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Uncontrollable Inaccuracy in Inverse Problems

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Abstract**—**In this paper the influence of errors of function derivatives in initial time which have been obtained by experiment (uncontrollable inaccuracy) to the results of inverse problem solution was investigated. It was shown that these errors distort the inverse problem solution as a rule near the beginning of interval where the solutions are analyzed. Several methods for removing the influence of uncontrollable inaccuracy have been suggested.

Keywords**—**Inverse problems, uncontrollable inaccuracy, filtration.

I. Introduction

THE inaccuracy is inevitable in experimental measuring of physical values. It consists of inaccuracy of measuring instruments, noise value and inaccuracy of visual means. The value of this inaccuracy can be evaluated by technical indicators of measuring instruments. They do not exceed 5-10 percent as a rule.

The experimental measuring is chosen as initial data for the following calculations with the use of mathematical models in many practical important problems. For example, the inverse problems for evolution process as [1], [2], the control problems with the use of experimental data as [3], [4] belong to this class.

Let us consider the certain dynamic system the motion of which is described by

$$\dot{X} = AX + BZ\,, \tag{1}$$

with initial conditions

$$X(0) = X^0\,, \tag{2}$$

where $Z(t)$ is the vector-function of external loads, $X(t)$ is the vector-function of state variables, A is the matrix of system, B is the matrix of control. The vector-function $Z(t)$ is given in direct problems. The matrices A and B are also given. The vector-function $X(t)$ is an unknown function. The initial conditions (2) X^0 have been given. The solution of system (1) can be presented in the form

$$X(t) = F(Z,\, X^0) \tag{3}$$

If we consider the inverse problems, for example, when the vector-function $Z(t)$ is searched, then we use the vector-function $X(t)$, values x_j^0 (components of vector X^0, $j = \overline{1,n}$) and matrixes A, B as initial data. If we have all components $x_j(t)$ of vector-function $X(t)$ then we have the all values $x_j^0 = x_j(0)$. But as a rule in practices we can't measuring all functions $x_j(t)$. One or two components of vector-function $X(t)$ are measured usually, for example, $x_1(t)$ only. Then it is necessary to have the values $\dot{x}_1(0), \ddot{x}_1(0), \ldots, x_1^{(n-1)}(0)$ for the search of vector-function $Z(t)$ But the inaccuracy of $\dot{x}_1(0), \ddot{x}_1(0), \ldots, x_1^{(n-1)}(0)$ can't be evaluated in principle as the function $x_1(t)$ was obtained by experimental way with errors. This inaccuracy equals infinity in general case. It leads to approximate solution will be equal zero. The indicated inaccuracy was called the uncontrollable inaccuracy as [5], [6].

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II. The Statement of a Problem

As an example let us consider the inverse problem of unbalance evaluation of deformable rotor characteristics which has two supports as [5], [7]. The physical significance of symbols and parameters in equations will not be interpreted. The main purpose is simply to preserve the structure of expressions.

The rotor motion is described by the system of ordinary differential equations of 18th order as [5], [7]:

$$\begin{aligned}
& m\ddot{\eta}(t) = P_2 + z_2(t)\,; \\
& T_1\ddot{\gamma}(t) + T_z^1(\dot{\varphi}\dot{\psi} + \ddot{\varphi}\psi) = M_1 + h z_2(t)\,; \\
& T_1\ddot{\psi}(t) - T_z^1(\dot{\varphi}\dot{\gamma} + \ddot{\varphi}\gamma) = M_2 + h z_1(t)\,; \\
& T_1\ddot{\varphi}(t) = M_3\,; \\
& m_A\ddot{\xi}_A + b_A^1\dot{\xi}_A + c_A^1\xi_A = l_1(bP_1 + M_2)\,; \\
& m\ddot{\eta}_A + b_A^2\dot{\eta}_A + c_A^2\eta_A = l_1(bP_2 + M_1)\,; \\
& m_B\ddot{\xi}_B + b_B^1\dot{\xi}_B + c_B^1\xi_B = l_1(aP_1 - M_2)\,; \\
& m_B\ddot{\eta}_B + b_B^2\dot{\eta}_B + c_B^2\eta_B = l_1(aP_2 + M_1)\,;
\end{aligned} \tag{4}$$

where

$P_1 = -c_1\tilde{\xi} + c_2\tilde{\psi}$, $M_1 = c_1\tilde{\xi} + c_3\tilde{\psi}$,

$P_2 = -c_1\tilde{\eta} + c_2\tilde{\gamma} + mg$, $M_2 = c_2\tilde{\eta} - c_3\tilde{\gamma}$,

$\tilde{\xi} = \xi - \xi_0$, $\tilde{\eta} = \eta - \eta_0$, $\tilde{\gamma} = \gamma - \gamma_0$, $\tilde{\psi} = \psi - \psi_0$,

$\xi_0 = l_1(a\xi_B + b\xi_A)$, $\eta_0 = l_1(a\eta_B + b\eta_A)$, $\psi_0 = l_1(\xi_B - \xi_A)$,

$\gamma_0 = l_1(\eta_A - \eta_B)$, $l_1 = (a+b)^{-1}$,

$c_1, c_2, c_3, m, m_A, m_B, T_1, T_1^z, b_A^1, b_A^2, b_B^1, b_B^2, c_A^1, c_A^2, c_B^1, c_B^2, a, b$ - const.

The unknown unbalance characteristics are

$z_1(t) = m_u r\dot{\varphi}^2 \sin(\vartheta + \varphi)$, $z_2(t) = -m_u r\dot{\varphi}^2 \cos(\vartheta + \varphi)$,

$z_3(t) = -h\, z_1(t)$.

The total characteristics of unbalance can be obtained by the use of these functions:

$$m_u = (z_1^2 + z_1^2) r^{-1} \dot{\varphi}^{-2},\ h = -\frac{z_3(t)}{z_1(t)}, \vartheta = arc\,tg\,(-\frac{z_1}{z_2}),$$

where m_u is mass of unbalance, h is arm of unbalance, ϑ is angle of unbalance in plane of correction.

Let us suppose that only functions $\ddot{\eta}_A(t), \ddot{\eta}_B(t)$ (the vibration of rotor supports A, B in horizontal direction) were obtained from experiment.

The equations for determination of functions $z_1(t), z_2(t), z_3(t)$ have a form (the inverse problem):

$$\int_0^t (t-\tau)\, z_i(\tau)\, d\tau = u_i(t), \qquad (i = 1,2,3) \ , \tag{5}$$

or

$$A z_i = u_i(t) \tag{6}$$

where $u_i(t)$ are the known functions obtained from experiment, A is a linear integral operator. For example, the function $u_2(t)$ has the form:

$$u_2(t) = \int_0^t \left\{ \left[\sum_{j=1}^4 (t-\tau)^{j-1} N_j^A \right] \ddot{\eta}_A(\tau) + \right.$$
$$\left. + \left[\sum_{j=1}^4 (t-\tau)^{j-1} N_j^B \right] \ddot{\eta}_B(\tau) \right\} d\tau + N_5^A \ddot{\eta}_A(t) +$$
$$+ N_5^B \ddot{\eta}_B(t) + N_6 + N_7 t + N_8 t^2 + N_9 t^3 , \tag{7}$$

where

$N_1^A = \frac{m}{\Delta} c_{12}(c_3 - ac_2), N_1^B = \frac{m}{\Delta} c_{22}(c_3 + bc_2),$

$N_2^A = \frac{m}{\Delta} c_{13}(c_3 - ac_2) + \frac{mb}{l} - c_{11},$

$N_2^B = \frac{m}{\Delta} c_{23}(c_3 + bc_2) + \frac{ma}{l} - c_{21},$

$N_3^A = -c_{12}, N_3^B = -c_{22}, N_4^A = -c_{13}, N_4^B = -c_{23},$

$N_5^A = \frac{m}{\Delta} c_{11}(c_3 - ac_2),\ N_5^B = \frac{m}{\Delta} c_{21}(c_3 + bc_2),$

$N_6 = -\frac{m}{\Delta}[c_{11}(c_3 - ac_2)\,\dddot{\eta}_A(0) + c_{21}(c_3 + bc_2)\,\dddot{\eta}_B(0)],$

$N_7 = -\frac{m}{\Delta}[c_{11}(c_3 - ac_2)\,\ddddot{\eta}_A(0) + c_{21}(c_3 - a\,c_2)\,\ddddot{\eta}_A(0) +$
$+ c_{21}(c_3 + b\,c_2)\,\ddddot{\eta}_B(0) + c_{22}(c_3 + b\,c_2)\,\dddot{\eta}_B(0)]$,

$N_8 = -\frac{1}{\Delta}[c_{12}\,\dot{\eta}_A(0) + c_{22}\,\dot{\eta}_B(0) + c_{13}\,\eta_A(0) + c_{23}\,\eta_B(0) - mg]$,

$N_9 = -\frac{1}{b}[c_{13}\,\dot{\eta}_A(0) + c_{23}\,\dot{\eta}_B(0)]$.

It is assumed that the errors of values

$\Delta\eta_A^3, \Delta\eta_B^3, \Delta\eta_A^2, \Delta\eta_B^2, \Delta\eta_A^1, \Delta\eta_B^1, \Delta\eta_A, \Delta\eta_B$

have appeared when values of

$\dddot{\eta}_A(0), \dddot{\eta}_B(0), \ddot{\eta}_A(0), \ddot{\eta}_B(0), \dot{\eta}_A(0), \dot{\eta}_B(0), \eta_A(0), \eta_B(0)$

are measured.

We will determine the influence of errors upon the solution of inverse problem (5). The right part of (1) represents the output of system (4) - $\ddot{\eta}_A(t), \ddot{\eta}_B(t)$ initiated by unknown action $z_2(t)$ only. In right part of (5) were excluded all the external actions and initial conditions. The uncontrollable inaccuracy in initial conditions leads to appearance of the supplementary terms $\mu_i y_i(t)$ in the expression for $u_2(t)$, where $y_1(t)$ is the solution of homogeneous system (4) $(z_1(t), z_2(t), z_3(t) = 0)$ with nonzero initial conditions, μ_i - const.

Using (4) we obtain the linear expression with regard to $\ddot{\eta}_A(t), \ddot{\eta}_B(t)$:

$$\gamma_{14}\,\eta_A^{(4)} + \gamma_{13}\,\eta_A^{(3)} + \gamma_{12}\,\ddot{\eta}_A + \gamma_{11}\,\dot{\eta}_A + \gamma_{10}\,\eta_A + \gamma_{24}\,\eta_B^{(4)} + \gamma_{23}\,\eta_A^{(3)} +$$
$$\gamma_{22}\,\ddot{\eta}_B + \gamma_{21}\,\dot{\eta}_B + \gamma_{20}\,\eta_B + \gamma = z_2(t)\,, \tag{8}$$

where γ_{ik}, γ are constant.

Let us consider two functions $\ddot{\eta}_A(t), \ddot{\eta}_B(t)$ which satisfy (8) identically when $z_2(t) \equiv 0$ for $t > 0$ and which satisfy the zero initial conditions:

$$\dddot{\eta}_A(0) = \dddot{\eta}_B(0) = \ddot{\eta}_A(0) = \ddot{\eta}_B(0) = \dot{\eta}_A(0) = 0 \quad ,$$

$$\dot{\eta}_B(0) = \eta_A(0) = \eta_B(0) = 0 . \tag{9}$$

Let the functions $\tilde{\ddot{\eta}}_A(t), \tilde{\ddot{\eta}}_B(t)$ coincide with functions $\ddot{\eta}_A(t), \ddot{\eta}_B(t)$ when $t > 0$ and satisfy the initial conditions:

$$\tilde{\dddot{\eta}}_A(0) = \Delta\eta_A^3,\ \tilde{\ddot{\eta}}_A(0) = \Delta\eta_A^2, \tilde{\dot{\eta}}_A(0) = \Delta\eta_A^1, \tilde{\eta}_A(0) = \Delta\eta_A,$$

$$\tilde{\dddot{\eta}}_B(0) = \Delta\eta_B^3,\ \tilde{\ddot{\eta}}_B(0) = \Delta\eta_B^2, \tilde{\dot{\eta}}_B(0) = \Delta\eta_B^1, \tilde{\eta}_B(0) = \Delta\eta_B .$$

The function $\tilde{z}_2(t)$ in (8) will differ from zero when $\tilde{\ddot{\eta}}_A(t), \tilde{\ddot{\eta}}_B(t)$ are substituted into (8). So, it is possible to reduce the investigation of influence of uncontrollable inaccuracy to the analysis of function $\tilde{z}_2(t)$.

The coincidence of $\ddot{\eta}_A(t), \ddot{\eta}_B(t)$ and $\tilde{\ddot{\eta}}_A(t), \tilde{\ddot{\eta}}_B(t)$ when $t > 0$ leads to

$$\tilde{\ddot{\eta}}_A(t) = \ddot{\eta}_A(t)\sigma_+(t),\ \tilde{\ddot{\eta}}_B(t) = \ddot{\eta}_B(t)\sigma_+(t) ;$$

where $\sigma_+(t)$ is the asymmetric single step-function as [8]:

$$\sigma_+(t) = \begin{cases} 0, & t \le 0; \\ 1, & t > 0. \end{cases}$$

By substitution of $\tilde{\ddot{\eta}}_A(t), \tilde{\ddot{\eta}}_B(t)$ into (8) by $t > 0$ we get

$$\begin{aligned}
&\gamma_{14}[\eta_A^{(4)}(t) + 2\Delta\eta_A^3\,\delta_+(t) + \Delta\eta_A^2\,\delta_+'(t)] + \\
&+ \gamma_{13}[\eta_A^{(3)}(t) + \Delta\eta_A^2\,\delta_+(t)] + \gamma_{12}\,\tilde{\eta}_A(t) + \\
&+ \gamma_{11}[\dot{\eta}_A(t) + \Delta\eta_A^1] + \gamma_{10}\,[\eta_A(t) + \Delta\eta_A t + \Delta\eta_A^1] + \\
&+ \gamma_{24}[\eta_B^{(4)}(t) + 2\Delta\eta_B^3\,\delta_+(t) + \Delta\eta_B^2\,\delta_+'(t)] + \\
&+ \gamma_{23}[\eta_B^{(3)}(t) + \Delta\eta_B^2\,\delta_+(t)] + \gamma_{22}\,\ddot{\eta}_B(t) + \\
&+ \gamma_{21}[\dot{\eta}_B(t) + \Delta\eta_B^1] + \gamma_{20}\,[\eta_B(t) + \Delta\eta_B^2 t + \Delta\eta_B^1] + \gamma = \\
&\tilde{z}_2(t) = d_1\,\delta_+'(t) + d_0\,\delta_+(t) + c_0 + c_1 t + z_2(t) \text{ , when } t > 0 ,
\end{aligned}$$

where $d_1 = \gamma_{14}\,\Delta\eta_A^2 + \gamma_{24}\,\Delta\eta_B^2,$

$$d_0 = 2\gamma_{14}\,\Delta\eta_A^3 + \gamma_{13}\,\Delta\eta_A^2 + 2\gamma_{24}\,\Delta\eta_B^3 + \gamma_{23}\,\Delta\eta_B^2,$$

$$c_0 = \gamma_{11}\,\Delta\eta_A + \gamma_{10}\,\Delta\eta_A^1 + \gamma_{21}\,\Delta\eta_B + \gamma_{20}\,\Delta\eta_B^1$$

$$c_1 = \gamma_{10}\,\Delta\eta_A + \gamma_{20}\,\Delta\eta_A ,$$

$\delta_+(t)$ is the asymmetric impulse-function as [8].

It is evident that uncontrollable inaccuracy leads to essential change the solution of (5).

Let us consider the influence of uncontrollable inaccuracy on solution of inverse problem of astrodynamics as [2].

The solution of this problem satisfies the integral equation of kind (5) with right part:

$$u_{jk}(t) = G^{-1}(\vec{r}_{0j}(t))_k - \int_0^t (t-\tau)\,\vec{\mu}_{jk}(\tau)\,d\tau - \frac{d(\vec{r}_{oj}(0))_k}{G\,dt}t - \\ - \frac{(\vec{r}_{oj}(0))_k}{G} , \tag{10}$$

where $\vec{\mu}_{jk}(t) = \sum_{\substack{i=1 \\ i \ne j}}^{n-1} m_i[\vec{r}_{0i}(t) - \vec{r}_{0j}(t)]_k \cdot \left|\vec{r}_{0i}(t) - \vec{r}_{0j}(t)\right|^{-3}$, G is the gravitation constant.

The uncontrollable inaccuracy is defined by term $\dfrac{d(\vec{r}_{oj}(0))_k}{G\,dt}$ in this problem.

Let us consider the motion of celestial bodies in projection to coordinate axis with number k :

$$\frac{d^2(\vec{r}_{oj}(0))_k}{G\,dt^2} = \sum_{\substack{i=1 \\ i \ne j}}^{n-1} m_i[\vec{r}_{0i}(t) - \vec{r}_{0j}(t)]_k \cdot \left|\vec{r}_{0i}(t) - \vec{r}_{0j}(t)\right|^{-3} + f_{jk}(t) \tag{11}$$

Let the functions $(\vec{r}_{0j}(t))_k$, $j = \overline{1, n-1}$ satisfy the expression (11) for $t > 0$ and satisfy the zero initial conditions: $(\vec{r}_{0j}(0))_k = 0\ (k = \overline{1, n-1}), \dfrac{d(\vec{r}_{oj}(0))_k}{dt} = 0$ by accurate defined function $f_{jk}(t)$.

Let the real functions $(\tilde{\vec{r}}_{0j}(t))_k$, $k = \overline{1, n-1}$ coincide with functions $(\vec{r}_{0j}(t))_k$, $k = \overline{1, n-1}$ for $t > 0$ and satisfy the initial condition:

$$(\tilde{\vec{r}}_{0j}(0))_k = \Delta_{jk},\ \frac{d(\tilde{\vec{r}}_{oj}(0))_k}{dt} = \Delta_{jk}^1,\ k = \overline{1, n-1}$$

where $\Delta_{jk}, \Delta_{jk}^1$ are the errors of initial conditions. Then $(\tilde{\vec{r}}_{0j}(t))_k = (\vec{r}_{0j}(t))_k\,\sigma_+(t)$,

$$\frac{d^2(\tilde{\vec{r}}_{oj})_k}{G\,dt^2} = \frac{d}{dt}\left[\frac{d(\vec{r}_{oj})_k}{dt}\sigma_+(t) + (\vec{r}_{oj}(0))_k\,\delta_+(t)\right] = \\ = \frac{d^2(\vec{r}_{oj})_k}{dt^2} + \frac{d(\tilde{\vec{r}}_{oj}(0))_k}{dt}\delta_+(t) + (\tilde{\vec{r}}_{oj}(0))_k\,\delta_+'(t) .$$

Substituting the functions $(\widetilde{\vec{r}}_{0j}(t))_k,\ k=\overline{1,n-1}$ and its second derivative into (11), we get

$$\frac{d^2(\vec{r}_{oj})_k}{Gdt^2}+\frac{\Delta_j\delta_+(t)}{G}+\frac{\Delta_{jk}\,\delta'_+(t)}{G}=$$
$$=\sum_{\substack{i=1\\ i\neq j}}^{n-1}(\vec{r}_{0i}-\vec{r}_{0j})_k\cdot\left|\vec{r}_{0i}(t)-\vec{r}_{0j}(t)\right|^{-3}+f_{jk}(t),$$

$$\widetilde{f}_{jk}(t)=f_{jk}(t)-\Delta_jG^{-1}\delta_+(t)-\Delta_{jk}G^{-1}\delta'_+(t). \qquad (12)$$

This statement remains true for nonlinear problems but the character of influence can be more complicated.

III. The Filtration of Initial Data

As has been shown above the uncontrollable inaccuracy distorts the unknown solution in the beginning of interval $[0,T]$ where the solution is studied as a rule.

The exclusion of some interval $[0,\varepsilon]$ (ε is the small value) from solution where it do not true is the single way to remove the influence of this inaccuracy for problems kind inverse problem of astrodynamics. Moreover, where it is possible, it is necessary to set the initial condition for the solution of inverse problem to correspond the state of rest. Then the all items which determine uncontrollable inaccuracy ought to be set equal to zero according to physical sense.

The following method of influence removal of uncontrollable inaccuracy on result of inverse problem solution is suggested: the items which determine the uncontrollable values of initial conditions are excluded from function $u_{jk}(t)$ in (5) by means of the special filtration as [6].

The components of $a_0,\ a_1t,\ a_2t^2,\ a_3t^3$ kind (a_0,a_1,a_2,a_3 are constants) are excluded from function $u_2(t)$ in problem of unbalance identification (components of a_0,a_1t kind correspondingly from the function $u_{jk}(t)$ in inverse problem of astrodynamics) because the errors of very these terms cannot be evaluated. The functions $u_{jk}(t)$, $u_2(t)$ are defined on interval $[0,T]$ and $u_{jk}(0)=0$, $u_2(0)=0$. These functions are continued on interval $[-T,0]$ by odd way. Here we used the properties of Legander's polynomials as [8]. Let us define the values of a_0,a_1,a_2,a_3 from expressions

$$a_0=\widetilde{a}_0-\frac{1}{2}\widetilde{a}_2,\ a_1=\widetilde{a}_1-\frac{3}{2}\widetilde{a}_3, a_2=\frac{3}{2}\widetilde{a}_2, a_3=\frac{5}{2}\widetilde{a}_3,$$

where $\widetilde{a}_0,\widetilde{a}_1,\widetilde{a}_2,\widetilde{a}_3$ are the coefficients of Fourier for Legander's polynomials:

$$\widetilde{a}_0=\frac{1}{2T}\int_{-T}^{T}u_{jk}(t)\,dt,\ \widetilde{a}_1=\frac{3}{2T^2}\int_{-T}^{T}t\,u_{jk}(t)\,dt,$$

$$\widetilde{a}_2=\frac{5}{4T^3}\int_{-T}^{T}(3t^2-T^2)\,u_{jk}(t)\,dt,$$

$$\widetilde{a}_3=\frac{7}{4T^4}\int_{-T}^{T}(5t^3-3tT^2)\,u_{jk}(t)\,dt.$$

Then the filtered function $u_{jk}^f(t)$ is substituted into right-hand side of integral equation of inverse problem of astrodynamics instead of $u_{jk}(t)$:

$$u_{jk}^f(t)=u_{jk}(t)-a_0-a_1t.$$

The following function $u_2^f(t)$ is used for problem of unbalance definition into right-hand side of (5):

$$u_2^f(t)=u_2(t)-a_0-a_1t-a_2t^2-a_3t^3.$$

The test of numerical computations demonstrates the ample efficiency of suggested method.

IV. Conclusion

The negative influence of errors of function derivatives which have been measured by experiment (uncontrollable inaccuracy) to the result of inverse problem solution was considered. It was shown that this inaccuracy distorts qualitatively the inverse problem solution in the beginning of examined interval of time as a rule. Several methods of influence removal of uncontrollable inaccuracy were suggested. In particular the method of special filtration of inverse problem initial data was described.

References

[1] A. Prilepko, D. Orlovsky, "About inverse problems for nonsteady systems", *Different. Equations and their Applications*, Vilnus, no. 34, pp.85-92, 1983.

[2] Yu. L. Menshikov, "The inverse Problem of Astrodynamics". *Different. Equations and their Applications in Physics*, Dniepropetrovsk, University, Dniepropetrovsk, Ukraine, pp.19-26, 1990.

[3] C. J. Radcliffe, Jr. C. D. Mote, "Identification and control of rotating disk Vibration", *Trans. ASME: Int. Symp. Syst., Meas. and Control*, pp.120-128, 2002.

[4] St. Dubowsky, "Active control of mechanical systems: the state - of the art for robotics manipulators", *26th Struct., Struct. Dyn. and Mater. Conf.*, Orlando, Fla, Apr.15-17, 1985, Pt. 2 .Coll. Techn. Pap. / New York, N.Y. pp.258-261, 1985.

[5] Yu. L. Menshikov, N. V. Polyakov, "Operative evaluation of unbalance characteristics of a deformable rotor". In *Proc.8th Int. Symp. on Technical Diagnostics (IMEKO)*, Dresden, 23-25 Sept. 1992, pp.399-408.

[6] Yu. L. Menshikov, "The influence of uncontrollable inaccuracy of initial data on diagnostics results of mechanical objects", *Vibrodiagnostics of machines and mechanisms. The Methods and Tools.* Zaporozhye, Ukraine, pp.65-66, 1985.

[7] Yu. L. Menshikov,"The Evaluation of the Unbalance Characteristics of a Rotor by the New Operative Method", in *Proc. of GAMM,* 1996, vol.4, pp.103-105.

[8] G. Korn, T. Korn, *Mathematical handbook for scientists and engineers*, N.-Y., Toronto, London, 1961.

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Last publications: 1. Menshikov Yu. Synthesis of Adequate Mathematical Description as Solution of Special Inverse Problems. European Journal of Mathematical Sciences, vol 2, No 3, 2013, p.256-271. 2. Menshikov Yu. Identification of Mathematical Model Parameters of Stationary Process. Journal of Applied Mathematics and Physics, v.2, n. 5, April 2014, 189-193. 3. Menshikov Yu. One approach to solutions of measurement's inverse problems. J. Mathematical Inverse Problems, Vol.1, No.2, 2014, USA., p.71-85.