Double Manifold Sliding Mode Observer for Sensorless Control of Multiphase Induction Machine under Fault Condition

Mohammad Jafarifar

***Abstract*—**Multiphase Induction Machine (IM) is normally controlled using rotor field oriented vector control. Under phase(s) loss, the machine currents can be optimally controlled to satisfy certain optimization criteria. In this paper we discuss the performance of double manifold sliding mode observer (DM-SMO) in Sensorless control of multiphase induction machine under unsymmetrical condition (one phase loss). This observer is developed using the IM model in the stationary reference frame. DM-SMO is constructed by adding extra feedback term to conventional single mode sliding mode observer (SM-SMO) which proposed in many literature. This leads to a fully convergent observer that also yields an accurate estimate of the speed and stator currents. It will be shown by the simulation results that the estimated speed and currents by the method are very well and error between real and estimated quantities is negligible. Also parameter sensitivity analysis shows that this method is rather robust against parameter variation.

***Keywords*—**Multiphase induction machine, field oriented control, sliding mode, unsymmetrical condition, manifold.

I. Introduction

INDUCTION MACHINES (IM) are widely used in industry. They have a simple and robust rotor construction and offer high efficiency, low cost and maintenance. In torque control, the dynamic requirements are satisfied often by field oriented control. In applications where accurate speed or torque control is required, the IM is usually fed by a three-phase inverter and is controlled using field-oriented methods (vector control). Field-oriented control allows accurate control of the motor torque and speed and is preferred over scalar (V/Hz) control. The most popular method for IM vector control is based on rotor field orientation. To implement the rotor field-oriented control scheme, the angle of the rotor flux is required. It is typical to estimate the angle of rotor flux of the IM model using an observer. Many such observer designs are available [1]–[10]. Depending on the approach, these are obtained using linear, non-linear or sliding mode (SM) design methods and perform state or state and speed estimation. Some methods combine estimation with adaptation [2], whereas others also involve parameter estimation [2]. A separate class is that of SM observers (SMOs) [2]-[3], [5], [10]; in this case, the estimation is done using discontinuous feedback terms. The SM motion may be enforced on single or multi-dimensional manifolds–SM methods allow for order reduction and provide robustness to uncertainties.

Mohammad Jafarifar is with Department of Electrical Engineering, Andimeshk Branch, Islamic Azad University, Andimeshk, Iran (e-mail: mohammad.jafarifar@gmail.com)

Nowadays, high power electric machine drive systems which control multiphase induction machine by field oriented method, have many application such as those found pumps, fans, compressor, rolling mills, cement mills and mine hoists just to name a few. In multi-phase drive systems, the electric machine has more than three phases in the stator and the same numbers of inverter legs are in the inverter side. So the current per phase in the machine and inverter is reduced. The most common multi-phase machine-drive structure is the symmetrical six-phase induction machine (SPIM), which has two sets of three-phase windings, spatially phase shifted by 60 electrical degrees. Multiphase drive has many advantages over three phase drive. One of these advantages is the improved reliability [1]. If one phase is open-circuited due to a fault, the machine can still be operated satisfactorily [1], in contrast to three-phase machines. In many literature shows that most of the method that used for control of three phase machine, can control symmetrical multiphase induction machine very well. But some of these methods haven't good performance in unsymmetrical condition.

In this paper we show two type sliding mode observers which can control unsymmetrical multiphase induction machine very well. In the first part, an SMO based on a single compound manifold is presented. It is shown in [2] that this observer does not fully converge; despite that, it yields the correct fluxes and a relatively accurate speed estimate. In the second part, the single-manifold design is augmented with additional feedback terms and this is transformed into a double-manifold SMO (DM-SMO). The paper examines the properties of this design and shows that the estimates converge and the speed obtained is fully accurate. This paper is organized in seven sections. In the next section dynamical model of multiphase induction machine under symmetrical and unsymmetrical condition is presented. SM-SMO and DM-SMO are presented in sections three and four. In sections five and six, simulation results and parameter sensitivity analysis are shown. Some conclusions and perspectives will be discussed in the last section.

II. Model of Multiphase Induction Machine

A. Symmetrical Six Phase Induction Machine

The model of SPIM under normal conditions has been given in [1]. The SPIM in the normal mode is a double-six-dimensional system (six stator and six rotor variables). It is shown in [1] that the SPIM model can be decomposed into three double two dimensional orthogonal subspaces, α-β, z_1-z_2, z_3-z_4 by the following transformation:

$$[T6]=\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & \cos(\gamma) & -\frac{1}{2} & \cos(2\pi/3+\gamma) & -\frac{1}{2} & \cos(4\pi/3+\gamma) \\ 0 & \sin(\gamma) & \frac{\sqrt{3}}{2} & \sin(2\pi/3+\gamma) & -\frac{\sqrt{3}}{2} & \sin(4\pi/3+\gamma) \\ 1 & \cos(\pi-\gamma) & -\frac{1}{2} & \cos(\pi/3-\gamma) & -\frac{1}{2} & \cos(5\pi/3-\gamma) \\ 0 & \sin(\pi-\gamma) & -\frac{\sqrt{3}}{2} & \sin(\pi/3-\gamma) & \frac{\sqrt{3}}{2} & \sin(5\pi/3-\gamma) \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \tag{1}$$

In this matrix, γ is the electrical angle between the two three phase stator windings that is 60 electrical degrees. The decoupled model is as follows.

1.Machine Model in α-β Subspace

The stator and rotor voltage equations are:

$$\begin{bmatrix} v_{s\alpha} \\ v_{s\beta} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s+L_s p & 0 & Mp & 0 \\ 0 & r_s+L_s p & 0 & Mp \\ Mp & \omega_r M & r_r+L_r p & \omega_r L_r \\ -\omega_r M & Mp & -\omega_r L_r & r_r+L_r p \end{bmatrix} . \begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \\ i_{r\alpha} \\ i_{r\beta} \end{bmatrix} \tag{2}$$

with: $Ls=L_{ls}+M$, $Lr=L_{lr}+M$, $M=3L_{ms}$.

This model is similar to the three phase machine model in the stationary reference frame.

2.Machine Model in z_1-z_2 Subspace

$$\begin{bmatrix} v_{sz1} \\ v_{sz2} \end{bmatrix} = \begin{bmatrix} r_s+L_{ls}p & 0 \\ 0 & r_s+L_{ls}p \end{bmatrix} . \begin{bmatrix} i_{sz1} \\ i_{sz2} \end{bmatrix}$$
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_r+L_{lr}p & 0 \\ 0 & r_r+L_{lr}p \end{bmatrix} . \begin{bmatrix} i_{rz1} \\ i_{rz2} \end{bmatrix} \tag{3}$$

3.Machine Model in z_3-z_4 Subspace

$$\begin{bmatrix} v_{sz3} \\ v_{sz4} \end{bmatrix} = \begin{bmatrix} r_s+L_{ls}p & 0 \\ 0 & r_s+L_{ls}p \end{bmatrix} . \begin{bmatrix} i_{sz3} \\ i_{sz4} \end{bmatrix}$$
$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_r+L_{lr}p & 0 \\ 0 & r_r+L_{lr}p \end{bmatrix} . \begin{bmatrix} i_{rz3} \\ i_{rz4} \end{bmatrix} \tag{4}$$

As it can be seen from these three subsystems, the electromechanical energy conversion takes place only in α-β subsystem, and the other subsystems have not any contribution in the energy conversion. The z_1-z_2 and z_3-z_4 subsystems produce only losses, so they should be controlled to be as small as possible. It can be concluded that the torque and speed controller synthesis and analysis for six-phase induction machines is almost the same as for three-phase induction machines: it is done with the equivalent model in α-β subspace.

B. Unsymmetrical Six Phase Induction Machine

In this section we open phase s_1 of machine. Whatever the open phases are, the stator and rotor voltage equations can be written as:

$$[V_s]=[R_s].[I_s]+\frac{d}{dt}([L_{ss}].[I_s]+[L_{sr}].[I_r])$$
$$[V_r]=[R_r].[I_r]+\frac{d}{dt}([L_{rr}].[I_r]+[L_{rs}].[I_s]) \tag{5}$$

When the phase s_1 is opened, the current and voltage vectors are:

$$\begin{aligned} V_s &= [v_{s2} \quad v_{s3} \quad v_{s4} \quad v_{s5} \quad v_{s6}]^T \\ I_s &= [i_{s2} \quad i_{s3} \quad i_{s4} \quad i_{s5} \quad i_{s6}]^T \\ V_r &= [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^T \\ I_r &= [i_{r1} \quad i_{r2} \quad i_{r3} \quad i_{r4} \quad i_{r5} \quad i_{r6}]^T \end{aligned} \tag{6}$$

In these equations, $[R_s]$, $[R_r]$, $[L_{ss}]$, $[L_{rr}]$, $[L_{sr}]$, and $[L_{rs}]$ are matrices including the model parameters described in [1].

The drawback of this model lies in the coupled nonlinear expressions of the inductance matrices. In order to obtain a decoupled model, it is shown in [1] that two transformation matrices are required. These matrices, so-called $[T_5]$ and $[T_6]$, split the original model into two decoupled subspaces: α-β subspace and z subspace. The first one represents the electromechanical energy conversion subspace while the other one gives only losses. The transformation matrix decomposing the model given (6) into α-β subspace and z subspace will be presented. The electromechanical energy conversion takes place only in α-β subspace which means that the MMF produced by five stator phases is equivalent to the MMF produced by two windings on the axes α and β with the currents $i_{s\alpha}$ and $i_{s\beta}$ respectively. These currents are defined by:

$$\begin{bmatrix} i_{s\alpha} \\ i_{s\beta} \end{bmatrix} = [T_c][I_s] \qquad [T_c] = \begin{bmatrix} [\alpha]/\|\alpha\| \\ [\beta]/\|\beta\| \end{bmatrix} \tag{7}$$

When s_1 is opened and $\gamma = 60^\circ$, the matrices[α], [β], $[T_5]$ and $[T_6]$ are:

$$[\alpha]=[\cos(\gamma) \quad \cos(\frac{2\pi}{3}) \quad \cos(\gamma+\frac{2\pi}{3}) \quad \cos(\frac{4\pi}{3}) \quad \cos(\gamma+\frac{4\pi}{3})]^T$$
$$[\beta]=[\sin(\gamma) \quad \sin(\frac{2\pi}{3}) \quad \sin(\gamma+\frac{2\pi}{3}) \quad \sin(\frac{4\pi}{3}) \quad \sin(\gamma+\frac{4\pi}{3})]^T \tag{8}$$

$$[T_5] = \begin{bmatrix} 0.5 & -0.5 & -1 & -0.5 & 0.5 \\ 0.866 & 0.866 & 0 & -0.866 & -0.866 \\ 0.5704 & -0.4102 & 0.6602 & -0.0898 & 0.25 \\ 0.5469 & 0.2265 & -0.2265 & 0.7735 & 0 \\ -0.0235 & 0.6367 & 0.1133 & -0.1367 & 0.75 \end{bmatrix} \tag{9}$$

$$[T_6] = \begin{bmatrix} 1 & 0.5 & -0.5 & -1 & -0.5 & 0.5 \\ 0 & 0.866 & 0.866 & 0 & -0.866 & -0.866 \\ 0.384 & -0.42 & 0.762 & -0.14 & 0.096 & 0.237 \\ 0.577 & 0.105 & -0.105 & 0.788 & -0.105 & 0.105 \\ 0.192 & 0.535 & 0.131 & -0.070 & 0.798 & -0.131 \\ -0.384 & 0.4296 & 0.237 & 0.140 & -0.096 & 0.762 \end{bmatrix} \tag{10}$$

By applying the transformation matrices $[T_5]$ and $[T_6]$ to the model of machine, the following formulation is obtained:

$$[T_5].[V_s]=[T_5].[R_s].[T_5]^{-1}.[T_5].[I_s]+\frac{d}{dt}([T_5].[L_{ss}].[T_5]^{-1}.[T_5].[I_s]+[T_5].[L_{sr}].[T_6]^{-1}.[T_6].[I_r])$$
$$[T_6].[V_r]=[T_6].[R_r].[T_6]^{-1}.[T_6].[I_r]+\frac{d}{dt}([T_6].[L_{rr}].[T_6]^{-1}.[T_6].[I_r]+[T_6].[L_{rs}].[T_5]^{-1}.[T_5].[I_s]) \quad (11)$$

1.Equations in α-β Subspace and z Subspaces:

From (11), the decoupled SPIM model gives the stator and rotor voltage equations:

$$v_{s\alpha}=R_s i_{s\alpha}+\frac{d}{dt}\lambda_{s\alpha} \qquad 0=R_r.i_{r\alpha}+\frac{d}{dt}\lambda_{r\alpha}+\omega_r.\lambda_{r\beta}$$
$$v_{s\beta}=R_s i_{s\beta}+\frac{d}{dt}\lambda_{s\beta} \qquad 0=R_r.i_{r\beta}+\frac{d}{dt}\lambda_{r\beta}-\omega_r.\lambda_{r\alpha} \quad (12)$$

$\lambda_{s\alpha}$, $\lambda_{s\beta}$, $\lambda_{r\alpha}$ and $\lambda_{r\beta}$ are respectively α-β components of the stator and rotor flux described as:

$$\lambda_{s\alpha}=L_{sd}i_{s\alpha}+M_d i_{r\alpha} \qquad \lambda_{r\alpha}=L_r i_{s\alpha}+M_d i_{s\alpha}$$
$$\lambda_{s\beta}=L_{sq}i_{s\beta}+M_q i_{r\beta} \qquad \lambda_{r\beta}=L_r i_{r\beta}+M_q i_{s\beta} \quad (13)$$

The other parameters are:

$$L_{sd}=L_{ls}+\|\alpha\|^2 L_{ms} \qquad M_d=\sqrt{3}.\|\alpha\|.L_{ms}$$
$$L_{sq}=L_{ls}+\|\beta\|^2 L_{ms} \qquad M_q=\sqrt{3}.\|\beta\|.L_{ms}$$

The stator voltage equations in z-subspace are:

$$v_{sz1}=R_s.i_{sz1}+L_{ls}\frac{di_{sz1}}{dt}$$
$$v_{sz2}=R_s.i_{sz2}+L_{ls}\frac{di_{sz2}}{dt} \quad (14)$$
$$v_{sz3}=R_s.i_{sz3}+L_{ls}\frac{di_{sz3}}{dt}$$

III. Single-Manifold SM Observer for IM Speed and State Estimation

The model of the induction motor in the stationary reference frame can be rewritten as [2]:

$$\frac{d\lambda_\alpha}{dt}=-\eta\lambda_\alpha-\omega_r\lambda_\beta+\eta L_m i_\alpha$$
$$\frac{d\lambda_\beta}{dt}=\omega_r\lambda_\alpha-\eta\lambda_\beta+\eta L_m i_\beta$$
$$\frac{di_\alpha}{dt}=\eta\beta\lambda_\alpha+\omega_r\beta\lambda_\beta-\gamma i_\alpha+\frac{1}{\sigma L_s}v_\alpha \quad (15)$$
$$\frac{di_\beta}{dt}=-\omega_r\beta\lambda_\alpha+\eta\beta\lambda_\beta+-\gamma i_\beta+\frac{1}{\sigma L_s}v_\beta$$

The parameters σ, β and γ are defined as:

$$\sigma=1-\frac{L_m^{\ 2}}{L_s L_r} \quad \beta=\frac{L_m}{\sigma L_s L_r} \quad \gamma=\frac{1}{\sigma L_s}\left(\frac{L_m^{\ 2}}{L_r^{\ 2}}R_r+R_s\right)$$

The equations of the single-manifold SM observer are:

$$\frac{d\hat{\lambda}_\alpha}{dt}=-\eta\hat{\lambda}_\alpha-\hat{\omega}_r\hat{\lambda}_\beta+\eta L_m i_\alpha$$
$$\frac{d\hat{\lambda}_\beta}{dt}=\hat{\omega}_r\hat{\lambda}_\alpha-\eta\hat{\lambda}_\beta+\eta L_m i_\beta$$
$$\frac{d\hat{i}_\alpha}{dt}=\eta\beta\hat{\lambda}_\alpha+\hat{\omega}_r\beta\hat{\lambda}_\beta-\gamma i_\alpha+\frac{1}{\sigma L_s}v_\alpha \quad (16)$$
$$\frac{d\hat{i}_\beta}{dt}=-\hat{\omega}_r\beta\hat{\lambda}_\alpha+\eta\beta\hat{\lambda}_\beta+-\gamma i_\beta+\frac{1}{\sigma L_s}v_\beta$$

The speed estimate $\hat{\omega}_r$ is of the form:

$$\hat{\omega}_r=\omega_0.sign(s_1) \quad (17)$$

where ω_0 is a constant design gain. The manifold S_1 is:

$$s_1=\hat{\lambda}_\alpha\bar{i}_\beta-\hat{\lambda}_\beta\bar{i}_\alpha \quad (18)$$

In (16), the voltages v_α and v_β and currents i_α and i_β are measured quantities. The mismatches are defined as:

$$\bar{\lambda}_\alpha=\hat{\lambda}_\alpha-\lambda_\alpha,\ \bar{\lambda}_\beta=\hat{\lambda}_\beta-\lambda_\beta,\ \bar{i}_\alpha=\hat{i}_\alpha-i_\alpha,\ \bar{i}_\beta=\hat{i}_\beta-i_\beta$$

By subtracting (15) from (16), the mismatches are:

$$\frac{d\bar{\lambda}_\alpha}{dt}=-\eta\bar{\lambda}_\alpha-\hat{\omega}_r\hat{\lambda}_\beta+\omega_r\lambda_\beta$$
$$\frac{d\bar{\lambda}_\beta}{dt}=\hat{\omega}_r\bar{\lambda}_\alpha-\omega_r\lambda_\alpha-\eta\bar{\lambda}_\beta$$
$$\frac{d\bar{i}_\alpha}{dt}=\eta\beta\bar{\lambda}_\alpha+\hat{\omega}_r\beta\hat{\lambda}_\beta-\omega_r\beta\lambda_\beta \quad (19)$$
$$\frac{d\bar{i}_\beta}{dt}=-\hat{\omega}_r\beta\hat{\lambda}_\alpha+\omega_r\beta\lambda_\alpha+\eta\beta\bar{\lambda}_\beta$$

To study the existence of the SM motion, s_1 is differentiated, this gives:

$$\dot{s}_1=\frac{d\hat{\lambda}_\alpha}{dt}\bar{i}_\beta+\hat{\lambda}_\alpha\frac{d\bar{i}_\beta}{dt}-\frac{d\hat{\lambda}_\beta}{dt}\bar{i}_\alpha-\hat{\lambda}_\beta\frac{d\bar{i}_\alpha}{dt} \quad (20)$$

In (20), replace the derivatives from (16) and (19), the result is:

$$\dot{s}_1=-\eta(\hat{\lambda}_\alpha\bar{i}_\beta-\hat{\lambda}_\beta\bar{i}_\alpha)-\hat{\omega}_r(\hat{\lambda}_\alpha\bar{i}_\alpha+\hat{\lambda}_\beta\bar{i}_\beta)+\eta L_m(i_\alpha\bar{i}_\beta-i_\beta\bar{i}_\alpha)$$
$$+\beta\omega_r(\hat{\lambda}_\alpha\lambda_\alpha+\hat{\lambda}_\alpha\lambda_\alpha)+\beta\eta(\hat{\lambda}_\alpha\bar{\lambda}_\beta-\hat{\lambda}_\beta\bar{\lambda}_\alpha)-\beta(\hat{\lambda}_\alpha^2+\hat{\lambda}_\beta^2)\hat{\omega}_r \quad (21)$$

Note that in (21), the speed estimation $\hat{\omega}_r$ appears twice. The term $-\beta(\hat{\lambda}_\alpha^2+\hat{\lambda}_\beta^2)\hat{\omega}_r$ is very important. Assuming that this observer estimates the fluxes, $-\beta(\hat{\lambda}_\alpha^2+\hat{\lambda}_\beta^2)=-\beta\left|\hat{\lambda}\right|^2<0$ Then, since this coefficient is reliable (non-zero and always negative), with high ω_0, the term $-\beta(\hat{\lambda}_\alpha^2+\hat{\lambda}_\beta^2)\omega_0 \text{sign}(s_1)$ can be made as large as desired – this should enforce the SM motion on s_1. On the other hand, the term $-\hat{\omega}_r(\hat{\lambda}_\alpha \bar{i}_\alpha+\hat{\lambda}_\beta \bar{i}_\beta)$ may be undesirable since the presence of a second switching term on the right side of (21) could disturb the SM motion already in place. However, if the observer converges or if the current mismatches are small, the term $(\hat{\lambda}_\alpha \bar{i}_\alpha+\hat{\lambda}_\beta \bar{i}_\beta)$ should be zero or very small. By rewriting (21), this becomes:

$$\dot{s}_1 = f(\eta,\beta,\hat{\lambda}_\alpha,\hat{\lambda}_\beta,\bar{i}_\alpha,\bar{i}_\beta) \\ -\left[\beta(\hat{\lambda}_\alpha^2+\hat{\lambda}_\beta^2)+(\hat{\lambda}_\alpha \bar{i}_\alpha+\hat{\lambda}_\beta \bar{i}_\beta)\right]\omega_0 sign(s_1) \tag{22}$$

The function f is:

$$f=-\eta(\hat{\lambda}_\alpha \bar{i}_\beta-\hat{\lambda}_\beta \bar{i}_\alpha)+\eta L_m(i_\alpha \bar{i}_\beta-i_\beta \bar{i}_\alpha)+\beta\omega_r(\hat{\lambda}_\alpha\lambda_\alpha+\hat{\lambda}_\alpha\lambda_\alpha) \\ +\beta\eta(\hat{\lambda}_\alpha\bar{\lambda}_\beta-\hat{\lambda}_\beta\bar{\lambda}_\alpha) \tag{23}$$

Since f cannot tend to infinity (f has a finite upper estimate), if $(\hat{\lambda}_\alpha \bar{i}_\alpha+\hat{\lambda}_\alpha \bar{i}_\alpha)+\beta(\hat{\lambda}_\alpha^2+\hat{\lambda}_\beta^2)>0$ then, the SM motion can be enforced on manifold s_1. If the designs gain ω_0 is chosen to satisfy $\left[(\hat{\lambda}_\alpha \bar{i}_\alpha+\hat{\lambda}_\beta \bar{i}_\beta)+\beta(\hat{\lambda}_\alpha^2+\hat{\lambda}_\beta^2)>0\right]\omega_0>|f|$ the manifold and its derivative will have opposite signs $\dot{s}_1\times s_1<0$ .As a result, the manifold is attractive and $s_1\rightarrow 0$. After SM occurs, $s_1 = 0$ identically. It means that:

$$\hat{\lambda}_\alpha\bar{i}_\beta-\hat{\lambda}_\beta\bar{i}_\alpha=0 \tag{24}$$

From a theoretical point of view, $s_1 = 0$ does not necessarily imply that $\bar{i}_\alpha=0$ and $\bar{i}_\beta=0$. It can be conclude immediately that $s_1=0$ can be obtained with a set of non-zero fluxes and current mismatches.

IV. Double Manifold SM Observer for IM Speed and State Estimation

The proposed observer is obtained by augmenting the single-manifold observer (16) with additional feedback terms –these are also switching terms [2]. The objective is to obtain an observer that enforces the SM motion at the intersection of the two manifolds. If SM occurs and both manifolds tend zero, this may improve the estimation properties of (16). The manifolds chosen are:

$$s_1=\hat{\lambda}_\alpha\bar{i}_\beta-\hat{\lambda}_\beta\bar{i}_\alpha \\ s_2=\hat{\lambda}_\alpha\bar{i}_\alpha+\hat{\lambda}_\beta\bar{i}_\beta \tag{25}$$

If SM can be enforced on these manifolds, then $S_1=0$ and $S_2=0$. This is equivalent to:

$$\begin{bmatrix}-\hat{\lambda}_\beta & \hat{\lambda}_\alpha \\ \hat{\lambda}_\alpha & \hat{\lambda}_\beta\end{bmatrix}\begin{bmatrix}\bar{i}_\alpha \\ \bar{i}_\beta\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix} \tag{26}$$

Since $\det(\Lambda)=-(\hat{\lambda}_\alpha{}^2+\hat{\lambda}_\beta{}^2)\neq 0$ (assuming that the fluxes converge), (26) has unique solution:

$$\bar{i}_\alpha=0 \quad ,\bar{i}_\beta=0 \tag{27}$$

Therefore, the current estimates of the observer are guaranteed to converge. The equations of the observer are:

$$\begin{aligned}\frac{d\hat{\lambda}_\alpha}{dt}&=-\eta\hat{\lambda}_\alpha-\hat{\omega}_r\hat{\lambda}_\beta+\eta L_m i_\alpha\\ \frac{d\hat{\lambda}_\beta}{dt}&=\hat{\omega}_r\hat{\lambda}_\alpha-\eta\hat{\lambda}_\beta+\eta L_m i_\beta\\ \frac{d\hat{i}_\alpha}{dt}&=\eta\beta\hat{\lambda}_\alpha+\hat{\omega}_r\beta\hat{\lambda}_\beta-\gamma i_\alpha+\frac{1}{\sigma L_s}v_\alpha-k\hat{\lambda}_\alpha u_2\\ \frac{d\hat{i}_\beta}{dt}&=-\hat{\omega}_r\beta\hat{\lambda}_\alpha+\eta\beta\hat{\lambda}_\beta+-\gamma i_\beta+\frac{1}{\sigma L_s}v_\beta-k\hat{\lambda}_\beta u_2\end{aligned} \tag{28}$$

where k is a design parameter. The switching controls are:

$$\hat{\omega}_r=\omega_0.sign(s_1) \\ u_2=M.sign(s_2) \tag{29}$$

By subtracting the original equations, the mismatches are:

$$\begin{aligned}\frac{d\bar{\lambda}_\alpha}{dt}&=-\eta\bar{\lambda}_\alpha-\hat{\omega}_r\hat{\lambda}_\beta+\omega_r\lambda_\beta\\ \frac{d\bar{\lambda}_\beta}{dt}&=\hat{\omega}_r\bar{\lambda}_\alpha-\omega_r\lambda_\alpha-\eta\bar{\lambda}_\beta\\ \frac{d\bar{i}_\alpha}{dt}&=\eta\beta\bar{\lambda}_\alpha+\hat{\omega}_r\beta\hat{\lambda}_\beta-\omega_r\beta\lambda_\beta-k\hat{\lambda}_\alpha u_2\\ \frac{d\bar{i}_\beta}{dt}&=-\hat{\omega}_r\beta\hat{\lambda}_\alpha+\omega_r\beta\lambda_\alpha+\eta\beta\bar{\lambda}_\beta-k\hat{\lambda}_\beta u_2\end{aligned} \tag{30}$$

The two manifolds are differentiated, the derivatives are replaced. For S_1, this gives:

$$\dot{s}_1=-\eta(\hat{\lambda}_\alpha\bar{i}_\beta-\hat{\lambda}_\beta\bar{i}_\alpha)-\hat{\omega}_r(\hat{\lambda}_\alpha\bar{i}_\alpha+\hat{\lambda}_\beta\bar{i}_\beta)+\eta L_m(i_\alpha\bar{i}_\beta+i_\beta\bar{i}_\alpha)+ \\ \beta\omega_r(\hat{\lambda}_\alpha\lambda_\alpha+\hat{\lambda}_\beta\lambda_\beta)+\beta\eta(\bar{\lambda}_\beta\hat{\lambda}_\alpha-\bar{\lambda}_\alpha\hat{\lambda}_\beta)-\beta(\hat{\lambda}_\alpha^2+\hat{\lambda}_\beta^2)\hat{\omega}_r \tag{31}$$

For S_2, the result is:

$$\dot{s}_2=-\eta(\hat{\lambda}_\alpha\bar{i}_\alpha+\hat{\lambda}_\beta\bar{i}_\beta)+\hat{\omega}_r(\hat{\lambda}_\alpha\bar{i}_\beta-\hat{\lambda}_\beta\bar{i}_\alpha)+\eta L_m(i_\alpha\bar{i}_\alpha+i_\beta\bar{i}_\beta)+ \\ \beta\eta(\hat{\lambda}_\alpha\bar{\lambda}_\alpha+\hat{\lambda}_\beta\bar{\lambda}_\beta)-\beta\omega_r(\lambda_\beta\hat{\lambda}_\alpha-\lambda_\alpha\hat{\lambda}_\beta)-k(\hat{\lambda}_\alpha^2+\hat{\lambda}_\beta^2)u_2 \tag{32}$$

Note that the switching term u_2 does not appear in (31). The expressions are rewritten as:

$$\dot{s}_1 = f_1 - \beta(\hat{\lambda}_\alpha{}^2 + \hat{\lambda}_\beta{}^2).\omega_0.sign(s_1)$$
$$\dot{s}_2 = f_2 - k(\hat{\lambda}_\alpha{}^2 + \hat{\lambda}_\beta{}^2).M.sign(s_2) \quad (33)$$

By inspecting (31) and (32), the functions f_1, f_2 are a combination of the estimates, the observer mismatches and also involve the speed and the motor parameters. If the estimates are finite, both functions have an upper estimate (they do not tend to infinity). Based on (33), with high enough values for ω_0 and M assuming that $\hat{\lambda}_\alpha^2+\hat{\lambda}_\beta^2 \neq 0$ the switching terms on the right side of (33) will be such that:

$$\beta(\hat{\lambda}_\alpha^2 + \hat{\lambda}_\beta^2).\omega_0 > |f_1|$$
$$\mathrm{k}(\hat{\lambda}_\alpha^2 + \hat{\lambda}_\beta^2).\mathrm{M} > |f_2| \quad (34)$$

Then, $\dot{s}_1$ and s_1 have opposite signs; the same for $\dot{s}_2$ and s_2. As a result, the manifolds are attractive and $s_1 \rightarrow 0$, $s_2 \rightarrow 0$.

V. Simulation Results

In this section, a five-phase induction machine, with the parameters given in Table I, is simulated to validate the double manifold sliding mode observer. The simulation is carried out using Matlab/Simulink. Fig. 1 shows the machine speed before applying vector control. It shows that the speed of machine isn't fixed and oscillates around 150 rpm without any control where the nominal speed is 2800 rpm. Stator currents of the machine in this case are completely unsymmetrical because of unsymmetrical supply. In order to show the behavior of the proposed observer in unsymmetrical operating condition, two tests are considered. In the first test, the motor is started towards a speed of 500 rpm and accelerates to 75% upper speed at t = 6 s. The simulations of this observer confirm the theoretical analysis. The SM motion occurs and s1 and s2 tends to zero. The estimated currents of the observer tend to the real currents. Fig. 2 shows the real and estimated speed of the motor under fault condition operation of the motor which shows the good performance of the observer. Fig. 3 shows the two manifold of the observer. As we can see when sliding mode motion occurs two manifolds tend to zero. Fig. 4 shows real and estimated stator current of the motor. The current estimates converge and the mismatches currents tend to zero as per (18). In the next test, we applied trapezoid reference speed to the machine control algorithm. In this test we want to examine the ability of the method in speed inversion condition. Fig. 5 shows that mismatch between references, real and estimated speeds are zero.

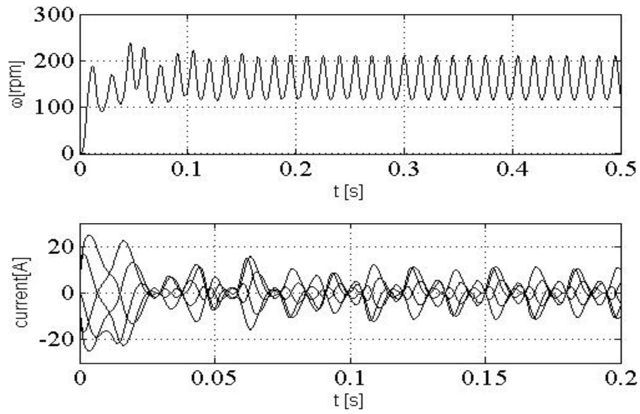

Fig. 1 Running up test without any applied control

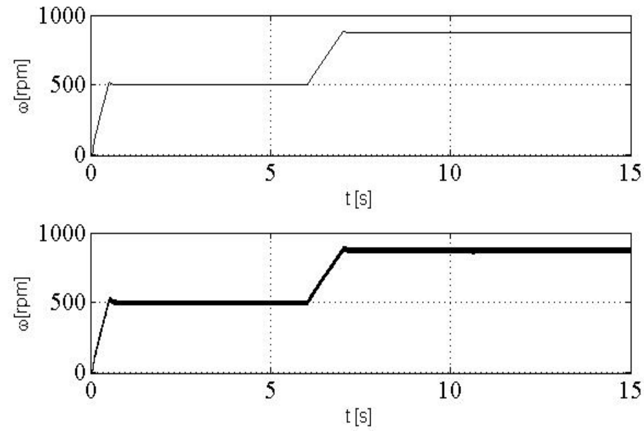

Fig. 2 Real and estimated speed in acceleration test

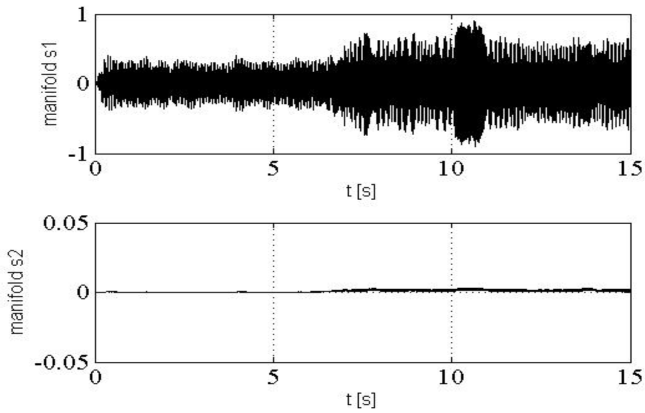

Fig. 3 Two manifolds of observer in acceleration test

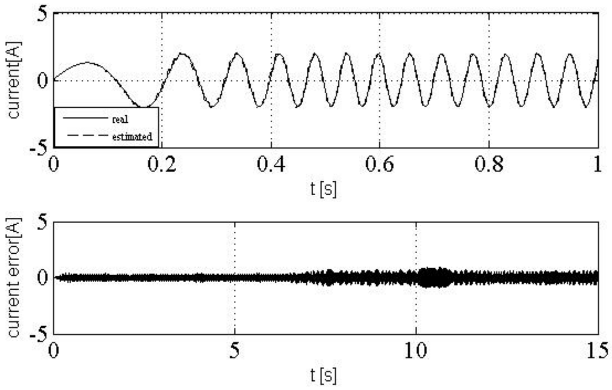

Fig. 4 Real and estimated current components in acceleration test

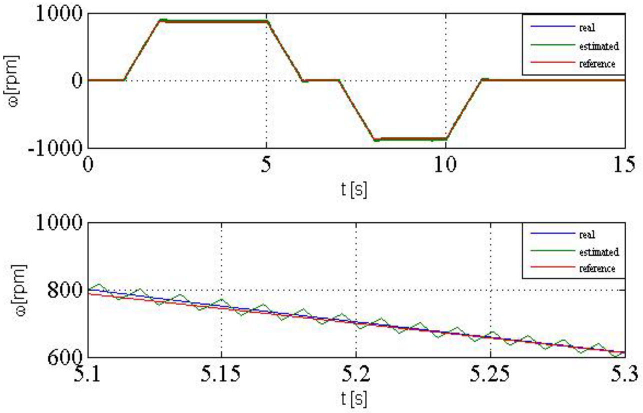

Fig. 5 Reference, real and estimated speed in speed inversion test

VI. Parameter Sensitivity Analysis

This section investigates the sensitivity of the observer to the variation of the IM parameters. The variation of the rotor resistance R_r and the magnetizing inductance L_m is considered. The variation of R_s is not considered (if needed, R_s can be estimated using stator-mounted thermistors).

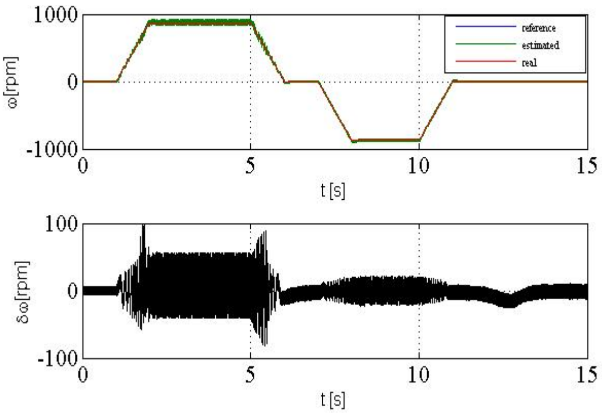

Fig. 6 L_m variation to half of rated value

The observer is implemented using the rated parameters while the machine is detuned: R_r is increased up to double the rated value; L_m is reduced to half its rated value (this corresponds to a practical situation). Simulations show that the observer is rather sensitive to L_m and is insensitive to R_r.

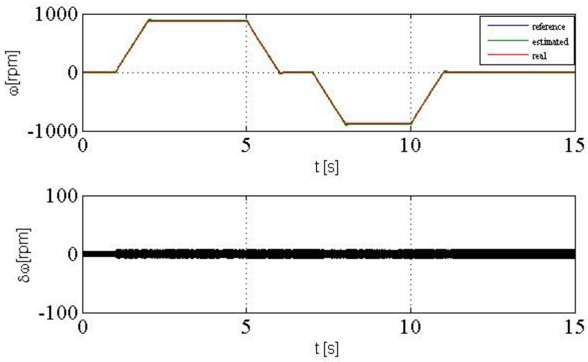

Fig. 7 R_r variation to double of rated value

VII. Conclusion

In this paper we discuss the performance and ability of double manifold sliding mode observer in Sensorless control of multiphase induction machine under fault condition. This observer is a modification of single manifold sliding mode observer which are discussed in many literature. Simulation results show that when sliding mode motion occurs observer can estimate machine currents and speed accurately and converge completely. On the other hand in speed inversion test, this observer work very well and error between real, estimated and reference speed is negligible. It is found that, under improper parameters, this observer is sensitive to the variation of the magnetizing inductance and is quite robust to the value of the rotor resistance.

TABLE I
Parameter of Simulated Machine

No. of poles	2
Rated output	90 W
Rated voltage	42 V
Rated current	2.6 A
Rated speed	2800 rpm
Rated torque	0.3 N.m
Stator resistance (Rs)	1.04 Ω
Stator inductance (Ls)	0.0127 mH
Rotor resistance (Rr)	0.4107 Ω
Rotor inductance (Lr)	0.0127 mH
Mutual inductance (M)	0.0115 mH

References

[1] R. Kianinezhad, B. Nahid-Mobarakeh, L. Baghli, F. Betin, G.A. Capolino, "Modeling and Control of Six-Phase symmetrical Induction Machine Under Fault Condition Due to Open Phases", *IEEE Trans on Industrial Electronics*, Vol. 55, No. 5, MAY 2008.

[2] M. Comanescu, "Single and double compound manifold sliding mode observers for flux and speed estimation of the induction motor drive", *Published in IET Electric Power Applications*, October 2013.

[3] M. Jafarifar, R. Kianinezhad, S. GH. Seifosaadat, “Sliding Mode Sensorless Control of Symmetrical Six-Phase Induction Machines”, *6th international conference on electrical and electronic engineering* (ELECO), 2009, Turkey.

[4] R. Kianinezhad, B. Nahidmobarakeh, F. Betin and G. A. Capolino, “Sensorless field-oriented control for six-phase induction machines”, *IEEE Industry Applications Society Conference*, IAS 2005; 2-6 Oct. 2005, Hong Kong.

[5] A. Derdiyok, "Speed-Sensorless Control of Induction Motor Using a Continuous Control Approach of Sliding-Mode and Flux Observer", *IEEE Trans. Industrial Electronics*, Vol. 52, No. 4, pp: 1170-1176, AUGUST 2005.

[6] Y. Zhao, T. A. Lipo, “Modelling and control of a multi-phase induction machine with structural unbalance. Part II: Field-oriented control and experimental verification,” *IEEE Trans on Energy Conversion,* Vol. 11, no. 3, pp. 578–584, Sep. 1996.

[7] G. K. Singh, K. Nam, and S. K. Lim, “A simple indirect field-oriented control scheme for multiphase induction machine,” *IEEE Trans on. Industrial Electronics* vol. 52, no. 4, pp. 1177–1184, Aug. 2005.

[8] C. B. Jacobina, R. S. Miranda, M. B. de R. Corrêa, A. M. N. Lima, “Disturbance-free operation of a six-phase AC motor drive system”, *in Proc.35th Annu.PESC,* Aachen, Germany, Jun. 20–25, 2004, vol. 2, pp. 925–931.

[9] G. K. Singh, Vinay Pant, ”Analysis of a Multiphase Induction Machine Under Fault Condition in a Phase- Redundant A.C. Drive Syste”, *Electric Machines & Power Systems journal*, Nov 2010.

[10] M. A. Fnaiech, F. Betin, G. Capolino and F. Fnaiech, “Fuzzy Logic and Sliding-Mode Controls Applied to Six-Phase Induction Machine With Open Phases”, *IEEE Trans. on Industrial Electronics,* VOL. 57, NO. 1, January 2010.