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Characterization of InGaAsP/InP Quantum Well Lasers

K. Melouk, M. Dellakrachai

***Abstract*—**Analytical formula for the optical gain based on a simple parabolic-band by introducing theoretical expressions for the quantized energy is presented. The model used in this treatment take into account the effects of intraband relaxation. It is shown, as a result, that the gain for the TE mode is larger than that for TM mode and the presence of acceptor impurity increase the peak gain.

***Keywords*—**Laser, quantum well, semiconductor, InGaAsP.

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I. Introduction

THE quaternary alloy $In_{1-x}Ga_xAs_yP_{1-y}$ has recently received considerable attention as a possible material for the fabrication of heterojunction lasers, light-emitting diodes, and avalanche photodiodes in the near infrared region of the spectrum [1]. In order to satisfy the bandgap and lattice constant requirements simultaneously, several recent efforts have concentrated on quaternary crystalline solid solutions. Among them, an $In_{1-x}Ga_xAs_yP_{1-y}$ /InP lattice matched heterostructure has been studied extensively because it can cover the 1.0-1.6μm wavelength region and it has become a promising candidate for the light source of an optical fiber communication system where transmission losses and dispersions of optical fibers are both low in this region [1]-[3]. Quantum-well laser is expected as an excellent device succeeding to the double hetero (DH) laser is to have low threshold current level and stable characteristic for temperature variation, because it could have limited energy levels and narrow gain spectrum. Multi-quantum well (MQW) laser consists with several well layers gives large cross-section to transfer energy of electrons to optical field. Although a large number of investigations has been done up to now by means of experiment about the quantum-well laser, theoretical criterion for performing characteristic is not sufficient [4]-[8].

Energy levels in the multi-well structure are broadened due to coupling effect of the electron wave among the wells. Lasing gain in $In_{1-x}Ga_xAs_yP_{1-y}$ /InP single-quantum-well (SQW) are theoretically analyzed in this paper. It is calculated by taking into account broadening effect of the intraband relaxation of electron wave. Moreover, we derive analytical formula of the optical gain; the analysis is based on a simple parabolic-band gain model [1]-[3]. First we obtain analytical expressions for the optical transition wavelength between the quantized energy levels of electrons and holes in quantum-wells and the dipole moment. Then, using explicit approximations, we derive analytical expressions for the Fermi functions and then, the optical gain. As a numerical example, we calculate the linear gain of $In_{0.53}Ga_{0.47}As$/InP SQW laser and the difference between TE and TM modes taking into account the effects of intraband relaxation and doping. Finally, conclusions are given in the last section.

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II. Approximate Quantized Energy Levels in Multi-Quantum-Well Structure

A carrier (electron or hole) in a DH laser is confined in a three-dimensional potential well. The energy levels of such carriers are obtained by separating the Hamiltonien into three parts, corresponding to kinetic energies in the x-, y-, and z-directions each of which form a continuum of states. When the thickness of the active region becomes comparable to the Broglie wavelength (λ~h/P), the kinetic energy corresponding to the particle motion along the y-direction is quantized. The energy levels can be obtained by separating the Hamiltonien into energies corresponding to x-, y-, z-directions.

System of coordinates in this analysis is shown in Fig. 1; x in the direction along the width, y is in thickness direction perpendicular to the junction plane and z is in longitudinal direction of the laser cavity. The wells are made of $In_{1-x}Ga_xAs_yP_{1-y}$. Number of wells is M and width La. The barriers are made of InP whose width is L_B.The bands gaps of InGaAsP and InP are Eg and Eg_B, respectively.

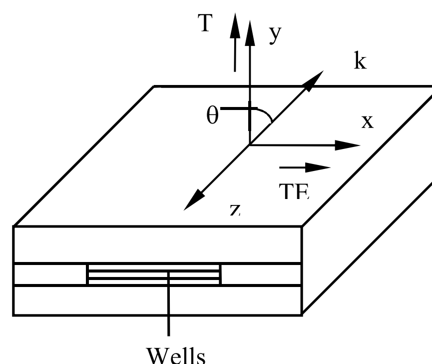

Fig. 1 The coordinate system in the analysis

The electron wave function in the semiconductor material is represented with the Bloch function

$$\rho_I(r) = A_I \exp[jkr]U_I(k,r) + B_I \exp[-jkr]U_I(-k,r) \quad (1)$$

where suffix I represents the conduction, the heavy hole and the light hole bands, A_I and B_I are arbitrary coefficients, k is the wavevector, and $U_I(k,r)$ is the function indicating distribution of the electron wave around a lattice point.

The energy states in quantum well structure are analyzed applying the so-called effective mass approximation. Supposing U_I as potential height, wave function $\rho_I(y)$ analysed by the following Schroidinger equation:

$$\frac{1}{m_I}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\rho_I + U_I\rho_I = E_I\rho_I \quad (2)$$

where m_I is the mass of the conduction electron or the heavy hole or the light hole.

The solution of (2) is given in general by (1) and the energy states E_I are given with components of the kinetic energy of an electron or hole along three spatial directions. Kinetic energies along x and z directions have continuous values as in the bulk material since no boundaries exist in these directions. However, the energy along the y direction reveals discrete values.

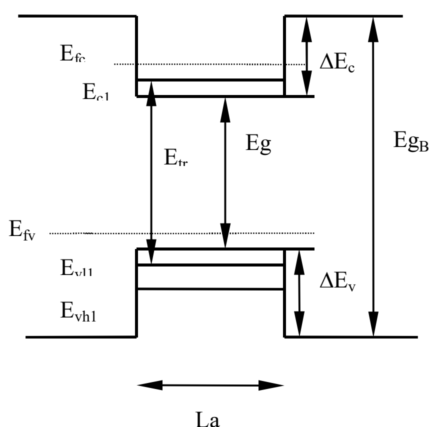

Fig. 2 Band model for a quantum well structure

A schematic energy band diagram representing the quantized levels is depicted in Fig. 2. The discontinuities of the band edges of conduction and valence bands at heterojonction are ΔE_c and ΔE_v respectively, E_{cn} and E_{vn} are the quantized energy levels in the conduction and valence band, respectively, and Etr is the transition energy between the quantized energy levels. The quasi-Fermi levels for electrons and holes in the well are Efc and Efv. We assume the origin of the energy levels is located at the bottom of the conduction band well, expected for the quantized levels for holes (E_{vn}) that are measured down-ward from the top of the valence band.

Using the parabolic band model [4]-[9] E_{In} is given by

$$E_{In}=\frac{\hbar^2\pi^2 n^2}{2m_I La^2} \quad (4)$$

where $\hbar$=h/2π and h is Planck's constant, La is the well width. The wavelength corresponding to a transition between the subbands in conduction and valence bands i.e., between the quantized levels, can be expressed as

$$\lambda_n=\frac{1.24}{Eg+E_{cn}+E_{vn}} \quad (5)$$

Calculation result of this wavelength for $In_{0.53}Ga_{0.47}As$ /InP for the first transition (n=1) is shown in Fig. 3 as a function of the well width. Values of the parameters used in this calculation were quoted from [1]-[3].

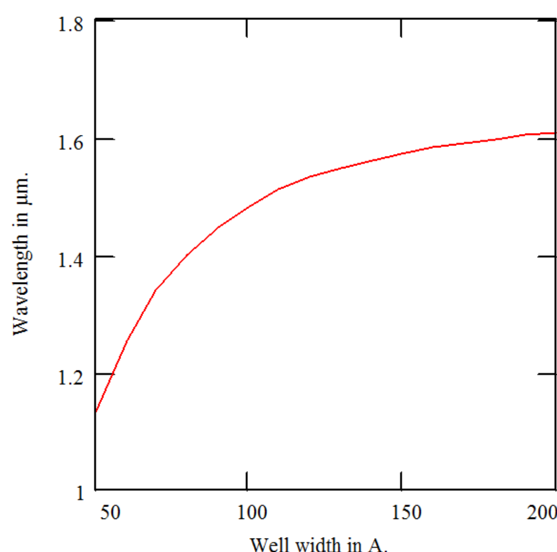

Fig. 3 Wavelength corresponding to the first transition between the quantized energy levels of conduction electrons and heavy holes as a function of well width calculated for $In_{0.53}Ga_{0.47}As$/InP

We define an energy component of a dipole moment vibrating along the y direction with E_{yn}, which consists of an electron and hole pair [9];

$$E_{yn}=E_{cn}+E_{vn} \quad (6)$$

Adding energy components $E_{x,y}$ along the x and z directions and the band gap energy Eg, the total energy of dipole is [7]

$$E_{cv}=Eg+E_{x,y}+E_{yn} \quad (7)$$

The dipole moment is represented as an inner product of the envelope functions and the term with the fine function between a conduction electron and a heavy hole as [4]-[9]:

$$R_{cJ}=\int \rho_c^{\ *}\rho_v^{\ *}\langle u_c(k,r)|er|u_J(k,r)\rangle. \quad (8)$$

where J is replaced by H or L for the heavy hole or the light hole respectively. Applying the so-called k.P perturbation approach developed by Kane, the dipole moment in quantum well structure is written as [4]-[9].

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$$R^2_{CH} = \begin{cases} (3/4)R_0^2(1+\cos^2\theta) & TE \text{ mode } (E \perp y) \\ (3/2)R_0^2 \sin^2\theta & TM \text{ mode } (E // y) \end{cases} \quad (9)$$

$$R^2_{CL} = 2R_0^2 - R^2_{CH} \quad (10)$$

where, θ is angle between the k-vector and the y-axis given with the energy components as

$$\cos^2\theta = \frac{E_{yn}}{E_{x,y} + E_{yn}} \quad (11)$$

R0 the value of the dipole moment in the bulk material which is given by [4]-[9],

$$|R_0|^2 = \frac{\hbar^2 e^2}{6 m_0 E_{cv}{}^2}\left(\frac{m_0}{m_c} - 1\right)\frac{Eg + (Eg + \Delta)}{Eg + (2/3)\Delta} \quad (12)$$

where Δ is the split-off energy in the valence band, m_c is mass of the conduction electron and m_0 the mass of a free electron.

III. Approximate Optical Gain

The optical gain of Asada et al. [9] is taken as a reference, where we assume that all subbands are parabolic and that optical transitions obey the k-selection rules. Taking into account the intraband relaxation, the optical gain for a multi-quantum well is expressed as [4]-[9]

$$g(\omega) = \omega\sqrt{\mu/\varepsilon}\,\frac{mr}{\pi\hbar^2 LaM}\cdot\sum_{n=1}^{4}\int_{Eg+E_{cn}+E_{vn}}^{\infty} \rangle R^2_{CH}\langle (fc - fv) F_\tau(E_{cv}) dE_{cv} \quad (13)$$

where ω is the angular frequency of light, μ the permeability, ε the dielectric constant, mr the reduced effective mass given by $\frac{m_c m_H}{m_c + m_H}$ with m_H the mass of the heavy hole. La is the well width, M is the number of well. In (13), the light whole band is neglected, and only the transition between the electrons and heavy holes are considered, since the density of states of this band is much smaller than that of the heavy hole band. $F_\tau(E_{cv})$ is a function expressing the transition broadening. Asada et al. [8], [9] used the Lorentzian function based on the density matrix formalism as;

$$F_\tau(E_{cv}) = \frac{\hbar/\tau_{in}}{(E_{cv} - \hbar\omega)^2 + (\hbar/\tau_{in})^2} \quad (14)$$

where τ_{in} is the intraband relaxation time, and fc and fv are the Fermi functions given by

$$fc = \frac{1}{1 + \exp\left(\frac{\varepsilon_{cn} - Efc}{kT}\right)} \quad (15\text{-a})$$

$$fv = \frac{1}{1 + \exp\left(\frac{\varepsilon_{vn} - Efv}{kT}\right)} \quad (15\text{-b})$$

where ε_{cn} and ε_{vn} are the total energies of electrons and holes for subbands n, k the Boltzman constant, and T the absolute temperature.

Efc and Efv are related to the densities of electrons and holes injected N and P, respectively, into wells

$$N \cong \left(\frac{m_c kT}{\pi\hbar^2 LaM}\right)\sum_n \int_{E_{cn}}^{\infty} fc\, d\varepsilon_{cn} = \frac{m_c kT}{\pi\hbar^2 LaM}\sum_n \ln\left[1 + \exp\left(\frac{Efc - E_{cn}}{kT}\right)\right] \quad (16\text{-a})$$

$$P \cong \left(\frac{m_H kT}{\pi\hbar^2 LaM}\right)\sum_n \int_{E_{vn}}^{\infty} fv\, d\varepsilon_{vn} = \frac{m_H kT}{\pi\hbar^2 LaM}\sum_n \ln\left[1 + \exp\left(\frac{-Efv - E_{vn} - Eg}{kT}\right)\right] \quad (16\text{-b})$$

IV. Analysis of Linear Gain in $In_{1-x}Ga_xAs_yP_{1-y}$/InP QW

As a numerical example, we have calculated the linear gain in $In_{1-x}Ga_xAs_yP_{1-y}$/InP SQW structure with y=1 and we have assumed τin to be 1.10^{-13}s. The calculated gain spectrum is shown in Fig. 4 for various carrier densities.

A. Effects of Impurities Doping

The presence of donors or acceptors can be accounted for (16) by replacing N in (16-a) by N+Nd and N in (16-b) by N+Na, where Nd and Na are the donor and acceptor densities respectively. The effect of p-type doping on gain spectral is shown in Fig. 5 for acceptor densities Na=5.10^{17} -1.10^{18}cm^{-3}. Similar plot is shown in Fig. 6 for n-type doping with donors densities Nd=5.10^{17}cm^{-3} -1.10^{18}cm^{-3}.

If we compared the two of figures the gain spectra is much larger in p-type doping than that in n-type doping. P-type doping effectively decreases fv since N is increased to N+Na, causing an increase in fc-fv. Therefore, p-type doping mostly increases the peak as shown in Fig. 5. On the other hand, n-type doping does not give rise to significant increase in fc-fv where N is increased to N+Nd.

B. Difference between TE and TM Modes

In the QW structure, a TE polarized optical wave whose electric vector lies in the plane of the QW layers can couple both with the electron to heavy hole transition and with the electron to light hole transition, but a TM polarized optical wave, whose electric vector is perpendicular to the plane of the QW layers is allowed to couple only with the latter transition, so that the optical gain for the TE wave is much larger than for TM wave since the n=1 heavy hole energy level has larger density of states than the n=1 light hole energy level. Figs. 7 and 8 show the linear gain of TM mode and the difference between TE and TM modes is shown in Fig. 8 for a

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carrier density of $2.10^{18}cm^{-3}$ and as seen, the peak gain for the TE mode is significantly larger than that for the TM mode.

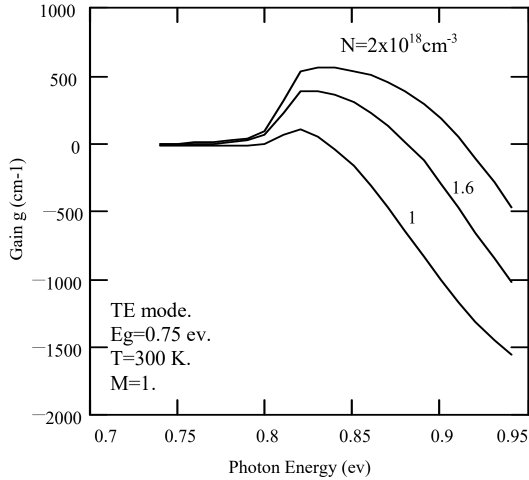

Fig. 4 Gain spectra calculated for $In_{0.53}Ga_{0.47}As/InP$

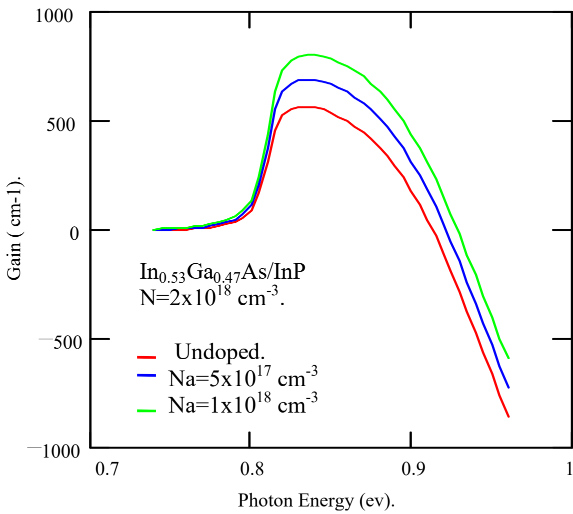

Fig. 5 Gain spectra at various acceptor densities Na

V. Conclusion

In this paper, we have established the theoretical dipole moment for approximate calculation of optical gain, which is based on a simple parabolic-band. Then we have shown that the donor acceptor increase the peak gain.

Finally, we have calculated the gain difference between TE and TM modes and it has been found that the optical gain is larger for TE mode than that TM mode.

In general, we have found that our results are in good agreement with Asada's and Makino's results.

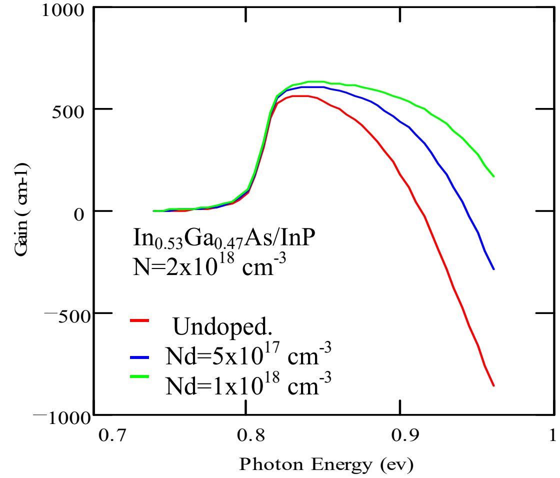

Fig. 6 Gain spectra at various donor densities Nd

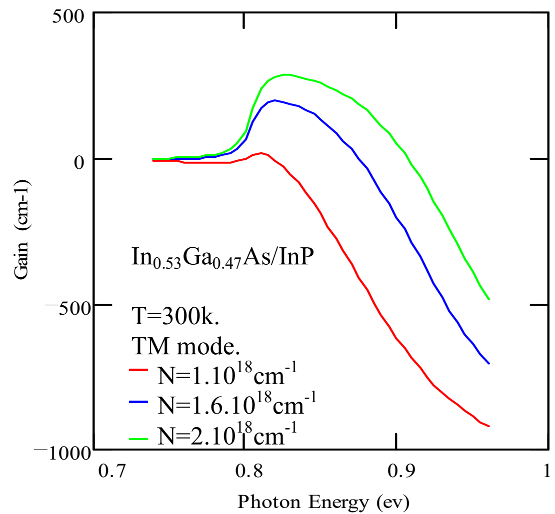

Fig. 7 Gain spectra for TM mode

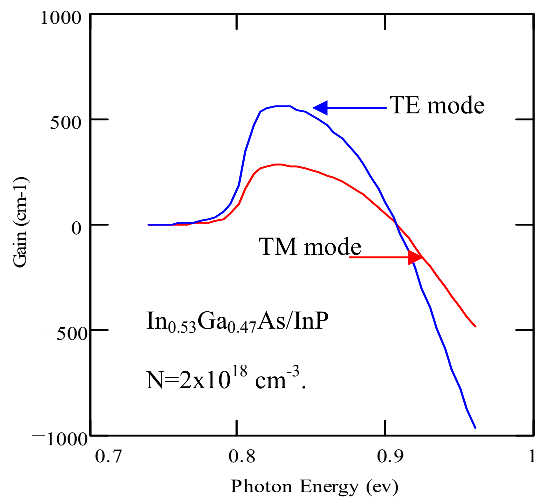

Fig. 8 Difference between TE and TM modes

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