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# Synchronization of Semiconductor Laser Networks

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***Abstract*—**In this paper, synchronization of multiple chaotic semiconductor lasers is achieved by appealing to complex system theory. In particular, we consider dynamical networks composed by semiconductor laser, as interconnected nodes, where the interaction in the networks are defined by coupling the first state of each node. An interest case is synchronized with master-slave configuration in star topology. Nodes of these networks are modeled for the laser and simulate by Matlab. These results are applicable to private communication.

***Keywords*—**Synchronization, chaotic laser, network.

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## I. Introduction

DURING the last decades, synchronization of two coupled chaotic systems has received great attention from mathematicians, physicists, biologists, control engineers, etc. see e.g. [1]-[7]. This interest has been greatly motivated by the possibility of encrypted information transmission by using a chaotic carrier, see e.g. [4], [8]-[17]; in which the confidential information is encrypted into the transmission chaotic signal by direct modulation, masking, or another technique. At the receiver end, if chaotic synchronization can be achieved, then it is possible to extract the hidden information from the transmitted signal. Chaotic circuits and lasers (see e.g. [18]–[26]) are ideal candidates for experimental realization in secure communications.

On the other hand, synchronization is required in complex networks with many coupled nodes. Particularly interesting is the scenario where the connected nodes have chaotic behavior.

Synchronization in complex dynamical networks has direct applications in different fields, see e.g. [12], [27]–[33]. Promising results on synchronization of coupled chaotic nodes in different topologies are reported in [8], [12], [27]–[32], [34]. However, synchronization in star coupled networks has direct application in communications, where is possible to transmit information from a single transmitter to multiple receivers. In particular, experimental realization of network synchronization of Chua's circuit like nodes is reported in [29].

Based on the importance of synchronization for optical communications, the aim of this paper is to study the on chaos synchronization of semiconductor laser network. In particular, this objective is achieved by synchronizing the semiconductor laser via complex networks theory.

The remainder of this paper is organized as follows: In Section II, Chaotic semiconductor laser description is given. In Section III, synchronization complex network is described. Section IV presents the synchronization of star topology using semiconductor laser. The paper is concluded with some remarks in Section V.

## II. Chaotic Semiconductor Laser

The mathematical model describing the dynamic behavior of a semiconductor laser (isolated node), for the following differential equations shown [34]:

$$\begin{aligned}\frac{dN}{dt} &= \frac{1}{\tau_C}\left[\frac{I_b+I_m sin\emptyset}{I_{th}} - N - \frac{N-\delta}{1-\delta}P\right] \\ \frac{dP}{dt} &= \frac{1}{\tau_P}\left[\frac{N-\delta}{1-\delta}(1-\epsilon P)P - P + \beta N\right] \\ \frac{d\emptyset}{dt} &= 2\pi f_m\end{aligned} \tag{1}$$

TABLE I
Units for Semiconductor Model

| Symbol | Quantity | Units |
|---|---|---|
| $\tau_C$ | Electron lifetime | $\tau_C = 3 \times 10^{-9}$ |
| $I_{th}$ | Threshold current | $I_{th} = 26 \times 10^{-9}$ |
| $I_b$ | Bias current | $I_b = 1.4 \times I_{th}$ |
| $I_m$ | Modulation amplitude | $I_m = .55 \times I_{th}$ |
| $\delta$ | Dimensionless constant | $\delta = 692 \times 10^{-3}$ |
| $\tau_P$ | Photon lifetime | $\tau_P = 6\,10^{-12}$ |
| $\epsilon$ | Nonlinear gain reduction | $\epsilon = 1 \times 10^{-4}$ |
| $\beta$ | Spontaneous emission factor | $\beta = 5 \times 10^{-5}$ |
| $f_m$ | Frequency of modulation | $f_m = 0.8 \times 10^{9}$ |

The parameters are $N$, the normalized carrier density; $P$, the normalized photon density; Im (mA), the amplitude of the modulation current; and fm (GHz) frequency of modulation. The system exhibits both chaotic vary the amplitude and frequency of modulation (Fig. 1).

## III. Synchronization of Complex Network

### A. Brief Review on Synchronization of Complex Networks

In this section, we give a brief review on complex dynamical networks, in particular of star coupling topology and its synchronization.

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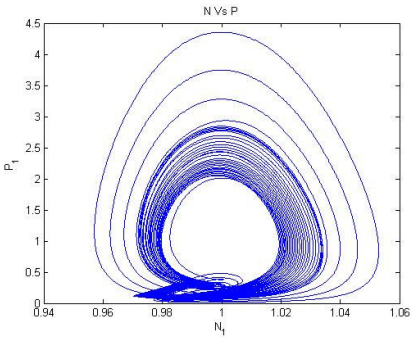



Fig. 1 Chaotic attractor (*N* vs *P*) of the master node

*B. Synchronization of Complex Network*

We consider a complex network composes of *N* identical nodes, linearly and diffusively coupled through the first state of each node. In this network, each node constitutes a n-dimensional dynamical system, described as

$$\dot{x}_i = f(x_i) + u_i, \qquad i = 1, 2, \dots, N \tag{2}$$

where $\boldsymbol{x}_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \in \mathbb{R}^n$ are the state variables of the node $i$, $u_i = u_{i1} \in \mathbb{R}$ is the input signal of the node i, and is defined by

$$u_i = c \sum_{j=1}^{N} a_{ij} \Gamma x_j, \quad i = 1, 2, \dots, N, \tag{3}$$

the constant c > 0 represents the coupling strength of the complex network, and $\Gamma \in \mathbb{R}^{n\times n}$ is a constant 0-1 matrix linking coupled state variables. For simplicity, assume that $\Gamma = \mathrm{diag}(r_1, r_2, \dots, r_n)$ is a diagonal matrix with $r_i = 1$ for particular $i$ and $r_j = 0$ for $j \neq i$. This means that two coupled nodes are linked through their $i$-th state variables. Whereas, $A = (a_{ij}) \in \mathbb{R}^{N\times N}$ is the coupling matrix, which represents the coupling topology of the complex network. If there is a connection between node i and node j, then $a_{ij} = 1$; otherwise, $a_{ij} = 0$ for $j \neq i$. The diagonal elements of coupling matrix A are defined as:

$$a_{ij} = -\sum_{j=1, i\neq j}^{N} a_{ij} = -\sum_{j=1, i\neq j}^{N} a_{ji}, \quad i = 1, 2, \dots, N, \tag{4}$$

If the degree of node $i$ is $d_i$, then $a_{ii} = -d_i$, i = 1,2,.. ,N.

Now, suppose that the complex network is connected without isolated clusters. Then, A is a symmetric irreducible matrix. In this case, it can be shown that zero is an eigenvalue of A with multiplicity 1 and all the other eigenvalues of A are strictly negative [27], [28].

Synchronization state of nodes in complex systems can be characterized by the nonzero eigenvalues of A. The complex network (1) and (2) is said to achieve (asymptotically) **synchronization**, if [28]:

$$\mathbf{x}_1(t) = \mathbf{x}_2(t) = \cdots = \mathbf{x}_N(t), \ as \ \ t \to \infty \tag{5}$$

The diffusive coupling condition (3) guarantees that the synchronization state is a solution, $s(t) \in \mathbb{R}^n$, of an isolated node, that is

$$\dot{\mathbf{s}}(t) = f(\mathbf{s}(t)) \tag{6}$$

where s(t) can be an equilibrium point, a periodic orbit, or a chaotic attractor. Thus, stability of the synchronization state,

$$\mathbf{x}_1(t) = \mathbf{x}_2(t) = \cdots = \mathbf{x}_N(t) = \mathbf{s}(t) \tag{7}$$

of complex network (1) and (2) is determined by the dynamics of an isolated node, i.e. – function $f$ and solution $\boldsymbol{s}(t)$ – the coupling strength c, the inner linking matrix $\Gamma$, and the coupling matrix **A**.

*C. Synchronization Conditions*

The following theorem gives the conditions to achieve synchronization of the network (1) and (2) as is established in (4).

**Theorem 1** [27], [28]. Consider the dynamical network (1) and (2). Let

$$0 = \boldsymbol{\lambda}_1 \geq \boldsymbol{\lambda}_2 \geq \boldsymbol{\lambda}_3 \geq \cdots \geq \boldsymbol{\lambda}_N \tag{8}$$

*be the eigenvalues of its coupling matrix A. Suppose that there exists a* $n \times n$ diagonal matrix $D > 0$ and two constants $\bar{d} < 0$ an $\tau > 0$, such that

$$\left[df(s(t)) + d\Gamma\right]^T \mathrm{D} + \mathrm{D}\left[df(s(t)) + d\Gamma\right] \leq -\tau \mathrm{I}_n \tag{9}$$

for all $d \leq \bar{d}$, where $\in \mathbb{R}^{n\times n}$ is an unit matrix. If, moreover,

$$c\lambda_2 \leq \bar{d}, \tag{10}$$

then, the synchronization state (7) of dynamical network (2) and (3) is exponentially stable.

Since $\lambda_2 < 0$ and $\bar{d} < 0$, inequality (9) is equivalent to

$$c \geq \left|\frac{\bar{d}}{\lambda_2}\right|. \tag{11}$$

Therefore, synchronizability of (2) and (3) with respect to a specific coupling topology can be characterized by the second-largest eigenvalue of A.

*D. Synchronization of Five Semiconductor Laser*

In this section, we describe the system to be constructed with N models using (1), which take the following form (according to (2) and (3)):

$$\begin{aligned} \frac{dN}{dt} &= \frac{1}{\tau_C}\left[\frac{I_b + I_m \sin\emptyset}{I_{th}} - \boldsymbol{N} - \frac{N-\delta}{1-\delta}\boldsymbol{P}\right] + \boldsymbol{u_{i1}} \\ \frac{dP}{dt} &= \frac{1}{\tau_P}\left[\frac{N-\delta}{1-\delta}(\mathbf{1} - \boldsymbol{\epsilon P})\boldsymbol{P} - \boldsymbol{P} + \boldsymbol{\beta N}\right] \\ \frac{d\emptyset}{dt} &= \boldsymbol{2\pi f_m} \end{aligned} \tag{12}$$

$$u_{i1} = c \sum_{j=1}^{N} a_{ij} \Gamma x_j. \tag{13}$$

If the coupling signal in (18) is $u_{i1} \equiv 0$ for i = 1,2, ... ,N, then we have the original set of *N* uncoupled semiconductor laser, which evolve according to their own dynamics. We consider, for illustrative purposes in this work N = 5, i.e. we

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have two coupled semiconductor laser like nodes to be synchronized in master-slaves topology and Γ = diag (1, 0, 0, 0, 0). In particular, we consider, a single master node N1 and four slave nodes $N_2$, $N_3$, $N_4$, $N_5$ for the physical implementation of this network (12)–(13), the topology is shown in Fig. 2.

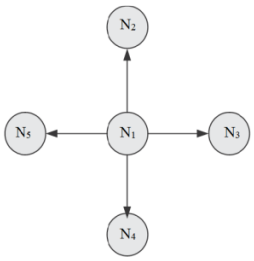



Fig. 2 Star network with master node $N_1$ and four slave nodes

Five isolated nodes ((12) and (13)) are considered to be synchronized in star network with master node $N_1$, and slave nodes $N_2$, $N_3$, $N_4$, $N_5$. The master node $N_1$ of the dynamical network is arranged as:

$$\dot{N}_1 = \frac{1}{\tau_C}\left[\frac{I_b+I_m sin\emptyset}{I_{th}} - N_1 - \frac{N_1-\delta}{1-\delta}P_1\right] + u_{11} \quad (14)$$
$$\dot{P}_1 = \frac{1}{\tau_P}\left[\frac{N_1-\delta}{1-\delta}(1-\epsilon P_1)P_1 - P_1 + \beta N_1\right]$$
$$\dot{\emptyset}_1 = 2\pi f_m$$
$$u_{11} = 0$$

the first slave node $N_2$ is given by

$$\dot{N}_2 = \frac{1}{\tau_C}\left[\frac{I_b+I_m sin\emptyset}{I_{th}} - N_2 - \frac{N_2-\delta}{1-\delta}P_2\right] + u_{11} \quad (15)$$
$$\dot{P}_2 = \frac{1}{\tau_P}\left[\frac{N-\delta}{1-\delta}(1-\epsilon P)P - P + \beta N\right]$$
$$\dot{\emptyset}_2 = 2\pi f_m$$
$$u_{21} = c(N_1 - N_2)$$

the second slave node $N_3$ by means of

$$\dot{N}_3 = \frac{1}{\tau_C}\left[\frac{I_b+I_m sin\emptyset}{I_{th}} - N_3 - \frac{N_3-\delta}{1-\delta}P_3\right] + u_{31} \quad (16)$$
$$\dot{P}_3 = \frac{1}{\tau_P}\left[\frac{N_3-\delta}{1-\delta}(1-\epsilon P_3)P_3 - P_3 + \beta N_3\right]$$
$$\dot{\emptyset}_3 = 2\pi f_m$$
$$u_{31} = c(N_1 - N_3)$$

the third slave node $N_4$ like

$$\dot{N}_4 = \frac{1}{\tau_C}\left[\frac{I_b+I_m sin\emptyset}{I_{th}} - N_4 - \frac{N_4-\delta}{1-\delta}P_4\right] + u_{41} \quad (17)$$
$$\dot{P}_4 = \frac{1}{\tau_P}\left[\frac{N_4-\delta}{1-\delta}(1-\epsilon P_4)P_4 - P_4 + \beta N_4\right]$$
$$\dot{\emptyset}_4 = 2\pi f_m$$
$$u_{41} = c(N_1 - N_4)$$

and the fourth slave node $N_5$ given by

$$\dot{N}_5 = \frac{1}{\tau_C}\left[\frac{I_b+I_m sin\emptyset}{I_{th}} - N_5 - \frac{N_5-\delta}{1-\delta}P_5\right] + u_{51} \quad (18)$$
$$\dot{P}_5 = \frac{1}{\tau_P}\left[\frac{N_5-\delta}{1-\delta}(1-\epsilon P_5)P_5 - P_5 + \beta N_5\right]$$
$$\dot{\emptyset}_5 = 2\pi f_m$$
$$u_{51} = c(N_1 - N_5)$$

Now, by using (14)–(18) as chaotic nodes, we have constructed the star network with master node $N_1$ to be synchronized according to Fig. 2. The corresponding coupling matrix is given by

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix} \quad (19)$$

with eigenvalues $\lambda_1 = 0,\ \lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = -1$.

## IV. Star Network Synchronization

The state $N_1$(t) of master $N_1$ versus state $N_2$(t) of slave $N_2$ is shown in Fig. 3, we can see synchronization between chaotic nodes $N_1$ and $N_2$.

For the other three slave nodes $N_3$, $N_4$, and $N_5$ it is the same as in node $N_2$, in Figs. 3-6 are shown the synchronization between master node $N_1$ and the slave nodes $N_3$, $N_4$, and $N_5$, respectively.

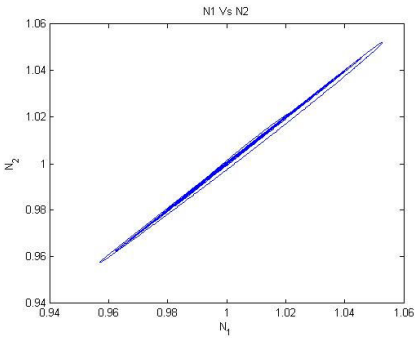



Fig. 3 ($N_1$, $N_2$) plane

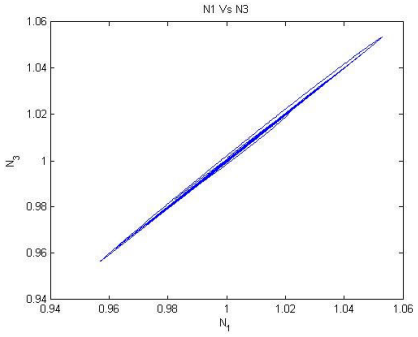



Fig. 4 ($N_1$, $N_3$) plane

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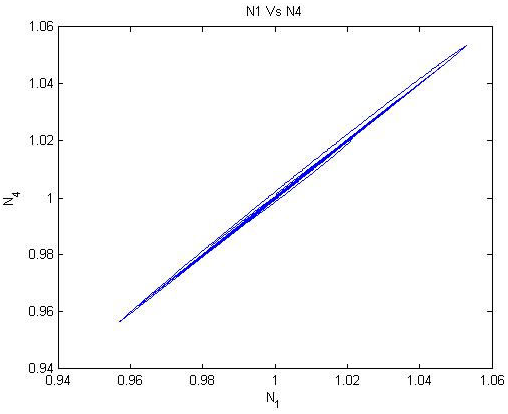



Fig. 5 ($N_1$, $N_4$) plane

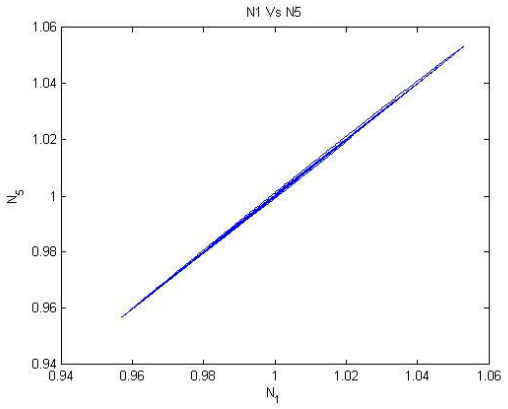



Fig. 6 ($N_1$, $N_5$) plane

## V. Conclusion

We have presented a simulate network synchronization among a master and four slave chaotic Semiconductor Laser by using a four communication channels. Star network synchronization was obtained by using complex systems theory. The obtained simulation results (for only illustrative purposes) have shown that it is possible network synchronization and a future application in the encrypted, transmission, and recovery of secret messages using semiconductor laser.

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