Heat and Mass Transfer of an Oscillating Flow in a Porous Channel with Chemical Reaction

Z. Neffah, H. Kahalerras

Abstract—A numerical study is made in a parallel-plate porous channel subjected to an oscillating flow and an exothermic chemical reaction on its walls. The flow field in the porous region is modeled by the Darcy–Brinkman–Forchheimer model and the finite volume method is used to solve the governing equations. The effects of the modified Frank-Kamenetskii (FK_m) and Damköhler (D_m) numbers, the amplitude of oscillation (A), and the Strouhal number (St) are examined. The main results show an increase of heat and mass transfer rates with A and St, and their decrease with FK_m and D_m.

Keywords—Chemical reaction, heat transfer, mass transfer, oscillating flow, porous channel.

I. INTRODUCTION

CONSIDERABLE attention has been given to the topic of simultaneous heat and mass transfer in reactive porous media due to their wide applications in nature and engineering practice such as catalytic and nuclear reactors, geothermal and petroleum reservoirs, oil delivery and so on. Reference [1] showed a considerable acceleration of the diffusion-controlled reaction due to convective transport in an unsaturated porous medium. Using macroscopic quantities describing multiphase flow with diffusion and chemical reactions in porous media, [2] established macroscopic balance equations for components, momentum and energy. Stability analysis of a first order isothermal and a zero order exothermic reaction of free convection in a porous medium were performed by [3]. Using linear stability analysis, [4] investigated the effect of zero-order exothermic chemical reaction on the onset of convective instability in a horizontal porous layer. Reference [5] considered a theoretical analysis of steady free convection along a vertical surface embedded in a fluid-saturated porous medium, where the flow is driven by a catalytic surface heating. The finite element method has been used by [6] to treat the problem of chemical instability in a fluid-saturated porous medium. Reference [7] developed a non-thermal equilibrium model to study the coupled heat and mass transfer in a porous medium undergoing a strong endothermic chemical reaction. The onset of convection in a horizontal porous layer with an exothermic surface reaction, described by the Arrhenius kinetics, on lower wall has been considered by [8]. Reference [9] carried out numerical analysis of coupled heat and mass transfer in a cylindrical duct filled with a reactive porous medium by considering the existence of a non-

thermal equilibrium between the solid and fluid phases. Reference [10] presented an exact solution for the problem of fully developed forced convection in a porous channel filled with a nanofluid and subjected to a constant heat flux and first order catalytic reaction.

Oscillating flows are frequently encountered in engineering applications, such as pulsed heat exchangers, reciprocating engines, refrigeration systems, etc. Several studies have been devoted to the effect of an oscillating flow through a porous medium as [11] which studied numerically the effect of oscillating inlet boundary conditions for pressure and temperature on the convective flow of a gas through a packed bed. Comparing the heat transfer rates obtained by pulsating and non-pulsating flows in a porous channel, [12] found an augmentation of the difference by increasing the amplitude of pulsation and by decreasing the frequency. Reference [13] found that the simultaneous use of porous medium and pulsating flow can lead to a significant augmentation of heat transfer and a reduction of pressure drop. Heat transfer in a channel filled with a porous medium and subjected to an oscillating flow was investigated experimentally by [14]. By performing experiments in metal foam heat sinks, [15] found that the higher heat transfer rates are obtained for low pore density and for oscillating flow. An experimental study of heat transfer in oscillating water flow through a porous media of steel balls was conducted by [16]. A general Nusselt number correlation independent of the fluid properties and the type of porous medium has been established. Reference [17] carried out numerical simulations to analyze the effect of simultaneous use of porous baffles and pulsating flows on a double pipe heat exchanger performance.

The present work is a contribution to the works cited above and its main objective is to analyze heat and mass transfer by adding an oscillating component to the mean flow when an exothermic and non-isothermal chemical reaction, governed by Arrhenius kinetics, takes place on the channel walls. This study can find application in the field of catalytic reactors.

II. MATHEMATICAL FORMULATION

The physical model (Fig. 1) consists of a two-dimensional parallel-plate channel filled with a porous medium of thickness e_p and porosity ε. The fluid enters the channel with a constant temperature T_i, a uniform species concentration C_i and an oscillating velocity $U_i = U_0(1+A sin2\pi ft)$, where A and f are the amplitude and frequency of oscillation respectively. An exothermic surface reaction is taking place on the walls, and can be represented by a single first-order, non-isothermal and one-step reaction governed by Arrhenius kinetics.

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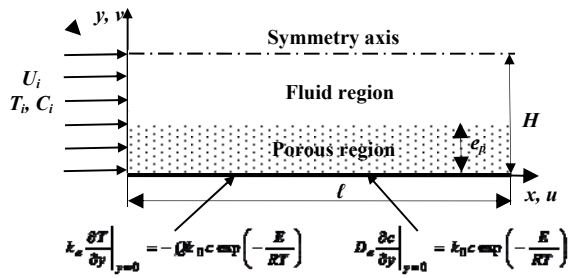International Journal of Aerospace and Mechanical Engineering
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Fig. 1 Physical domain

The forced convection flow is unsteady, two-dimensional, laminar and incompressible with no heat generation and neglecting viscous dissipation. The thermophysical properties of the fluid are assumed to be constant, and the porous medium is considered homogeneous, isotropic and saturated with a single phase fluid which is in local equilibrium with the solid matrix.

Under the above assumptions, and adopting the Brinkman-Forchheimer extended Darcy model [18] in the porous medium, the governing equations can be written as:

Continuity:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial X} = 0 \tag{1}$$

Momentum:

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{R_\mu}{Re}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \frac{\varepsilon}{Re\,Da}U \\ -\frac{\varepsilon^2 C_F}{\sqrt{Da}}\left|\vec{V}\right|U \tag{2}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{R_\mu}{Re}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - \frac{\varepsilon}{Re\,Da}V \\ -\frac{\varepsilon^2 C_F}{\sqrt{Da}}\left|\vec{V}\right|V \tag{3}$$

Energy:

$$R_{Cp}\frac{\partial \theta}{\partial \tau} + U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{R_k}{\varepsilon\,Pr\,Re}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

Concentration:

$$\frac{\partial C}{\partial \tau} + U\frac{\partial C}{\partial X} + V\frac{\partial C}{\partial Y} = \frac{1}{Le\,Pr\,Re}\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2}\right) \tag{5}$$

where U and V are the dimensionless velocity components in the axial (X) and transverse (Y) directions, $\left|\vec{V}\right| = \sqrt{U^2 + V^2}$, P is the dimensionless pressure, C_F the inertia coefficient, θ the dimensionless temperature and C the dimensionless concentration. In non-porous region: $\varepsilon = 1$, $R_k = 1$ and $Da \to \infty$.

The associated boundary conditions are:

At the inlet:

$$U = 1 + A\,sin2\pi St\tau,\ V = 0,\ \theta = 0,\ C = 1 \tag{6}$$

At the exit:

$$\frac{\partial U}{\partial X} = 0,\ V = 0,\ \frac{\partial \theta}{\partial X} = 0,\ \frac{\partial C}{\partial X} = 0 \tag{7}$$

On the axis:

$$\frac{\partial U}{\partial Y} = 0,\ V = 0,\ \frac{\partial \theta}{\partial Y} = 0,\ \frac{\partial C}{\partial Y} = 0 \tag{8}$$

At the wall:

$$U = V = 0,\ \left.\frac{\partial \theta}{\partial Y}\right|_{Y=0} = -FK_m\,C\exp\left(\frac{\theta}{\gamma\theta+1}\right),\ \left.\frac{\partial C}{\partial Y}\right|_{Y=0} = D_m\,C\exp\left(\frac{\theta}{\gamma\theta+1}\right) \tag{9}$$

At the porous-nanofluid interfaces: continuity of pressure, velocity components, temperature, concentration, and heat fluxes.

The following variables were used to convert the governing equations and boundary conditions to dimensionless form:

$$X = \frac{x}{H}; Y = \frac{y}{H}; U = \frac{u}{\varepsilon U_0}; V = \frac{v}{\varepsilon U_0}; P = \frac{p}{\rho U_0^2}; \tau = \frac{t}{H/U_0}; \theta = \frac{T - T_i}{RT_i^2/E}; C = \frac{c}{C_i}$$

The dimensionless parameters appearing in the above equations are defined as:

$$Re = \frac{H\rho U_à}{\mu},\ R_\mu = \frac{\mu_e}{\mu},\ Da = \frac{K}{H^2}, R_{Cp} = \frac{(\rho C_p)_e}{\rho C_p},\ R_k = \frac{k_e}{k},\ Pr = \frac{\mu C_p}{k}$$

$$Le = \frac{k}{\rho C_p D}, St = \frac{f H}{U_i},\ FK_m = \frac{Qk_0 C_i H}{k_e}\frac{E}{RT_i^2}\exp\left(-\frac{1}{\gamma}\right)$$

$$D_m = \frac{k_0 H}{D_e}\exp\left(-\frac{1}{\gamma}\right), \gamma = \frac{RT_i}{E}$$

where ρ is the density, μ is the dynamic viscosity, K is the permeability of the porous medium, C_p is the specific heat at constant pressure, k is the thermal conductivity, Re is the Reynolds number, Da is the Darcy number, Pr is the Prandtl number, Le is the Lewis number, St is the Strouhal number, FK_m is the modified Frank-Kamenetskii number, D_m is the Damköhler number and γ is the activation energy parameter. The subscript e indicates the parameters relative to porous medium (effective).

The local Nusselt and Sherwood numbers along the wall of the channel are evaluated as:

$$Nu = -R_k\frac{\left.\frac{\partial \theta}{\partial Y}\right|_{Y=0}}{\theta_w - \theta_m}\quad and\quad Sh = -\varepsilon\frac{\left.\frac{\partial C}{\partial Y}\right|_{Y=0}}{C_w - C_m} \tag{10}$$

The mean temperature θ_m and mean concentration C_m are calculated as follows:

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$$\theta_m = \frac{\int_0^1 U\,\theta\,dY}{\int_0^1 U\,dY} \quad and \quad C_m = \frac{\int_0^1 U\,C\,dY}{\int_0^1 U\,dY} \qquad (11)$$

The average Nusselt and Sherwood numbers are defined as:

$$Nu_m = \frac{1}{L}\int_0^L Nu\,dX \quad and \quad Sh_m = \frac{1}{L}\int_0^L Sh\,dX \qquad (12)$$

For an oscillating flow, the cycle averaged quantities for a period of oscillation $\tau^* = 1/St$ are defined as:

$$<\Phi> = St \int_{\tau_0}^{\tau_0 + 1/St} \Phi\,d\tau \quad where \quad \Phi = \theta_w, C_w, \theta_m, C_m, Nu_m \ or \ Sh_m \qquad (13)$$

III. NUMERICAL PROCEDURE

The finite volume method [19] is used to solve the governing equations using a pressure-velocity formulation. The power law scheme is employed in the discretizing procedure to treat the convection and diffusion terms. As a time integration method, the first-order fully implicit scheme is adopted. The obtained algebraic equations are solved using the line by line technique, combining between the tridiagonal matrix algorithm and the Gauss-Seidel method. A non-uniform grid (250×50 in X and Y directions, respectively) is employed in the transverse direction by locating the finer meshes near the axis of symmetry, the porous-fluid interface and the lower wall of the channel. For the convergence criteria of the iterative process, the relative variations of velocity components, temperature, and concentration between two successive iterations are required to be smaller than 10^{-6}. One period of oscillation is divided into 120 time steps such that the time resolution is equal to $\Delta\tau = \tau^*/120 = 1/120St$. The steady periodic solutions are obtained after approximately 10 cycles of oscillation.

IV. RESULTS

The numerical calculations are performed for air ($Pr = 0.7$), a porosity $\varepsilon = 0.95$, an inertia coefficient $C_F = 0.1$, a Darcy number $Da = 10^{-3}$, a Reynolds number $Re = 100$, a Lewis number $Le = 1$, an activation energy parameter $\gamma = 0.1$, a viscosity ratio $R_\mu = 1$, a thermal conductivity ratio $R_k = 1$ and a heat capacity ratio $R_{Cp} = 1$. The effects of the characteristics of the chemical reaction ($0.1 \le FK_m \le 20$ and $5 \le D_m \le 80$) and oscillating flow (amplitude: $0 \le A \le 0.9$ and frequency: $0.1 \le St \le 2$) are analyzed. The results averaged over a cycle of oscillation in the periodic steady state, are presented in terms of temperature and concentration distributions, and in terms of heat and mass transfer coefficients described by the Nusselt and Sherwood numbers respectively.

The impact of the first parameter characterizing the chemical reaction, i.e. the modified Frank-Kamenetskii number, on the cycle averaged mean temperature and concentration at different sections of the channel is illustrated

on Fig. 2. It can be noticed an increase of $<\theta_m>$ throughout the porous channel and with FK_m. Indeed, more this parameter is important; greater is the release of heat due to the exothermic nature of reaction, causing the increase of the medium temperature. Regarding the mean concentration $<C_m>$, it has an opposite trend than that of mean temperature where it decreases along the channel and with the increase of the modified Frank-Kamenetskii number whose effect is less pronounced in comparison to what has been found for $<\theta_m>$ because this parameter is present in the temperature boundary condition.
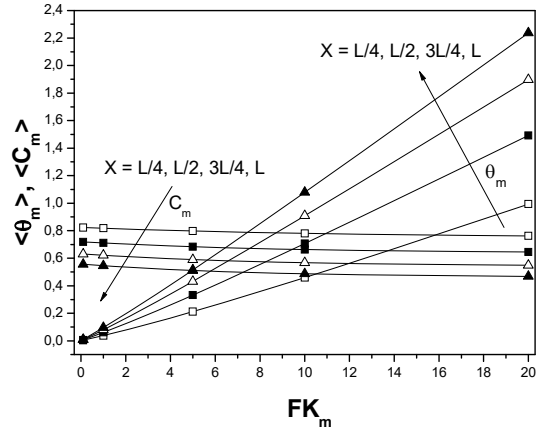

Fig. 2 Variation of $<\theta_m>$ and $<C_m>$ with FK_m

Fig. 3 depicts the influence of the chemical reaction through the modified Frank-Kamenetskii number on the cycle averaged wall temperature and concentration. As the reaction occurs on the surface of the channel walls and gives rise to heat generation, its temperature increases significantly with FK_m and in parallel the parietal concentration decreases quickly. Indeed, at high values of this parameter, the majority of the reactants are transformed into products and so concentration decreases towards zero.
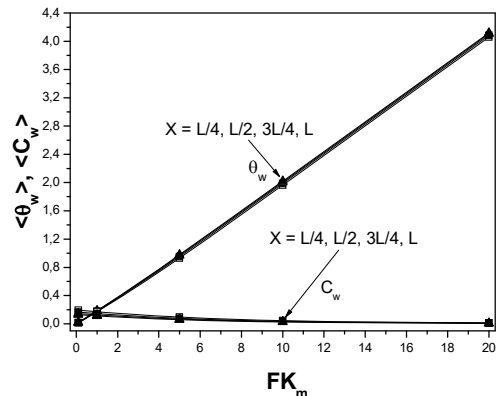

Fig. 3 Variation of $<\theta_w>$ and $<C_w>$ with FK_m

The evolution of the cycle averaged mean Nusselt and Sherwood numbers with FK_m are presented on Fig. 4 (a) which shows that the exothermic nature of the chemical reaction is detrimental to heat and mass transfer as $<Nu_m>$ and

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$<Sh_m>$ decrease with the modified Frank-Kamenetskii number. It is worth noting that the evolution of the two transfer coefficients are similar due to the taken value of the Lewis number ($Le = 1$).

(a)

(b)
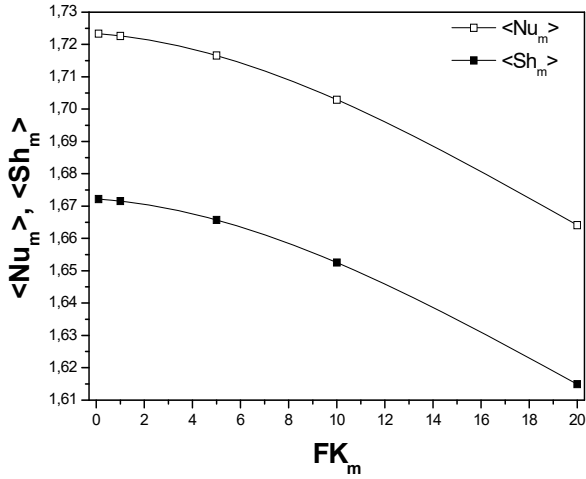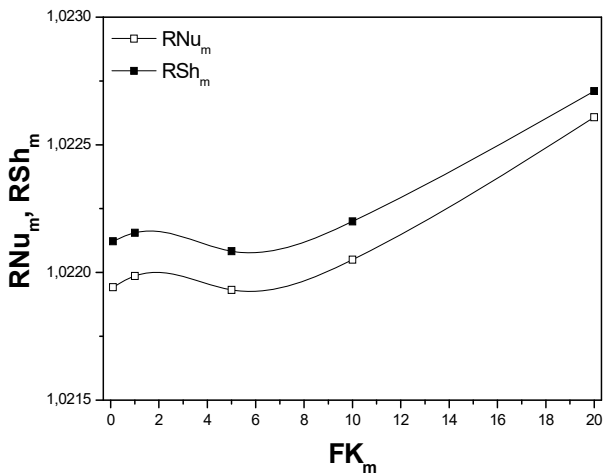
Fig. 4 $<Nu_m>$, $<Sh_m>$ (a) and RNu_m, RSh_m (b) versus FK_m

To show the impact of the oscillating flow on heat and mass transfer, we present on Fig. 4 (b) the variation of the ratios RNu_m and RSh_m, normalized by the steady non-oscillating values of mean Nusselt and Sherwood numbers, with FK_m. The addition of an oscillating component to the mean flow seems to be beneficial since these ratios are greater to the unity and vary very slightly with the modified Frank-Kamenetskii number. The mean enhancement rate is around 2.2%.

The effect of the chemical reactivity, expressed by the modified Damköhler number, on the mean temperature and concentration is illustrated on Fig. 5 where this factor influences considerably $<\theta_m>$ but moderately $<C_m>$. As the Damköhler number represents the ratio between the time scales of diffusion and reaction; at high values of D_m the reaction gets faster and diffusion becomes dominant resulting

in a reduction of the mean temperature and concentration as well as the parietal values $<\theta_w>$ and $<C_w>$ (Fig. 6) where the effect of this parameter is more apparent since the reaction occurs on the channel walls.

Fig. 7 shows that heat and mass transfer rates decrease with the modified Damköhler number and that the enhancement rate by adding an oscillating component is almost independent of D_m and is around 2.2%.

The second part of the results is devoted to the analysis of the effect of the oscillating flow that we began by the study of the influence of the amplitude. The variation of the cycle averaged mean Nusselt number $<Nu_m>$ and mean Sherwood number $<Sh_m>$, in the periodically established regime, with the amplitude of oscillation is illustrated in Fig. 8. On one hand, there is an increase of heat and mass transfer rates with the augmentation of A, and on the other hand, this figure reveals an enhancement in comparison to the oscillating flow where the highest ratios RNu_m and RSh_m are obtained at $A=0.9$ and reach the value of approximately 5%. Due to the chosen value of Lewis number ($Le = 1$), the curves of these ratios are confounded. Another parameter to consider in the analysis of an oscillating flow is the Strouhal number, whose variation refers that of the frequency. Fig. 9 shows that there is a rise of heat and mass transfer by increasing St. The comparison with the non-oscillating case reveals that the enhancement achieved by addition of an oscillating component to the mean flow is perceived beyond $St \approx 0.15$. The highest rate of enhancement is obtained at large values of the frequency of oscillation and is around 6.5%. The improvement obtained with A and St can be explained by the fact that the oscillating aspect of the flow allows a better mixing of the fluid.
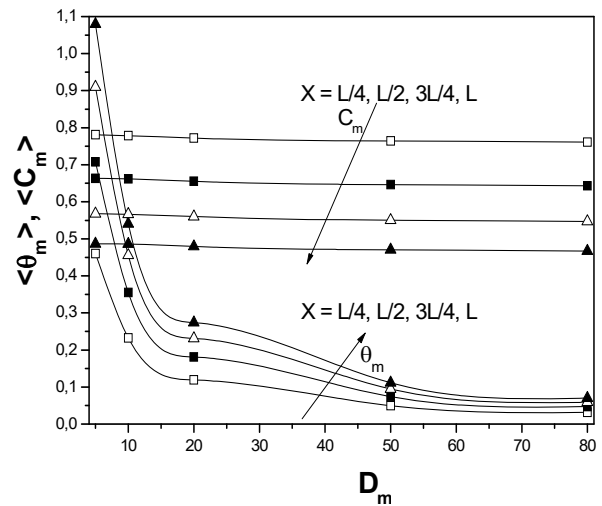

Fig. 5 Variation of $<\theta_m>$ and $<C_m>$ with D_m
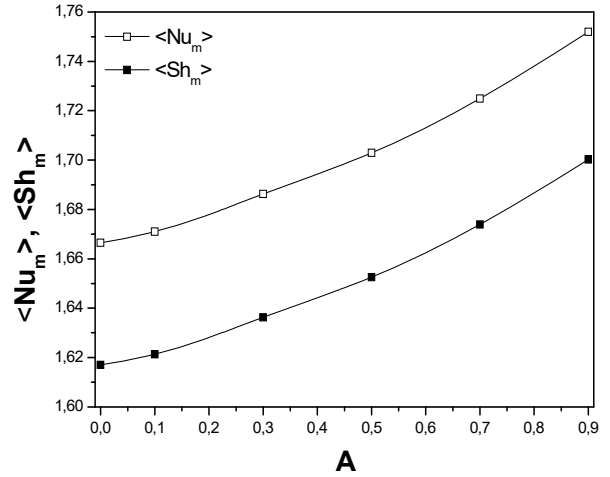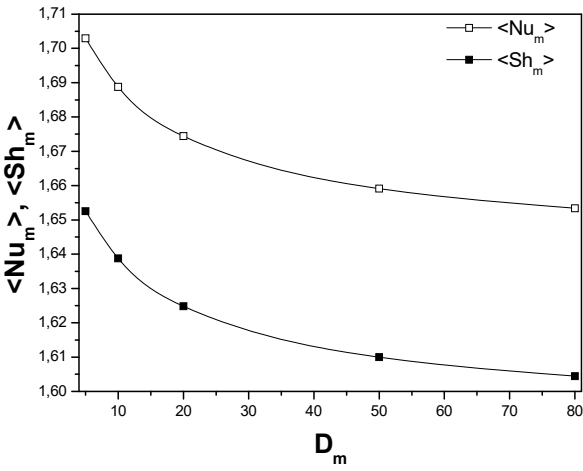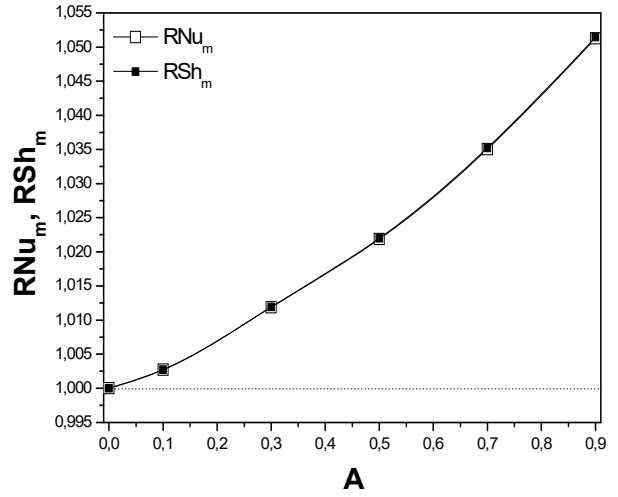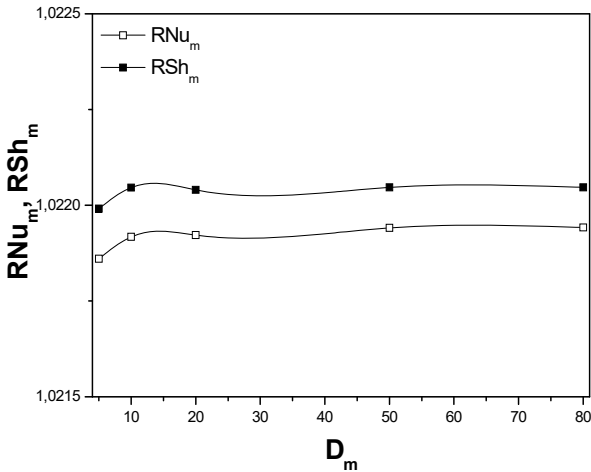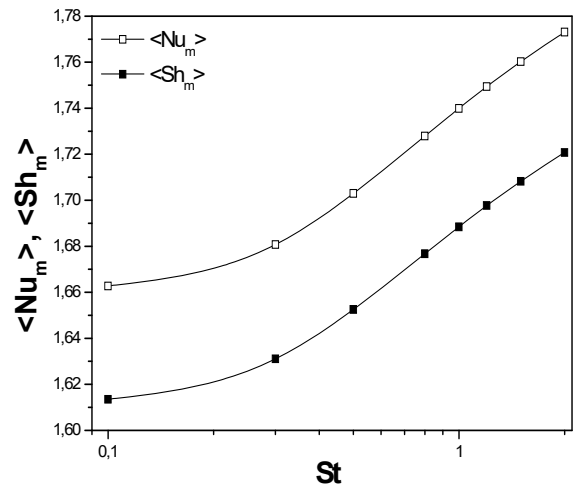
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Fig. 6 Variation of $<\theta_w>$ and $<C_w>$ with D_m

(a)

(b)

Fig. 7 $<Nu_m>$, $<Sh_m>$ (a) and RNu_m, RSh_m (b) versus D_m

(a)

(b)

Fig. 8 $<Nu_m>$, $<Sh_m>$ (a) and RNu_m, RSh_m (b) versus A

(a)

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(b)
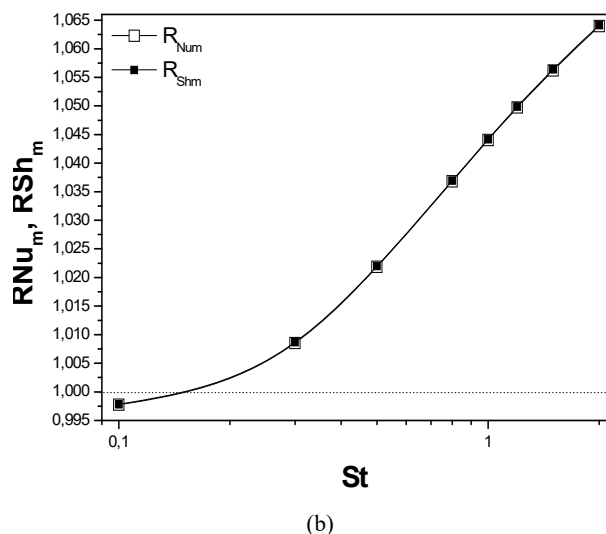
Fig. 9 $<Nu_m>$, $<Sh_m>$ (a) and RNu_m, RSh_m (b) versus St

V. CONCLUSION

Heat and Mass transfer of an oscillating flow with surface chemical reaction in a porous channel are studied numerically. The effect of various parameters related to the chemical reaction (modified Frank-Kamenetsii number FK_m and modified Damköhler number D_m) and oscillating flow (amplitude A and frequency St) on mean and wall temperature and concentration as well as mean Nusselt and Sherwood numbers is analyzed. The mean and wall temperatures and concentrations are strongly affected by the exothermic nature of the chemical reaction leading to a decrease of heat and mass transfer rates with the augmentation of the modified Frank-Kamenetskii and Damköhler numbers. The results show also that the addition of an oscillating component to the mean flow is beneficial since the ratios RNu_m and RSh_m are globally greater than the unity, increase with the amplitude and frequency of oscillation and are almost independent of the chemical reaction parameters.

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