Worm Gearing Design Improvement by Considering Varying Mesh Stiffness

A. H. Elkholy, A. H. Falah

Abstract—A new approach has been developed to estimate the load share and distribution of worm gear drives, and to calculate the instantaneous tooth meshing stiffness. In the approach, the worm gear drive was modelled as a series of spur gear slices, and each slice was analyzed separately using the well-established formulae of spur gear loading and stresses. By combining the results obtained for all slices, the entire envolute worm gear set loading and stressing was obtained.

The geometric modelling method presented allows tooth elastic deformation and tooth root stresses of worm gear drives under different load conditions to be investigated. Based on the slicing method introduced in this study, the instantaneous meshing stiffness and load share are obtained. In comparison with existing methods, this approach has both good analysis accuracy and less computing time.

Keywords—Gear, load/stress distribution, worm, wheel, tooth stiffness, contact line.

I. INTRODUCTION

THE load carrying capacity and service life are the primary requirements for worm gears. Furthermore, the ever-improving techniques in the gear manufacturing industry are creating new possibilities for realizing gears with special shape geometry to improve their running behaviour. Due to the manufacturing, assembly errors and elastic deflections, a theoretically correct design procedure of a gear set does not normally provide desirable performance. In fact, because of the tooth load distribution due to end effect, gears that are designed using traditional methods are often unsatisfactory and require unnecessarily high manufacturing precision. A reliable method for calculating loads and stresses in worm gearing is comprehended as an effective remedy for a better design.

The performance of a pair of gears depends on the loading magnitude and distribution. A great deal of research effort has been devoted to the analysis and optimization of the strength and running behaviours of worm gears. Most of the earlier approaches [1] were mainly based on experimentation and simplified mechanics, as accurate mechanical analysis was almost impossible without the use of high performance computers. A factor, known as a load-distribution factor, was

employed in the empirical formulae to account for the effect of the non-uniformly distributed loading across the facewidth. This load-distribution factor was chosen in consideration of the misalignment of the axes and the elastic deflections of the shaft assembly. Later, more analytical and accurate approaches to stress analysis were derived and adopted in industry, such as those described in [2]. These methods are based on a simple thin-slice model of involute gears. Therefore, the load distribution across the facewidth can be analytically calculated using simplified formulae. This model, however, assumes constant mesh stiffness at all phases of engagement, which fails to comply with the practical property of the gearing process. With the recent development of computer technology, finite element analysis methods have been adopted in gear analysis [3]. Using a full 3D model, accurate determination of gear load and stress becomes possible [4], [5]. Nevertheless, the demand of high computer power is hardly justified for most small and middle scale companies.

This paper introduces a new approach, which is more efficient and less computer intensive than those based on a full 3D FEA model and more accurate than the traditional methods. It takes into account the varying mesh stiffness and elastic deformations (bending, shear, foundation and contact). In order to improve the computational efficiency, this method also discretizes a gear with a number of thin slices, so that the 3D mechanical properties can be efficiently approximated using accurate 2D analyzing tools. Since the fundamental geometry of a worm gear set can be represented by its transverse section, a 2D model is therefore incorporated to ensure the computational precision.

II. CONFIGURATION OF THE MESH PLANE

As shown in Fig. 1, the mesh plane of a pair of worm gears is taken as the plane containing the worm axis and its orientation varies with the variable angle θ shown in the figure. If angle θ is given a small increment, a slice of a spur gear set will result. If angle θ is given a further increment, an adjacent slice will result. This slicing procedure should continue using the slicing planes shown in the figure to cover the entire worm gear set. The transverse projection of a typical slice is shown in Fig. 2, which is simply a "spur gear set" whose face width depends upon the increments assigned to angle θ. The more these increments, the better the modeling of the worm gear set. By analyzing the spur gear slices for load and stresses, the same can be obtained for the entire worm gear set. The number and position of the contact lines at nay

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phase of mesh can be readily worked out from the geometric relationship on the slicing planes.
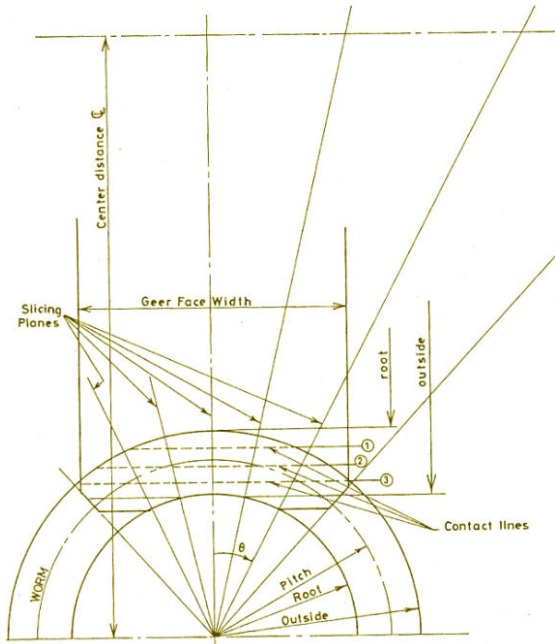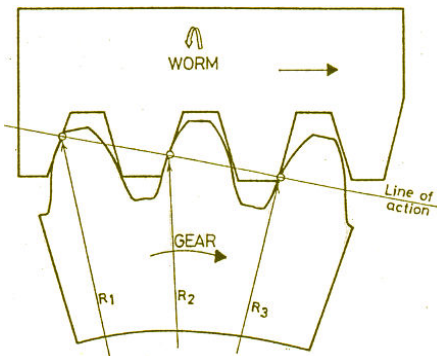

Fig. 1 Contact lines and slicing planes

Fig. 2 Line of action intersecting teeth of a slice

A worm gear is a three dimensional object and a spur gear is conventionally regarded as two dimensional, as the basic geometry can be clearly represented by its transverse section. On this ground, a worm gear can also be approximated as a series of spur gear slices whose face-width is relatively small. Each slice is then treated as a spur gear and viewed as a 2D model. Under this simplification, all theories suitable for 2D gears will be applicable. It does not only simplify the process of analysis, but more importantly, increases the computational efficiency.

III. MESH STIFFNESS

The primary objective of this analysis process is to obtain the load distribution and the stresses of a pair of engaging worm gears. The load distribution depends upon the tooth geometry and elastic deformations of the teeth and gear bodies. It is clear that at any instant of the engagement, the total force being transmitted must balance the input force. But the load borne at an individual point of a tooth flank is determined by the mesh stiffness throughout the number of contact lines at any moment.

Theoretically, the mesh stiffness of a contact point between a pair of teeth is a function of the phase of mesh and the position along the tooth width, or equivalently, it is a function of the co-ordinates on the mesh plane. In practice, it is usually acceptable for the load distribution to be made discrete [6]. Therefore, the mesh plane is modelled by a matrix of points. Each column of the matrix represents a gear slice.

The mesh stiffness is composed of the bending, contact and shear stiffness as well as foundation stiffness of the engaged teeth. Instead of deriving it directly, we will first compute the stiffness of each gear slice and then the stiffness of the entire worm gear set.

IV. SLICE STIFFNESS AND DEFLECTION CALCULATION

Since both the worm and gear are represented by a series of spur gear slices, each slice on either the worm or gear is modeled as a straight spur gear whose main dimensions depend upon the location of the slice on the line of contact between meshing gears, and along the line of action of the slice. If a slice is subjected to a normal load F_{ij}, then the total deflection δ_{ij} is calculated from the equation given in [7] which accounts for bending, shear, Hertzian contact and foundation deformations.

$$\delta_{ij} = \delta_{B_{ij}} + \delta_{S_{ij}} + \delta_{G_{ij}} + \delta_{P_{ij}} \tag{1}$$

where, $\delta_{B_{ij}}$: bending deflection, $\delta_{S_{ij}}$: shearing deflection, $\delta_{G_{ij}}$: tooth foundation deflection, $\delta_{P_{ij}}$: contact deflection. The slice stiffness K_{ij} then becomes

$$K_{ij} = \frac{F_{ij}}{\delta_{ij}} \tag{2}$$

The stiffnesses of all slices on the worm and the corresponding ones on the gear can therefore be determined. The slice stiffness K_{ij} on a given line of contact depends upon the point of load application on the slice and the slice geometry.

V. LOAD CALCULATION

Due to the fact that the mesh stiffness varies from point to point along a tooth flank, it is reasonable to consider each contact point as a small spring associated with a stiffness coefficient. At point i, on a contact line whose order is j, the normal load is F_{ij} and the normal stiffness of the spring is $K_{eq_{ij}}$ which accounts for the stiffness at point i of both the worm K_{ij} and the gear K_{ij}^*. The governing equation is:

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$$K_{eq_{ij}} = \frac{K_{ij}\, K_{ij}^*}{K_{ij} + K_{ij}^*} \qquad \begin{array}{l} i = 1,\, 2, ...n \\ j = 1,\, 2, ...m \end{array} \qquad (3)$$

where, n is the total number of points along a contact line, and m is the total number of slices. The load carried by point i is then given by

$$F_{ij} = K_{eq_{ij}} \cdot \delta_{ij} \qquad \begin{array}{l} i = 1,\, 2, ...n \\ j = 1,\, 2, ...m \end{array} \qquad (4)$$

The total load F_{tot} transmitted by the entire gear set is then computed from:

$$F_{tot} = \sum_{i=1}^{n} \sum_{j=1}^{m} F_{ij} \qquad (5)$$

Equations (4) and (5) were solved to determine the individual loads F_{ij} as:

$$F_{ij} = \frac{K_{eq_{ij}}}{\sum_{k=1}^{n} \sum_{\ell=1}^{m} K_{eq_{k\ell}}} (F_{tot}) \qquad (6)$$

Once the loads transmitted by the slices and the geometry of each slice are known, the gear set stresses can be easily determined as discussed in the following section.

VI. TOOTH FILLET STRESS

Tooth fillet tensile stress σ_{ft} for both the worm and gear were determined for each tooth slice through the path of contact in order to obtain the stress distribution as the teeth roll into, and out of contact, Aida and Terauchi formula [7] for fillet stress calculation was applied to each slice as:

$$\sigma_{ft} = \left(1 + 0.08\frac{t}{r}\right)\left[0.66\sigma_b + 0.4\sqrt{\sigma_b^2 + 36\tau^2} + 1.15\sigma_c\right]\frac{F_{ij}}{\Delta b} \qquad (7)$$

where σ_b, τ and σ_c are the tooth bending, shear and compressive stresses, respectively, as given in [7], and t, r and Δb are the tooth circular thickness, fillet radius and slice face width, respectively.

VII. RESULTS AND DISCUSSION

A worm gear set whose main dimensions are given in Table I and subjected to an input torque of 9.8 N.m was used in this study.

A. Stiffness Distribution

The stiffness of all the spur gear slices that represent the actual gear set, are calculated for the worm and gear using (2). The results show that tooth stiffness increases close to the form and decreases gradually towards the tip. This is because the tooth experiences less deflection close to the form than it does at its tip. The equivalent stiffness which is the combined

effect of stiffness of the worm and gear slices at the lines of contact is plotted in the same figure.

TABLE I
GEAR SPECIFICATIONS

		Worm	Gear
Module	(mm)	6.2	
Pressure angle	(deg)	20	
Lead angle	(deg)	17.896	
Center distance	(mm)	125	
Number of teeth		3	31
Pitch circle diameter	(mm)	57.6	192.4
Averaged diameter	(mm)	57.8	192.2
Outside diameter	(mm)	68.0	204.8
Root diameter	(mm)	41.2	178.0
Face width	(mm)	--	46

B. Load Distribution

The transmitted (contact) loads along the lines of contact were calculated from equation (6) and presented in Fig. 3 against the worm radius. It is clear that the transmitted loads along all the lines of contact are larger in the middle than at the tooth edge. This is due to the fact that the stiffness in the middle of the contact lines is greater than at the edges.
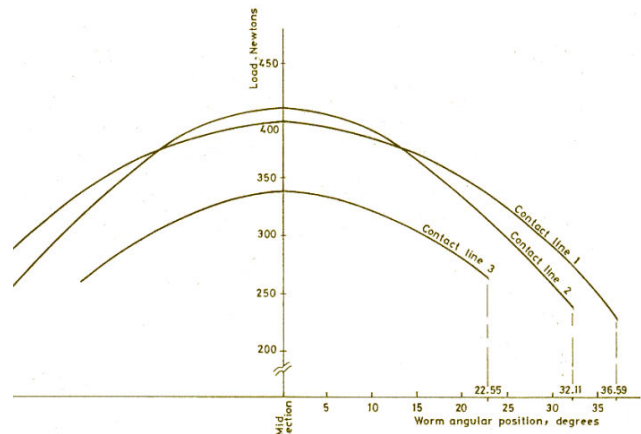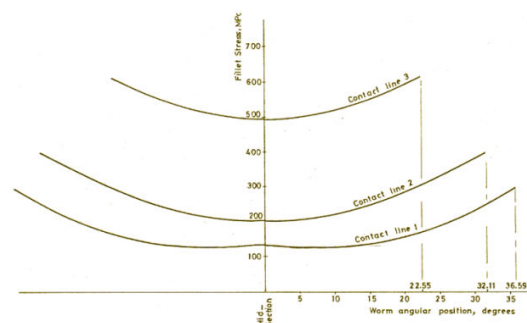

Fig. 3 Load distribution

Fig. 4 Fillet stress distribution

C. Fillet Stress Distribution

The distribution of tensile tooth fillet stress of the worm during the meshing action is plotted in Fig. 4, which indicates that the fillet stress is larger close to the tip area and decreases

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towards the form area. This is mainly due to the increased moment arm of load at the tip area.

VIII. CONCLUSION

Load and stress analysis is very important for the design of worm gears. However, the existing stress analysis procedures of such gears adopted in industry are much too simplified to account for the tooth shape geometry and deflection accurately enough. They need to be refined to meet the ever-increasing industrial demand. On the other hand, although a full 3D FEA model makes it possible to compute gear load and stress more precisely, it is too computationally costly to be justified for many applications. The approach presented in this paper overcomes these disadvantages.

In order to improve computing efficiency, similar to some existing approaches, a discrete model is adopted and each gear slice is viewed as a thin spur gear, so that a full 3D problem is substituted by a group of 2D ones. A comprehensive two dimensional model was then incorporated to take into account the geometric details including tooth geometry and elastic deflections. More importantly, instead of assuming a constant mesh stiffness, this approach fully considers the variation of the mesh stiffness of the contact points at any phase of mesh, which further improves the prediction of the loading situations. A great deal of CPU time is saved. Running on a Pentium PC micro-computer, it takes less than 5 min to compute the loads and stresses. This approach is, therefore, particularly suitable for small and middle scale companies.

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