Second Order Sliding Mode Observer Using MRAS Theory for Sensorless Control of Multiphase Induction Machine

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***Abstract*—**This paper presents a speed estimation scheme based on second-order sliding-mode Super Twisting Algorithm (STA) and Model Reference Adaptive System (MRAS) estimation theory for Sensorless control of multiphase induction machine. A stator current observer is designed based on the STA, which is utilized to take the place of the reference voltage model of the standard MRAS algorithm. The observer is insensitive to the variation of rotor resistance and magnetizing inductance when the states arrive at the sliding mode. Derivatives of rotor flux are obtained and designed as the state of MRAS, thus eliminating the integration. Compared with the first-order sliding-mode speed estimator, the proposed scheme makes full use of the auxiliary sliding-mode surface, thus alleviating the chattering behavior without increasing the complexity. Simulation results show the robustness and effectiveness of the proposed scheme.

***Keywords*—**Multiphase induction machine, field oriented control, sliding mode, super twisting algorithm, MRAS algorithm.

I. INTRODUCTION

THE three-phase induction machine is extensively used in many industrial drives due to its advantages such as simplicity, robustness, and reliability. In some applications such as electric ship propulsion, aircraft drives, locomotive traction, and high power industrial applications, high power ratings for both the motor and converter are required. However, converter ratings cannot be increased above certain range due to semiconductor devices power rating limitations [4], [6]-[10]. As a solution to this problem, multilevel inverters with a conventional three-phase Induction machine could be used. Alternatively, a multiphase machine fed from multiphase inverter drive could be used with fewer phases current or designed with a lower voltage for the same total power as the multilevel inverter drive system [6]-[10]. Multiphase (more than three phases) drives possess several advantages over conventional three-phase drives, such as reducing the amplitude and increasing the frequency of torque pulsations, reducing the rotor harmonic currents, reducing the current per phase without increasing the voltage per phase, lowering the dc-link current harmonics, and higher reliability. By increasing the number of phases it is also possible to increase the power /torque per ampere for the same volume machine [6]-[9]. Some publications have surveyed multiphase machines [4]-[10], covering topics such as properties [8], modeling [6], [7], applications [12], performance with different control techniques [11], and advantages [12]. As we know, many different methodologies have been developed for control of induction machine such as: extended Kalman Filter, sliding mode, adaptive control, artificial intelligence and model based method. These methods are also applied for speed estimation of multiphase machines and various Sensorless techniques have been developed [1]-[12]. A common approach for control and state estimation of induction machine is based on the classical sliding mode technique [1]-[5]. The concepts and principles of sliding-mode (SM) control applied to electrical motors are introduced in [1]-[3]. The success of this type of control for electric drives is mainly due to its disturbance rejection, strong robustness, and simple implementation, as shown by a large number of papers on Sensorless IM drives [1], that use the standard approach of SM control. The main obstacle to put this technique into industrial application is the chattering behavior, which consists in a high-frequency oscillation when the sliding mode takes place. The higher order sliding-mode algorithm is one of the solutions to alleviate the chattering behavior with the robustness remained unchanged [1]. The super twisting algorithm (STA) is a well-known second-order sliding-mode algorithm, and it has been widely used for observation [1], [2], control [1]-[3], and robust exact differentiation [1]-[3].

This paper presents a speed estimation scheme based on second-order sliding-mode STA and MRAS theory. A stator current observer is designed based on STA, which takes the place of the reference voltage model of the standard MRAS. By making full use of the auxiliary sliding-mode surfaces, the proposed observer successfully alleviates the chattering behavior and is insensitive to variations of rotor resistance. Then, a MRAS speed estimator is obtained according to Popov's hyper stability theory.

This paper is organized in seven sections. In the next section dynamical model of multiphase induction machine is presented. Second order sliding mode observer is presented in section three. In section four we see rotor speed estimation scheme. In sections five and six, simulation results and parameter sensitivity analysis are shown. Some conclusions and perspectives will be discussed in the last section.

II. MODEL OF MULTIPHASE INDUCTION MACHINE

The model of six phase induction machine (SPIM) under normal conditions has been given in [4], [6]. The SPIM in the normal mode is a double-six-dimensional system (six stator

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and six rotor variables). It is shown in [4] that the SPIM model can be decomposed into three double two dimensional orthogonal subspaces, α-β, z1-z2, z3-z4 by the following transformation (1). In this matrix, γ is the electrical angle between the two three phase stator windings that is 60 electrical degrees in this paper. The decoupled model is as:

$$[T6]=\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & \cos(\gamma) & -\frac{1}{2} & \cos(2\pi/3+\gamma) & -\frac{1}{2} & \cos(4\pi/3+\gamma) \\ 0 & \sin(\gamma) & \frac{\sqrt{3}}{2} & \sin(2\pi/3+\gamma) & -\frac{\sqrt{3}}{2} & \sin(4\pi/3+\gamma) \\ 1 & \cos(\pi-\gamma) & -\frac{1}{2} & \cos(\pi/3-\gamma) & -\frac{1}{2} & \cos(5\pi/3-\gamma) \\ 0 & \sin(\pi-\gamma) & -\frac{\sqrt{3}}{2} & \sin(\pi/3-\gamma) & \frac{\sqrt{3}}{2} & \sin(5\pi/3-\gamma) \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \quad (1)$$

A. Machine Model in α-β Subspace

The stator and rotor voltage equations are:

$$\begin{bmatrix} v_{s\alpha} \\ v_{s\beta} \\ 0 \\ 0 \end{bmatrix}=\begin{bmatrix} r_s+L_s p & 0 & Mp & 0 \\ 0 & r_s+L_s p & 0 & Mp \\ Mp & \omega_r M & r_r+L_r p & \omega_r L_r \\ -\omega_r M & Mp & -\omega_r L_r & r_r+L_r p \end{bmatrix}\cdot\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ i_{r\alpha} \\ i_{r\beta} \end{bmatrix} \quad (2)$$

with: Ls=Lls+M, Lr=Llr+M, M=3Lms.

B. Machine Model in z_1-z_2 Subspace

$$\begin{bmatrix} v_{sz1} \\ v_{sz2} \end{bmatrix}=\begin{bmatrix} r_s+L_{ls}p & 0 \\ 0 & r_s+L_{ls}p \end{bmatrix}\cdot\begin{bmatrix} i_{sz1} \\ i_{sz2} \end{bmatrix}$$
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}=\begin{bmatrix} r_r+L_{lr}p & 0 \\ 0 & r_r+L_{lr}p \end{bmatrix}\cdot\begin{bmatrix} i_{rz1} \\ i_{rz2} \end{bmatrix} \quad (3)$$

C. Machine Model in z_3-z_4 Subspace

$$\begin{bmatrix} v_{sz3} \\ v_{sz4} \end{bmatrix}=\begin{bmatrix} r_s+L_{ls}p & 0 \\ 0 & r_s+L_{ls}p \end{bmatrix}\cdot\begin{bmatrix} i_{sz3} \\ i_{sz4} \end{bmatrix}$$
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix}=\begin{bmatrix} r_r+L_{lr}p & 0 \\ 0 & r_r+L_{lr}p \end{bmatrix}\cdot\begin{bmatrix} i_{rz3} \\ i_{rz4} \end{bmatrix} \quad (4)$$

As it can be seen from these three subsystems, the electromechanical energy conversion takes place only in α-β subsystem, and the other subsystems have not any contribution in the energy conversion. The z_1-z_2 and z_3-z_4 subsystems produce only losses, so they should be controlled to be as small as possible. It can be concluded that the torque and speed controller synthesis and analysis for six-phase induction machines is almost the same as for three-phase induction machines: it is done with the equivalent model in α-β subspace.

III. Second-Order Sliding Mode Observer

The SPIM model in α-β subspace can be written in terms of stator current and rotor flux as follows [4]: In this model $i_{s\alpha}$, $i_{s\beta}$, $\psi_{r\alpha}$, $\psi_{r\beta}$, and $u_{s\alpha}$, $u_{s\beta}$ are respectively stator currents, rotor fluxes, and stator voltages. ω_r is the angular velocity. Rs and Rr are the stator and rotor resistances. L_s and L_r are respectively the stator inductance and rotor inductance. L_m is the mutual inductance. σ is the leakage coefficient. T_r is the rotor time constant.

$$\begin{bmatrix} pi_{s\alpha}=-(k_1.k_2+R_s.k_3).i_{s\alpha} \\ \quad +k_2.(\psi_{r\alpha}/T_r+\omega_r.\psi_{r\beta})+k_3.u_{s\alpha} \\ pi_{s\beta}=-(k_1.k_2+R_s.k_3).i_{s\beta} \\ \quad +k_2.(\psi_{r\beta}/T_r-\omega_r.\psi_{r\alpha})+k_3.u_{s\beta} \\ p\psi_{r\alpha}=k_1.i_{s\alpha}-\psi_{r\alpha}/T_r-\omega_r.\psi_{r\beta} \\ p\psi_{r\beta}=k_1.i_{s\beta}-\psi_{r\beta}/T_r+\omega_r.\psi_{r\alpha} \end{bmatrix} \quad (5)$$

where:

$$\sigma=1-L_m^2/(L_sL_r) \qquad k_1=L_m/T_r$$
$$k_2=L_m/(\sigma L_sL_r) \qquad k_3=1/(\sigma L_s)$$

A. STA

The stability of STA has been proved by Lyapunov methods in [1]. The simplest form of STA can be written as:

$$\begin{bmatrix} p\hat{x}_1=f(\hat{x}_2)+\lambda.|x_1-\hat{x}_1|^{0.5}.\mathrm{sgn}(x_1-\hat{x}_1)+\rho_1 \\ p\hat{x}_2=\delta.\mathrm{sgn}(x_1-\hat{x}_1)+\rho_2 \end{bmatrix} \quad (6)$$

where xi denotes the state variables, λ and δ are switching gains, ρ_1 and ρ_2 represents the perturbation terms. According to [1], it is well known that the STA is robustly stable to perturbations globally bounded by:

$$\rho_1=0, \quad |\rho_2|\le L$$

For any positive constant L when the gains are appropriately selected.

B. STA-Based Observer

The third and fourth terms of (5) can be substituted into the first and second terms; thus (5) can be rewritten as:

$$\begin{bmatrix} pi_{s\alpha}=-R_s.k_3.i_{s\alpha}-k_2.p\psi_{r\alpha}+k_3.u_{s\alpha} \\ pi_{s\beta}=-R_s.k_3.i_{s\beta}-k_2.p\psi_{r\beta}+k_3.u_{s\beta} \\ p\psi_{r\alpha}=k_1.i_{s\alpha}-\psi_{r\alpha}/T_r-\omega_r.\psi_{r\beta} \\ p\psi_{r\beta}=k_1.i_{s\beta}-\psi_{r\beta}/T_r+\omega_r.\psi_{r\alpha} \end{bmatrix} \quad (7)$$

Considering a variable substitution:

$$z_1=i_{s\alpha} \qquad z_3=-p\psi_{r\alpha}$$
$$z_2=i_{s\beta} \qquad z_4=-p\psi_{r\beta} \quad (8)$$

where z_1-z_4 are the intermediate variables. Substituting (8) into (7) yields:

$$\begin{bmatrix} pz_1=-R_s.k_3.z_1+k_2.z_3+k_3.u_{s\alpha} \\ pz_2=-R_s.k_3.z_2+k_2.z_4+k_3.u_{s\beta} \end{bmatrix} \quad (9)$$

By applying the STA to the IM model (5), a current observer can be constructed as:

$$\begin{bmatrix} p\hat{z}_1 = -R_s.k_3.\hat{z}_1 + k_2.\hat{z}_3 + k_3.u_{s\alpha} \\ \quad + \lambda_1.|e_1|^{0.5}.\mathrm{sgn}(e_1) \\ p\hat{z}_3 = \delta_1.\mathrm{sgn}(e_1) \\ p\hat{z}_2 = -R_s.k_3.\hat{z}_2 + k_2.\hat{z}_4 + k_3.u_{s\beta} \\ \quad + \lambda_2.|e_2|^{0.5}.\mathrm{sgn}(e_2) \\ p\hat{z}_4 = \delta_2.\mathrm{sgn}(e_2) \end{bmatrix} \quad (10)$$

where $\hat{z}_1, \hat{z}_2, \hat{z}_3$ and $\hat{z}_4$ are the observations. λ_1, λ_2 and δ_1, δ_2 are respectively the gains of the primary and auxiliary sliding mode surfaces. The sgn () represents the sign function, $e_1$and $e_2$are the errors, which are defined as: $e_1=z_1-\hat{z}_1$ and $e_2=z_2-\hat{z}_2$. Rs, k_2 and k_3 are treated as constants in the observer. According to (8), there exists a simple relationship between the observations $\hat{z}_3$ and $\hat{z}_4$ and the derivatives of rotor flux:

$$\left[\begin{bmatrix} -\hat{z}_3 \\ -\hat{z}_4 \end{bmatrix} = \begin{bmatrix} p\hat{\psi}^z_{r\alpha} \\ p\hat{\psi}^z_{r\beta} \end{bmatrix} = p\hat{\psi}^z_r\right] \quad (11)$$

where $p\hat{\psi}^z_r$ represents the estimated results with the physical significance of derivative of rotor flux.

IV. Estimation Scheme

Since we obtain the derivatives of rotor flux components, i.e., from the proposed observer, an adaptive mechanism of speed based on these observations is required. The current model of IM can be written as:

$$\left[p\psi^i_r = k_1.i_s + \begin{bmatrix} -1/T_r & -\omega_r \\ \omega_r & -1/T_r \end{bmatrix}.\psi^i_r \right] \quad (12)$$

In (12), there is the presence of the rotor speed variable. Regarding k_1 and T_r as constants, the observer equation can be constructed as:

$$\left[p\hat{\psi}^i_r = k_1.\hat{i}_s + \begin{bmatrix} -1/T_r & -\hat{\omega}_r \\ \hat{\omega}_r & -1/T_r \end{bmatrix}.\hat{\psi}^i_r \right] \quad (13)$$

where:

$$i_s = \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} \quad \psi^i_r = \begin{bmatrix} \psi^i_{r\alpha} \\ \psi^i_{r\beta} \end{bmatrix} \quad \hat{i}_s = \begin{bmatrix} \hat{z}_1 \\ \hat{z}_2 \end{bmatrix} \quad \hat{\psi}^i_r = \begin{bmatrix} \hat{\psi}^i_{r\alpha} \\ \hat{\psi}^i_{r\beta} \end{bmatrix}$$

Symbols ψ^i_r and $\hat{\psi}^i_r$ are respectively the actual and estimated rotor flux vectors calculated by the current model. By subtracting (12) from (13), we get:

$$\left[pe^i_\psi = A.e^i_\psi - \Delta\omega_r J.\hat{\psi}^i_r \right] \quad (14)$$

where:

$$e^i_\psi = \begin{bmatrix} e^i_{\psi\alpha} \\ e^i_{\psi\beta} \end{bmatrix} = \begin{bmatrix} \psi^i_{r\alpha} - \hat{\psi}^i_{r\alpha} \\ \psi^i_{r\beta} - \hat{\psi}^i_{r\beta} \end{bmatrix}$$

$$A = \begin{bmatrix} -1/T_r & -\omega_r \\ \omega_r & -1/T_r \end{bmatrix} \quad J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Differentiating (14) and simplifying the terms of differential errors, we obtain:

$$\left[p(pe^i_\psi) = A.p(e^i_\psi) - \Delta\omega_r J.p(\hat{\psi}^i_r) \right] \quad (15)$$

The stability of the above system has been discussed in [1]. For speed estimation, the output of the reference model is regarded as equal to the actual rotor flux vector, and hence:

$$pe^i_\psi = \begin{bmatrix} p\psi^i_{r\alpha} - p\hat{\psi}^i_{r\alpha} \\ p\psi^i_{r\beta} - p\hat{\psi}^i_{r\beta} \end{bmatrix} = \begin{bmatrix} p\hat{\psi}^z_{r\alpha} - p\hat{\psi}^i_{r\alpha} \\ p\hat{\psi}^z_{r\beta} - p\hat{\psi}^i_{r\beta} \end{bmatrix}$$

Eventually, the estimation equation of rotor speed is presented as:

$$\left[\hat{\omega}_r = (K_p + K_i / p).e_\omega \right] \quad (16)$$

where:

$$\begin{bmatrix} e_\omega = (pe^i_\psi)^T.J.p\hat{\psi}^z_r \\ = \left[p\hat{\psi}^z_{r\alpha} - p\hat{\psi}^i_{r\alpha} \quad p\hat{\psi}^z_{r\beta} - p\hat{\psi}^i_{r\beta} \right].\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.\begin{bmatrix} p\hat{\psi}^i_{r\alpha} \\ p\hat{\psi}^i_{r\beta} \end{bmatrix} \\ = -p\hat{\psi}^z_{r\alpha}.p\hat{\psi}^i_{r\beta} + p\hat{\psi}^z_{r\beta}.p\hat{\psi}^i_{r\alpha} \end{bmatrix} \quad (17)$$

Compared with the standard MRAS speed estimation shown in [1] the proposed algorithm eliminates the integrators, thus simplifying the system and avoiding problems caused by integration.

V. Simulation Results

In this section, a six phase induction machine, with the parameters given in Table I, is simulated to validate performance of the proposed method. The simulation is carried out using Matlab/Simulink. In order to show the behavior of the proposed algorithm, two tests are considered. In the first test, the machine is started towards a speed of 500 rpm and accelerates to 75% upper speed at t = 6 s. Fig. 1 shows that real and estimated speeds follow reference speed very well which shows the good performance of the algorithm. The estimated currents of the STA tend to the real currents of the machine as we see in Fig. 2. In the next test, we applied trapezoid reference speed to the machine control algorithm. In this test we want to examine the ability of the method in speed inversion condition. Fig. 3 shows that mismatch between references, real and estimated speeds are zero. The estimated currents of the STA tend to the real currents of the machine in this test too.

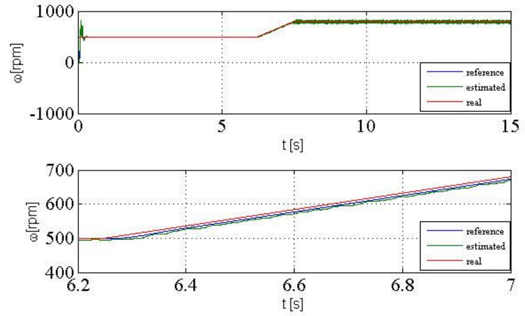

Fig. 1 Real and estimated speed in acceleration test

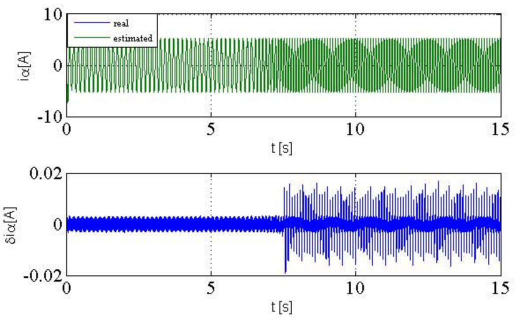

Fig. 2 Real and estimated current components in acceleration test

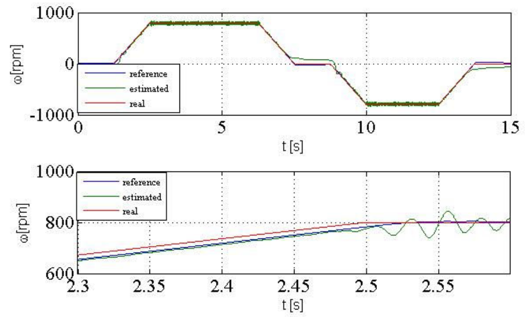

Fig. 3 Reference, real and estimated speed in speed inversion test

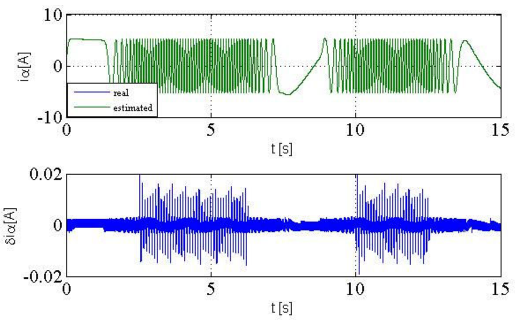

Fig. 4 Real and estimated current components in speed inversion test

VI. Parameter Sensitivity Analysis

This section investigates the sensitivity of the observer to the variation of the IM parameters. The variation of the rotor resistance Rr and the magnetizing inductance Lm is considered. The variation of Rs is not considered (if needed, Rs can be estimated using stator-mounted thermistors). The observer is implemented using the rated parameters while the machine is detuned: Rr is increased up to double the rated value; Lm is reduced to half its rated value (this corresponds to a practical situation). Simulations show that the observer is insensitive to Lm and Rr variation.

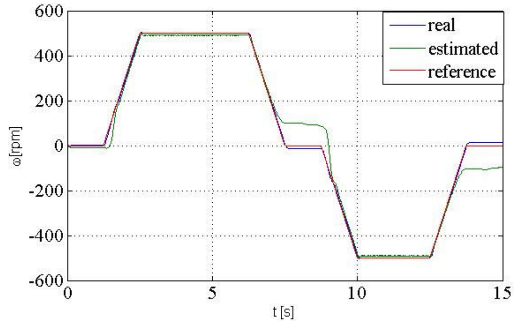

Fig. 5 Lm variation to half of rated value

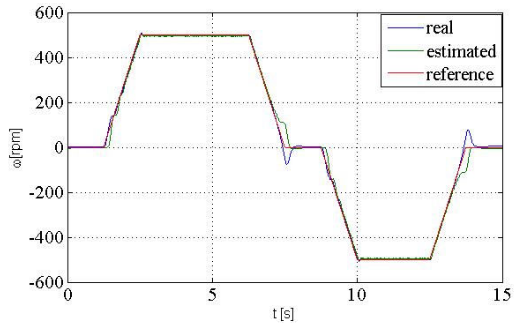

Fig. 6 Rr variation to double of rated value

TABLE I
Parameter of Simulated Machine

No. of poles	2
Rated output	90 W
Rated voltage	42 V
Rated current	2.6 A
Rated speed	2800 rpm
Rated torque	0.3 Nm
Stator resistance (R_s)	1.04 Ω
Stator inductance (L_s)	0.0127 mH
Rotor resistance (R_r)	0.4107 Ω
Rotor inductance (L_r)	0.0127 mH
Mutual inductance (M)	0.0115 mH

VII. Conclusion

This paper presents a modified speed Sensorless control scheme for multiphase induction machine based on second-order sliding-mode STA and MRAS estimation theory. The estimation scheme has been obtained by combining a second-order sliding-mode current observer with a parallel speed estimator based on rotor flux-based MRAS. The STA-based observer is utilized to take the place of the reference voltage model of the standard MRAS. Derivatives of rotor flux are obtained and designed as the state of MRAS, thus eliminating the integration. Moreover, by making full use of auxiliary surfaces, the observations are insensitive to rotor resistance and magnetizing inductance variation with the alleviation of chattering behavior at the same time. Simulation results show that when sliding mode motion occurs observer can estimate machine currents and speed accurately and converge completely. On the other hand in speed inversion test, this observer work very well and error between real, estimated and reference speed is negligible. It is found that, under improper parameters, this observer is insensitive to the variation of the magnetizing inductance and rotor resistance.

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