coordinates of virtual satellite. The flight time is 30000 seconds, with the J4 perturbation model. The initial absolute position and absolute velocity of the virtual satellite:

$$\begin{aligned} R_0 &= [6864.645062 \quad -180.790749 \quad 1199.501510] \text{ km} \\ V_0 &= [-1.315699 \quad -0.998198 \quad 7.379183] \text{ km/s} \end{aligned}$$

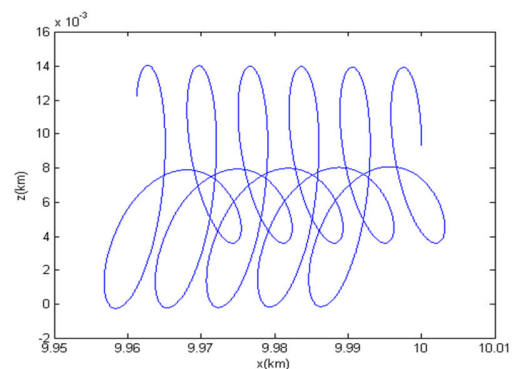


Fig. 2 The drift trajectory of the satellite, at the initial position $[x, y, z] = [10, 0, 0.009]$ km

We see that the drift in the diagram is small enough to ensure that the satellite is stable on the target point. The target point with excellent properties found through simulation serves as a valuable reference for practical application.

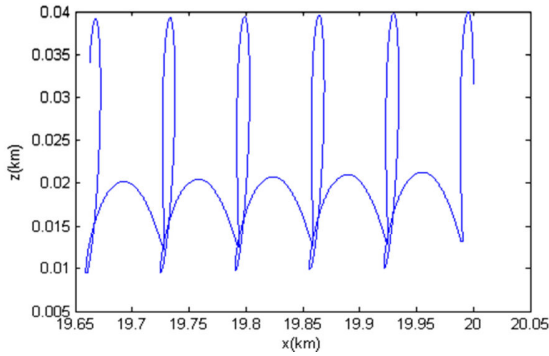


Fig. 3 The drift trajectory of the satellite, at the initial point $[x, y, z] = [20, 0, 0.0316]$ km

III. FURTHER ANALYSIS ON THE STRATEGY OF SATELLITE ORBIT AVOIDANCE

If the satellite encounters space debris, the satellite can avoid with a single maneuver. However, the satellite should be capable of maneuvering several times to avoid continuously in the situation of continuously hostile attacks by space-based laser weapons. This section renders a further analysis on the strategy of the satellite avoidance in order to reinforce the ability of satellite to evade.

Two factors should be reflected upon the strategy of the satellite avoidance: the satellite's maneuver capability and the field view scope of the tracking equipment. On one hand, the maneuver capability of satellite shall be improved, which is mainly dealt with the thruster and maneuver guidance law. On the other hand, maneuver form of the satellite is also considered as the satellite shall escape beyond the range of the field view of the hostile satellite. This relates to maneuver capability as well as maneuver form.

Now we plan the optimal path for satellite avoid. Set the target points as

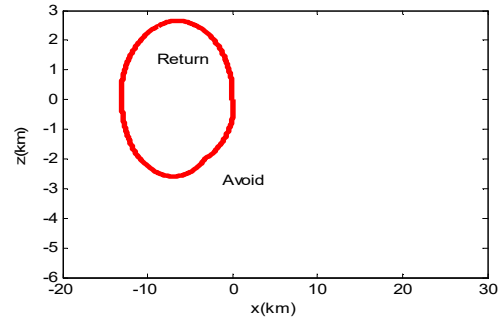
$$\begin{aligned} x_i &= x_{i-1} - a_i \\ y &= 0 \\ z_i &= \frac{0.65 x_i^2}{r_i} \\ i &= 1, 2, \dots, n \end{aligned} \quad (8)$$

The range of the satellite avoid depends on the size of a_i , the direction of the satellite avoid depends on whether a_i is more than zero.

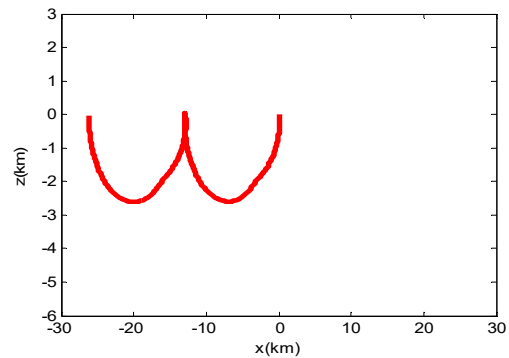
When a_i is more than zero, the satellite avoid to below relative to the virtual satellite. When a_i less than zero, the satellite avoid to ahead relative to the virtual satellite.

When a_i is always more than zero or less than zero, the satellite can be regarded as unidirectional maneuver; When a_i is sometimes more than zero and sometimes less than zero, the satellite can be regarded as bidirectional maneuver. Generally, the satellite takes up unidirectional maneuver. When necessary,

the satellite can operate the bidirectional maneuver which includes avoid and return, and also can be returned to the virtual satellite, as shown in Fig. 4. The origin of the coordinate in the diagram is the virtual satellite.



(a) Bidirectional maneuver

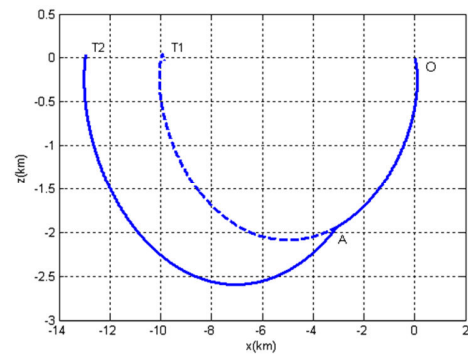


(b) Unidirectional maneuver

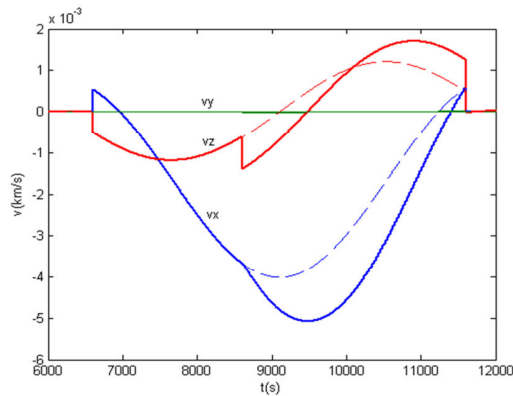
Fig. 4 Sketch of the avoid maneuver

In order to improve the ability of avoidance maneuver, the satellite can also avoid the new target point while maneuvering.

As shown in Fig. 5 (a), if the satellite does not change the target point at A point, the satellite can accurately reach the target point T1 $[-10 \ 0 \ 0.0093]$, which is a common avoidance way. By putting a velocity pulse at the point A, the satellite avoids further away to the point T2 $[-13.05 \ 0 \ 0.0158]$. The ability to avoid has been enhanced apparently.



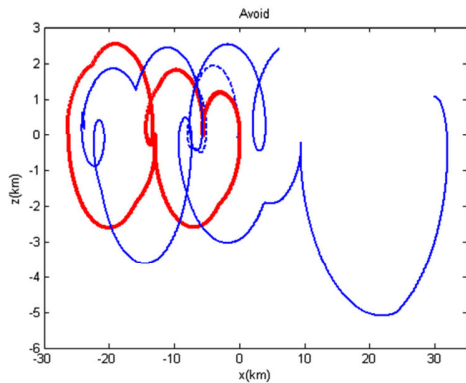
(a) Change target-point in midway



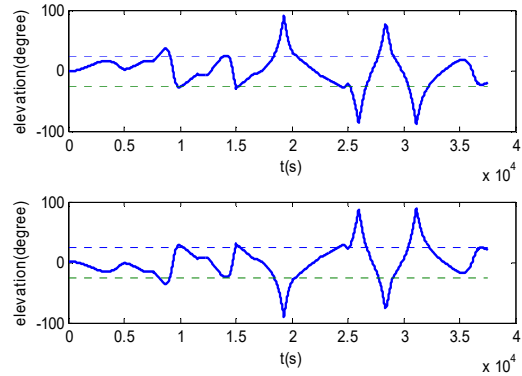
(b) The velocity of the satellite

Fig. 5 Sketch of the guidance of avoid

As shown in Fig. 5 (b), the dotted line is the situation when the satellite escapes to the target point T1, and the velocity increment is 1.468953m/s. The solid line is the situation when the satellite escapes to the target point T2, and the velocity increment is 2.913507m/s. It can be seen that the ability of avoidance maneuver in the second situation is greater than in the first situation. But the second situation costs more fuel. Moreover, it can improve the chances of success for the satellite avoidance if the satellite escapes out the range of the field view of the hostile satellite. As shown in Fig. 6, the satellite encounters the attack of the hostile space-based laser anti-satellite satellite, the satellite takes bidirectional maneuver, first avoid and then return, and at last return to the virtual satellite. In the process of the two-way avoidance maneuver, the elevation angle between the two satellites changes as shown in Fig. 4 (b), it is assumed that field of view (FOV) of the navigation sensor is ± 25 degrees. It can be seen that the elevation angle changes within 25 degrees in the process of unidirectional maneuver, while the elevation angle goes off 25 degrees in the process of bidirectional maneuver which increases the difficulty for the hostile space-based laser anti-satellite satellite tracking the our optical remote sensing satellite. The velocity increment consumption of the remote sensing satellite is 11.6740 (m/s).



(a) Diagram of satellite avoid



(b) The elevation between the two satellites

Fig. 6 Simulation of satellite avoid

Above all, it is probable to modify the ability of the satellite avoidance with bidirectional maneuver to multi-target-points based on the virtual satellite.

IV. CONCLUSIONS

This paper mainly discusses the strategy of the satellite avoidance based on the virtual satellite. The strategy with the characteristic of multi-target-points bidirectional maneuver improves the satellite avoidance ability and also enhances the space security of the on-orbit satellite, especially for the optical remote sensing satellite. On the target points in the avoidance strategy above, the satellites share the consistent cycle time and the same semi-major axis with the virtual satellite, which ensures the properties of the satellite's sun-synchronous orbit remain unchanged. Furthermore, the orbit can be restored back to the virtual satellite through bidirectional maneuver. By adopting strategies discussed above, the remote sensing satellite could get rid of the hostile space-based weapons attack and meanwhile fulfill its normal tasks on orbit.

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