Design of a Compact Herriott Cell for Heat Flux Measurement Applications

R. G. Ramírez-Chavarría, C. Sánchez-Pérez, V. Argueta-Díaz

Abstract—In this paper we present the design of an optical device based on a Herriott multi-pass cell fabricated on a small sized acrylic slab for heat flux measurements using the deflection of a laser beam propagating inside the cell. The beam deflection is produced by the heat flux conducted to the acrylic slab due to a gradient in the refractive index. The use of a long path cell as the sensitive element in this measurement device, gives the possibility of high sensitivity within a small size device. We present the optical design as well as some experimental results in order to validate the device's operation principle.

Keywords—Heat flux, herriott cell, optical beam deflection, thermal conductivity.

I. INTRODUCTION

A MULTI-PASS optical cell is a device with concave mirrors in which a light beam undergoes multiple reflections. Among the multi-pass cells most reported in literature, one can found the White cell [1] and Herriott cell [2]. Due to its versatility, multi path cells have been used as optical delay lines in antennas [4], also they have been widely used for liquids or gas detection by absorption spectroscopy where optical paths in the order of some meters have been obtained highly increasing sensitivity [3]. The Herriot cell has the advantage of been mechanically more stable since two mirrors are used instead of three as in the White cell. It is possible to distinguish three types of Herriott cells according to the mirrors used: spherical [2], astigmatic [5] and cylindrical [6]. Herriott cell based in cylindrical configuration implies the use of uniaxial curvature mirrors, so it becomes simpler to be fabricated with the same performance of the other two configurations with biaxial mirrors. In most of the reported works the cell cavity was formed in free space making the systems of big dimensions and mechanically delicate to be adjusted. We are not aware of Herriott cell integrating the cell elements and propagation media in one block.

In this work we present the design, fabrication and test of a Herriott cell for heat flux measurements using the photo-thermal laser beam deflection technique within a thermo-optic

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slab, this principle was previously reported by our group [7]. This technique was used to characterize heat conduction in biological tissue with good sensitivity to distinguish different tissues but the sample's size was big since the optical path required for measurements was of about 3 cm. The redesign of the sensor using a Herriot cell will allow us to highly reduce the size of the sample while keeping the same sensitivity or even to increase this last.

II. OPERATION PRINCIPLE

The proposed optical cell is formed by two cylindrical mirrors M_1 and M_2 facing each other and separated by a distance d as shown in Fig. 1. The mirrors were fabricated by manufacturing two faces of an acrylic block with a curvature radius R_1 and R_2 and later deposition of an aluminum reflecting layer by sputtering to form M_1 and M_2 respectively. When a laser beam enters the Herriott cell it will bounce several times within the cell and later exit after a specific number of passes. If a heat Q is applied to the multi-pass cell, the output beam will be deflected a certain angle (θ_{MR}) by the presence of a temperature gradient perpendicular to the optical axis direction (z). In this way by using a photo-detector with a knife-edge, blocking around of 50% of its sensitive surface, a lateral displacement of the beam due to the deflection could be measured as a change in the power detected.
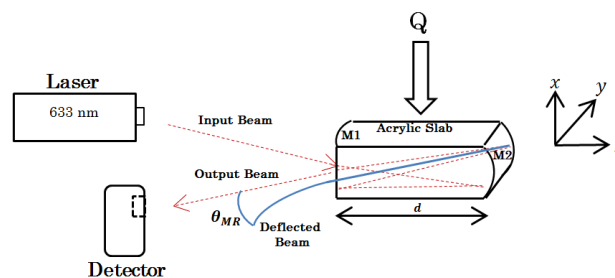

Fig. 1 Schematic diagram of the Herriott cell based device for heat flux measurements

III. DESIGN OF THE HERRIOTT CELL

The proposed Herriott multi-pass cell device consists of two cylindrical mirrors of equal curvature radii $R_1 = R_2 = R$ separated by a distance d less than or equal to two times R in order to ensure the stability criteria described by [6]. A laser beam enters the cell through a non-aluminized port in mirror M_1. The beam is periodically reflected and refocused between the two mirrors and then, after a designated number of passes N, exits through the input port with a slope that is the opposite of the entry slope. As a result, the total path length L

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traversed in the cell is approximately $L = N \cdot d$.

A. Matrix Ray Propagation Method

For the cylindrical configuration of the Herriott cell, one of the mirrors could be rotated a certain angle δ with respect to the other mirror for the output beam to exit at the same input port. One may calculate the ray trace by the matrix method using the paraxial approximation as it is done for thin lens systems [6]. This method allows calculating the propagation of light rays trough the Herriott cell, hence where the beam hits at the mirrors by direct matrix manipulation. Let us denote x_o and y_o as the initial position and η and γ as the slopes of the beam respectively, the bidimensional position can be represented by a vector $\mathbf{r} = [x_o\ \eta\ y_o\ \gamma]$. One may define the translation matrix $\mathbf{D}$ as:

$$\mathbf{D} = \begin{bmatrix} 1 & d/n & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d/n \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

with n as the refractive index of the propagation media, i.e. $n = 1.49$ for acrylic. There exists one reflection matrix for each mirror. If M_1 has a curvature radius R along x axis, $\mathbf{R_1}$ becomes its reflection matrix:

$$\mathbf{R_1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{-2n}{R} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

and if M_2 has its radius R oriented along the y axis, the reflection matrix $\mathbf{R_2}$ for the second mirror is:

$$\mathbf{R_2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \frac{-2n}{R} & 1 \end{bmatrix}. \tag{3}$$

As seen in above matrices, mirrors are orthogonally aligned so the angle between their curvatures is $\delta = 90°$.

After one round trip of the beam within the cell, one can calculate the total transfer matrix $\mathbf{C}$:

$$\mathbf{C} = \mathbf{R_1} \cdot \mathbf{D} \cdot \mathbf{R_2} \cdot \mathbf{D}. \tag{4}$$

In the case when $\delta \neq \pi/2$ it is necessary to modify the reflection matrix for M_2 in terms of a rotation matrix $\mathbf{T(\delta)}$ [8].

The positions of the beam after N passes, namely, when it reflects at both mirrors, can be computed to obtain a spot pattern diagram in terms of M_1 and M_2 by the vectors $\mathbf{r_1}$ and $\mathbf{r_2}$ respectively:

$$\mathbf{r_1} = \mathbf{D} \cdot \mathbf{R_2} \cdot \mathbf{D} \cdot \mathbf{C}^{0.5N-1} \cdot \mathbf{r} \quad \text{, and} \tag{5}$$

$$\mathbf{r_2} = \mathbf{D} \cdot \mathbf{C}^{0.5N-1} \cdot \mathbf{r}. \tag{6}$$

B. Herriot Cell Characteristic Parameters

There are also some characteristic parameters necessary to be taken in account; in order to design the Herriott cell. The reentrant angle ϕ_R, which refers to the angle between two successive reflections; since $R1 = R2 = R$ for mirrors radii, exists an expression that allows determining the angle between two consecutives reflections ϕ_R as a function of R and d,

$$\phi_R = \arccos\sqrt{\left(1 - \frac{d}{R}\right)} \tag{7}$$

This angle is related with the number of orbits M of the beam on the mirror´s surface and related with N as:

$$\phi_R = \pi M / N \tag{8}$$

M has valid solutions for even integer numbers, as in the case of $M = 2$ it implies that the sum of ϕ_R will complete an entire cycle, i.e. 360 °. An exceptional case occurs when $M = 1$ which represents a half orbit made by the spot pattern, 180°. The term N can be obtained by (8) if ϕ_R and M are previously known.

We evaluate (7) in order to calculate ϕ_R. Fig. 2 shows the values ϕ_R as function of d for mirror curvature radii R from 0 to 5 cm.
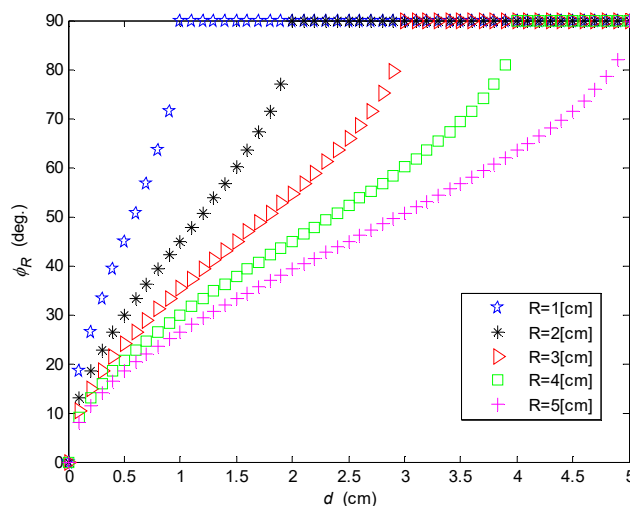

Fig. 2 Reentrant angle ϕ_R as a function of mirror separation d for different curvature radii R

It can be seen that ϕ_R increases as the value of d does. For $R=1$ cm, changes in reentrant angle are more significant than for bigger values. The flat region at the top of the plot represents the instability of the cell, this is for a distance between the mirrors $d \geq R$, the reentrant angle ϕ_R does not change its value, meaning that the beam gets a path out of the cell. Thus we can write an expression for stability criteria as: $0 < d < R$.

For calculating the total path length, first it is necessary to calculate N using (8), knowing ϕ_R and assuming in this case $M = 1$. Fig. 3 shows the behavior of L as a function of the mirror separation d for five different values of curvature radii R from 1 to 5 cm.

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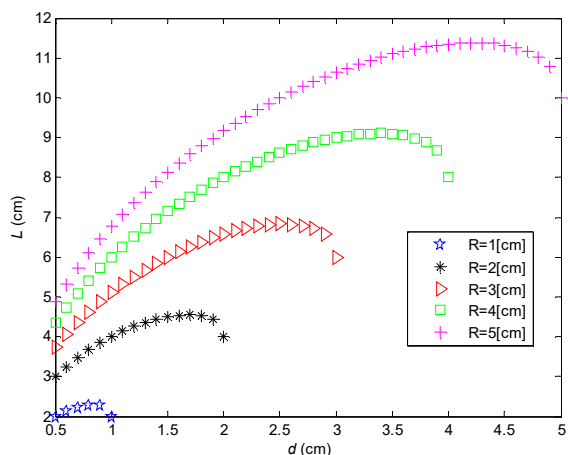Vol:9, No:8, 2015

Fig. 3 Total path length L as a function of mirror separation d for different curvature radii R

It is possible noting at Fig. 3 that, the value of L increases as the mirrors separation also increases, until is valid the stability criterion. For larger values of R, the change on L is also larger. Also it is important to note that, when $d \leq R/2$, the behavior of L can be seen as linear, but when d becomes larger than $d = R/2$ the change in the total path increases slowly.

The plots above can help us to obtain the possible values of R and d for target path length, so this is our design criteria. Our desired total path length must be around 4 cm, as we can obtain enough sensitivity for heat flux measurement applications.

The fabrication techniques available for manufacturing the acrylic slab, allow us to study the cases for $0.5 \leq d \leq 1.5$ cm and $2 \leq R \leq 5$ cm, in order to obtain the characteristic parameters of the Herriott cell. In Table I we resume three cases for selected values of d and R, including the performance parameters obtained by its analytical solutions.

TABLE I
HERRIOTT CELLS AND ITS CHARACTERISTIC PARAMETERS

Cell	d	R	δ	ϕ_R	N	M	L
A	1.5 cm	3 cm	90°	45°	4	1	6 cm
B	1 cm	2 cm	90°	45°	4	1	4 cm
C	0.5 cm	2 cm	90°	30°	11	1	5.5 cm

The analysis was made taking in account that we need a total path length near to 4 cm. According to results in Table I, it can be seen that cells B and C have the same value of R but the separation of the mirrors in C is half of B, resulting on a larger number of passes and therefore the total path length increases 37.5% respect to the path length for B, nevertheless a large amount of passes may represent larger losses on the beam due to reflective surfaces and attenuation of the acrylic. When comparing cells A and B, it can be seen that for A, mirror separation seems to be just 50% larger than B, but the mirror curvature radii now is 3 cm. With these parameters the numbers of passes for cell A stay at the same value than cell B, nevertheless the total path length increases 50%. The only trouble of the cell A is related with its dimensions because it

will be larger than other two cells, but it will be useful in order to demonstrate its application on the heat flux measurement principle. So, we consider cell A as the case of study due its fabrication facility as well as its characteristic parameters, $d = 1.5$ cm, $R = 3$cm, $N = 4$ and $L = 6$ cm.

In order to know the behavior of the beam within the cell, we obtained the spots positions on M_1 and M_2. In Fig. 4 is shown the spot pattern formed both mirrors of the cell A, obtained by the matrix transfer method.
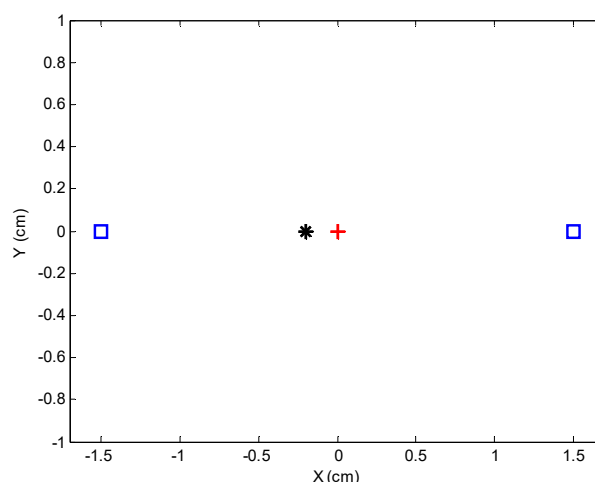

Fig. 4 Spot pattern formed on the mirrors of the Herriott cell obtained with matrix transfer method. Square bounces are the spots located on M_2, asterisk bounce is the spot on M_1 and the plus sign mark represents the input/output port

In the above figure, the squared marks represent the spots on M_2, it is possible to see that these spots are located near to the mirror edge such they cover almost the whole mirror's area. The one at right side represents the first incident spot, immediately after the beam enters to the cell. The spot at the left hand is the $N - 1$, reflection, thus is the last incidence, before the beam exits the cell. In the other hand, the asterisk point represents the incident spot on M_1 which is located near to intput/ouput port marked with a plus symbol.

The behavior of the beam well agrees with the analytical solutions, so with $d = 1.5$ cm and $R = 3$ cm the cell has 4 passes and 3 reflections, resulting on a total path length equal to 6 cm.

In summary, the cell A satisfies simultaneously two requirements, the total path length and the fabrication tolerances, so it will be adequate for obtaining experimental preliminary results for heat flux measurements.

IV. EXPERIMENTAL RESULTS

A. Fabrication of the Herriott Cell

To fabricate the cell we use a laser cut machine "40W/45W CO_2 Hobby Laser" of Full Spectrum Laser™, with resolution of 1000 dpi for cutting the acrylic slab with curved shape on two sides. The resolution of R that could be obtained is 2 cm and for the mirror separation d, the fewer value possible to fabricate is 0.5 cm, making possible to fabricate the three cells

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depicted on Table I.

The mirrors of the cell were fabricated using sputtering deposition method by coating the two curved faces of the acrylic slab with an aluminum layer of approximately 1 μm thick. The reflectance is near to 75%. In Fig. 5 we show the first compact integrated Herriott cell that we fabricated, according to the parameters of cell A from Table I.
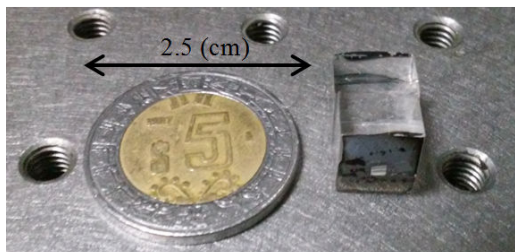

Fig. 5 Fabricated Herriott cell integrated on an acrylic slab with $d = 1.5$ cm and $R = 3$ cm

B. Heat Flux Measurement

To experimentally characterize heat flux response function of the cell, we used a λ=633 nm laser as the light source, considering the curvature of M_1 is on x direction and M_2 radius is on y axis. The beam enters the cell at the center of mirror, i.e. x_o=0 and y_o=0, with an initial slope γ=10° only at y direction (see Fig. 1).

In order to evaluate the heat flux response of the proposed device, we designed a measurement system for heat flux quantifying using the photo-thermal beam deflection technique [8], in which the deflection θ_i is described by:

$$\theta_i = \frac{1}{n} \frac{dn}{dT} \int \frac{\partial T}{\partial x} \, dz \qquad (9)$$

where n is the acrylic's refractive index, dn/dT refers to the thermo-optic coefficient of acrylic and $\partial T/\partial x$ is the thermal gradient between two faces of the acrylic slab induced by the heat source Q applied in x direction. Therefore at the output of the slab, the beam will be deviated from its linear trajectory a certain angle θ_i. In a multiple pass device like our Herriot cell, it is necessary to obtain the deflecting angle after N passes, which will be the sum of the deflection of the $i-th$ beam passing within the cell, hence giving higher sensitivity than in the heat flux measurements for a single pass device [8]. Let us consider a multiple reflection deflected angle denoted by θ_{MR} which can be calculated as:

$$\theta_{MR} = \sum_{i=1}^{N} \theta_i \qquad (10)$$

In order to induce a heat flux Q in the cell, we use a thermo element that can increase the temperature of the acrylic slab in its upper face, as shown in Fig. 1. We use a Peltier cell, in which one of its faces becomes colder and the other side becomes hotter when an electrical current flows on it, due to the Peltier and Joule effects [9]. To estimate heat flux applied, the dissipated power P at the hot side, must be divided by the area of dissipation A, Q $= P/A$ W/m^2.

At the output of the Herriott cell we measure the deflection angle θ_{MR} traduced as a lateral displacement Δx over the surface of a photodiode. Given that the laser beam has a Gaussian profile we can measure Δx as the changing of the optical power using the knife-edge technique [10]. The knife-edge is attached to a Si photodiode. We center the output beam just in edge of the knife so that almost the 50% of the total power of the output beam is detected by the photodiode. Fig. 6 presents the effect of displacement over the knife-edge-coupled photodiode. The resultant displacement of the beam causes a change in the optical power detected by the photodiode.
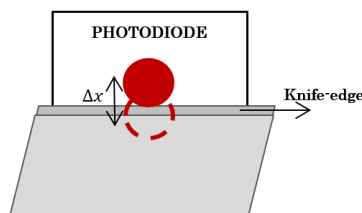

Fig. 6 Detection scheme for measuring the lateral displacements by the knife-edge technique

The heat flux measurement is doing as follows. The laser beam is traveling within the Herriott cell, after 20 s for stability proof, immediately the heat source is turned on, since a temperature gradient appears, the beam starts to deflect so the photodiode and knife-edge set, traduces this deflection of the output beam as a decrement of total beam power as the time of measurement goes. The heating is stopped after 420 s of elapsed time in order to well distinguish the maximum decrement. Then, where there is not induced, the output power will attempt to return at its original value, for appreciate this effect the measurement finishes at 1200 s. We measured the device response to different values of Q, as shown in Fig. 7.
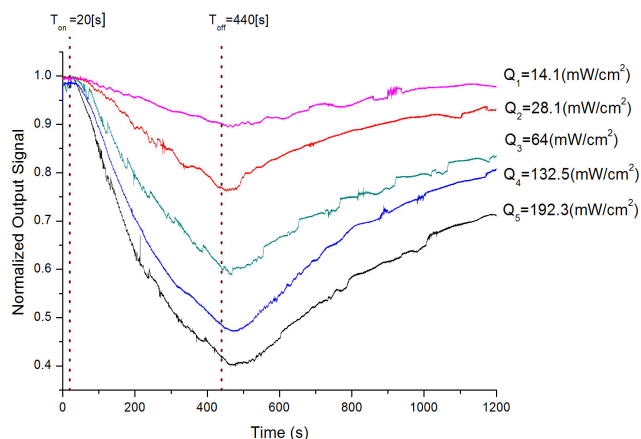

Fig. 7 Normalized photo-diode output signals for different heat flux Q's values

Fig. 7 shows that after the 20 s, the output power does not rapidly decrease, which is due to the thermal inertia because of the natural response of a thermal system. Also it can be seen that after 1200 s, the output response does not return to its

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original value, since the inverse effect of heat conduction is slower than the heating process. In addition, Fig 7 shows the heat flux measurement device response, where the output power is a function of the applied heat, so it is hoped that a larger value of heat will represent a larger decrement of the detected light power. It is important to remark that the output signal of the photo-diode is normalized to its maximum value in order to compare the signals from different experiments.

The measurement process gives us a plot of normalized output signal versus time, i.e. a transient response. So we can give a quantitative result in terms of the applied heat flux, for the largest value of $Q = 192.3$ mW/cm^2 the output power decreases about 65% of the total power whilst the smallest $Q = 14.1$ mW/cm^2 only represents the 10% decrease of the input power.

Finally, we demonstrated the initial approach from the point of view of heat conduction can be traduced as a decrease of light power detected in the photo-diode.

V. CONCLUSION

We presented the design of a compact Herriott cell integrated on an acrylic slab with 1.5 cm^2. The multiple-reflections scheme given by the Herriott cell, results a powerful device which allows reducing the size of an element by increasing the optical path length as well as the sensitivity in comparison with a single pass device.

We designed a preliminary system in which different heat fluxes were applied in order to demonstrate the measurement response.

The presented methodology could be considered to integrate a whole flux sensor in order to characterize thermal properties of materials like, thermal conductivity, effusivity, diffusivity and among others, with the main feature that the size of the sample under study could be small sized around 1 cm^2 and good sensitivity in the measurements. So it may be suitable for applications like, biomedicine, thermal engineering and materials' science.

ACKNOWLEDGMENT

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