

# Using Jumping Particle Swarm Optimization for Optimal Operation of Pump in Water Distribution Networks

R. Rajabpour, N. Talebbeydokhti, M. H. Ahmadi

**Abstract**—Carefully scheduling the operations of pumps can be resulted to significant energy savings. Schedules can be defined either implicit, in terms of other elements of the network such as tank levels, or explicit by specifying the time during which each pump is on/off. In this study, two new explicit representations based on time-controlled triggers were analyzed, where the maximum number of pump switches was established beforehand, and the schedule may contain fewer switches than the maximum. The optimal operation of pumping stations was determined using a Jumping Particle Swarm Optimization (JPSO) algorithm to achieve the minimum energy cost. The model integrates JPSO optimizer and EPANET hydraulic network solver. The optimal pump operation schedule of VanZyl water distribution system was determined using the proposed model and compared with those from Genetic and Ant Colony algorithms. The results indicate that the proposed model utilizing the JPSO algorithm is a versatile management model for the operation of real-world water distribution system.

**Keywords**—JPSO, operation, optimization, water distribution system.

## I. INTRODUCTION

ENERGY plays a crucial role in new modern world. With the increasing growth of population and a severe shortage in the energy sources, use of these sources has gained much significance. Pumping energy costs form a large part of the operational cost of water distribution systems worldwide. Even a small overall increase in operational efficiency would result in significant cost savings to the industry. Other benefits of operational optimization include improved water preservation and quality, ensuring compliance with water industry regulations, improved system management and benefits for future expansion such as automation.

Operation costs for a pumping station include energy and maintenance costs. The maintenance cost, related to the wear of the rotating equipment, is difficult to be quantified. However, it is true that the maintenance cost increases when the number of pump switches increases. This point must be considered that the maintenance cost is proportional to the

number of pump switches. A pump switch refers to “turning on a pump that was not operating in the previous period” [1].

Mathematically, the optimal design and operation of pumping stations is a large-scale nonlinear programming problem because of the size of the problem in terms of the number of the decision variables and nonlinearity of the constraints. The objective in a design and operating of irrigation pumping system problem is to minimize the annual depreciation cost of construction and operations while satisfying system constraints to account for the hydraulics behavior, bounding constraints on decision variables, and other constraints that may reflect the operator preferences or system limitations.

Many researchers have developed optimal control formulations to minimize the operating costs associated with water distribution pumping systems. Various optimization techniques have been applied to the operational optimization problem, including linear programming [2], [3], nonlinear programming [4], [5], dynamic programming [6], fuzzy logic [7], nonlinear heuristic optimization [8], [9], genetic algorithms [1], [10]-[12], particle swarm optimization [13], Ant colony optimization [14].

Finding optimal schedules for pumps in a water distribution network (WDN) is a difficult task for researchers and managers. A careful scheduling of pump operations may shift workload to cheaper electrical tariff periods and reducing the cost of energy consumed by pumps. Furthermore, energy savings can be accomplished by pumping water when tank levels are lower and combining the operations of several pumps efficiently. On the other hand, (future) pump maintenance costs caused by pump operations cannot be easily quantified, so surrogate measures are used to estimate it. The most common of such measures is the total number of pump switches: frequent switching (on/off) causes wear and tear of pumps and pressure surges throughout the network, and, hence, increases future maintenance costs. These maintenance costs can be considered in the optimization problem by limiting the number of pump switches [15]. In this paper, the optimal operation of pumping stations was determined using a Jumping Particle Swarm Optimization (JPSO) algorithm so that the minimum energy cost. The schedule for the operation of the water pump system can be a significant savings in the cost of energy to be achieved. Determine the optimum pump operation schedule an optimization model - simulation-based JPSO algorithm was developed. The model integrates JPSO optimizer and EPANET hydraulic network solver in

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MATLAB. The proposed model is applied to find the optimal pump operation schedule of VanZyl water distribution network. The optimization model is developed and subsequently solved with the JPSO on method. Their efficiency and effectiveness was compared with GA and ACO algorithm.

## II. DISCRETE PARTICLE SWARM OPTIMIZATION

The standard PSO considers a swarm  $S$  containing  $s$  particles ( $S = 1, 2, \dots, s$ ) in a  $d$ -dimensional continuous solution space [16]. Each  $i$ <sub>th</sub> particle of the swarm has a position  $X_i = (x_{i1}, x_{i2}, \dots, x_{id})$  and a velocity  $V_i = (v_{i1}, v_{i2}, \dots, v_{id})$ . The position  $X_i$  represents a solution to the problem, while the velocity  $V_i$  gives the rate of change for the position of particle  $i$  at the next iteration. The position of particle  $i$  in each iteration is adjusted according to:

$$X'_i = X_i + V'_i \quad (1)$$

Each particle  $i$  of the swarm communicates with a social environment or neighborhood,  $N(i) \subseteq S$ , representing the group of particles with which it communicates, and which could change dynamically. In nature, a bird adjusts its position in order to find a better position, according to its own experience and the experience of its companions. In the same manner, in each iteration, each particle "I" updates its velocity reflecting the attractiveness of its best position so far  $B_i = (b_{i1}, b_{i2}, \dots, b_{id})$  and the best position  $G^* = (g_1, g_2, \dots, g_d)$  of its social neighborhood  $N(i)$ , according to (2):

$$V'_i = \omega V_i + c_1 \text{rand}() (B_i - X_i) + c_2 \text{rand}() (G^* - X_i) \quad (2)$$

The parameters  $\omega$ ,  $c_1$  and  $c_2$  are positive constant weights representing the degrees of confidence of particle  $i$  in the different positions that influence its dynamics, while  $\text{rand}()$  refers to a random number with uniform distribution  $[0, 1]$  that is independently generated at each iteration.

The original PSO algorithm can only optimize problems in which the elements of the solution are continuous real numbers. Therefore several Discrete Particle Swarm Optimization (DPSO) methods have been proposed. In the DPSO proposed by [16] for problems with binary variables, the position of each particle is a vector  $X_i = (x_{i1}, x_{i2}, \dots, x_{id})$  of the  $d$ -dimensional binary solution space,  $X_i \in \{0,1\}^d$ , but the velocity is still a vector  $V_i$  of the  $d$ -dimensional continuous space,  $V_i \in \mathcal{R}^d$ . A different way to update the velocity was considered on [17].

A DPSO whose particles at iteration are affected alternatively by its best position and the best position among its neighbors was proposed in [18]. The multi-valued PSO (MVPSO) proposed in [19] deals with variables with multiple discrete values. The position of each particle is a mono dimensional array in the case of a continuous PSO, a 2 dimensional array in the case of a DPSO, and a 3-dimensional array for a MVPSO. Indeed, the position of particle  $i$  in the MVPSO is expressed by the term  $x$ , representing the probability that the  $i_{jk}$   $i$ <sub>th</sub> particle, in the  $j$ <sub>th</sub> iteration, takes the

$k$ <sub>th</sub> value.

A new DPSO proposed in [20] does not consider any velocity, from the lack of continuity of the movement in a discrete space, the notion of velocity loses sense; however they kept the attraction of the best positions. They interpret the weights of the updating equation as probabilities that, at each iteration, each particle has a random behavior or acts in away guided by the effect of an attraction. The moves in a discrete or combinatorial space are jumps from one solution to another. The attraction causes the given particle to move towards this attractor if it results in an improved solution. An inspiration from the nature for this process is found in frogs, which jump from a lily pad to a pad in a pool. Thus, this new discrete PSO is called Jumping Particle Swarm Optimization (JPSO). Reference [21] tested capabilities of JPSO by solving minimum labeling Steiner tree problem, an NP-hard graph problem. Based on their computational analysis, JPSO clearly outperformed all the other procedures, obtaining high-quality solutions in short computational running times. This confirms the ability of JPSO method to deal with NP-hard combinatorial problems.

## III. PUMP SCHEDULING PROBLEM

Given a water distribution network where demand patterns, initial tank levels, and electricity tariffs are specified, the goal is to find the best pump schedule over a typical operating cycle such that the total operational costs are minimized while guaranteeing a competent network service. Pump operational costs include cost of energy consumed by pumps and pump maintenance costs derived from the workload imposed on pumps. System constraints ensure feasibility of pump schedules, including that demands are supplied at adequate pressures and water supplied from tanks is recovered by the end of the scheduling period [22].

Pump energy costs depend on the energy price as well as on the amount of energy consumed. The price per unit of energy is given by electricity tariffs, which may vary during a scheduling period. In general, it is divided into an expensive peak and cheaper off-peak electricity tariffs. The actual amount of energy consumed by a pump depends on several parameters, including flow through the pump, head supplied by the pump, and wire-to water efficiency. These parameters can be calculated using a hydraulic simulator for a known pump schedule. Formally, operation of  $N_p$  pumps in a WDN is scheduled over a scheduling period  $T$ . This scheduling period is divided into a number of time intervals  $N_T$ . Given a particular schedule  $S$ , which represents pumps operating during each time interval, the total cost of energy is calculated as:

$$\text{MinCOST} = \sum_{n=1}^{N_p} \sum_{i=1}^{N_T} R_e(i) \cdot E_c(n,i) \cdot S(n,i) + \text{Penalty 1} \quad (3) \\ + \text{Penalty 2}$$

where  $N_p$ : number of pumps;  $N_T$ : number of time intervals;  $S(n,i)$ : duration for which pump  $n$  is operating during interval

$i(h)$ ;  $R_c(i)$  :energy tariff during interval  $i$  (\$/kWh);  $E_c(n,i)$ : energy consumption rate of pump  $n$  during interval  $i$  (kWh/h) [22].

The energy consumption rate of a pump depends on flow through the pump, head supplied by the pump, and efficiency at which it operates:

$$E_c(n,i) = \frac{0.01019 \cdot Q(n,i) \cdot h(n,i)}{e(n,i)} \quad (4)$$

where  $Q(n,i)$  flow rate through pump  $n$  during interval  $i$  (L/s);  $h(n,i)$ : total dynamic head (TDH) supplied by pump  $n$  during interval  $i$  (m); and  $e(n,i)$ : overall (wire-to-water) efficiency of pump  $n$  during interval  $i$  [22].

In this model, the energy cost is calculated by the Epanet model directly. The penalties include:

Penalty1: It is penalty to leave the choice of minimum and maximum allowable level in the reservoir is considered. The amount of the penalty is calculated from (5).

$$\text{Penalty 1} = \alpha \left\{ \begin{array}{l} \sum_{k=1}^{N_t} \sum_{t=1}^T \left( \frac{H_k^t}{H_{\min,k}} - 1 \right)^2 \\ + \\ \sum_{k=1}^{N_t} \sum_{t=1}^T \left( \frac{H_k^t}{H_{\max,k}} - 1 \right)^2 \end{array} \right\} \vee \begin{cases} H_k^t < H_{\min,k} \\ H_k^t > H_{\max,k} \end{cases} \quad (5)$$

where  $k$ : number of reservoirs;  $N_t$ : size of reservoirs;  $H_k^t$  : water level for which reservoir  $k$  is operating during interval  $t$ (h);  $H_{\min,k}$  and  $H_{\max,k}$ : minimum and maximum level of reservoir  $k$ .

To achieve a balance between water supplied and consumed from tanks, a viable schedule must ensure that tanks recover their levels by the end of scheduling period. That is, tank levels at the end of the scheduling period are not lower than those at the start. The lack of depth in the reservoir at the end than at the beginning of its depth can be controlled with a penalty 2. The amount of the penalty is calculated from (6):

$$\text{Penalty 2} = \beta \left\{ \sum_{k=1}^{N_t} \left( \frac{H_k^T}{H_{\min,k}} - 1 \right)^2 \right\} \quad (6)$$

where  $H_k^T$  :tank level at the end of the scheduling period;  $H_{\min,k}$ : initial tank level and  $\alpha$  and  $\beta$  are penalty coefficients.

On the other hand, maintenance costs are difficult to quantify and are usually measured using a surrogate objective, such as the number of pump switches. A pump switch is defined as turning a pump on that was off during the preceding time interval. Frequent switching of pumps causes wear and tear of pumps which, in turn, increases maintenance costs. Thus, it is assumed that minimization of the number of pump switches reduces maintenance costs [6]. Frequent switching causes wear and tear of pumps, which, in turn, increases maintenance costs. Moreover, pump switches generate pressure surges that cause an unspecified damage to network components, such as pipes and valves. Thus, the general practice is to minimize the number of pump switches in order to reduce both the wear of the pumps and the damage to pipes and valves, hence, minimizing future maintenance costs. Most works consider energy cost as the most important objective,

and add the number of pump switches as a constraint to the problem [6], [10], [22], [23]. Such constraint in the number of pump switches is implicitly enforced by the representation of schedules presented in this paper.

#### IV. CONSTRAINTS

In order to optimize, the model must satisfy system constraints that represent its performance criteria. These include hydraulic constraints representing conservation of mass and energy, minimum and maximum limits on tank storage levels, minimum pressures requirements at demand nodes and a balance between supply and demand from tanks. The hydraulic simulator implicitly handles some of these constraints, including hydraulic constraints and constraints on maximum and minimum tank levels [22].

To achieve a balance between water supplied and consumed from tanks, a viable schedule must ensure that tanks recover their levels by the end of scheduling period. That is, tank levels at the end of the scheduling period are not lower than those at the start [22].

Therefore, the optimization model must satisfy minimum pressure constraints at demand nodes:

$$H_{j,i} \leq H_j^{\min} \quad j = 1, \dots, N_d \quad (7)$$

where  $H_{j,i}$ : head supplied at demand node  $j$  during time period  $i$ ;  $H_j^{\min}$ : minimum head required at demand node  $j$ ; and  $N_d$ : number of demand nodes [22].

In order to reduce maintenance costs, an additional constraint on the number of pump switches is used. Thus, the number of pump switches is limited to a specified value:

$$N_s = N_p \times SW \quad (8)$$

where  $SW$ : constant to be specified and it is the maximum number of switches allowed per pump during a scheduling period. Schedules with a lower number of pump switches are also acceptable, and thus, constraint (7) may be relaxed as [22]:

$$N_s \leq N_p \times SW \quad (9)$$

#### V. VANZYL TEST NETWORK

Initially the test network, published in [23], was solved using the proposed model. It contains three pumps and two tanks. The layout of the network is shown in Fig. 1. Pumps 1A and 2B are identical pumps connected in parallel. When neither of these pumps is active, a booster pump 3B transfers water from Tank A to Tank B. In case one or both the pumps (1A and 2B) are active, Pump 3B boosts the flow to Tank B. Tank B has a higher elevation than Tank A, and thus water may flow by gravity from Tank B to Tank A through the pipes connected to the demand node. The pump scheduling period  $T$  (24 h) is divided into 24 1-hr intervals. All tanks in the network must be 95% full at the start of the peak electricity period (7:00 a.m.).

In this instance the demand charge is taken to be zero and the water available at the reservoir is assumed to be infinite.

The electricity cost is divided into two periods with a peak electricity tariff period from 7:00 to 24:00 and an off-peak tariff from 0:00 to 7:00. The demand pattern contains two peaks at 7:00 and 18:00. More details about the test instance are provided by [24].

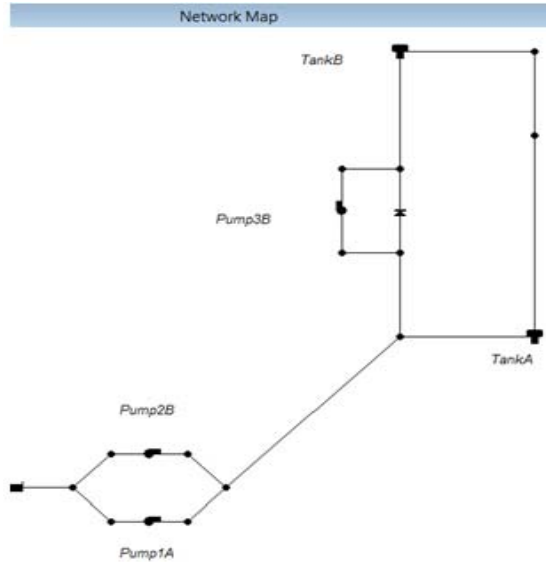


Fig. 1 VanZyl test network

For this example, 25 runs were conducted using different random seeds to assess the JPSO model's average performance. In order to enable a fair comparison with the results provided by [23], each run was continued until the same number of function evaluations, i.e., 6,000 function evaluations for the VanZyl instance. Results obtained are compared with those obtained using a simple genetic algorithm (SGA) with binary representation and Hybrid GA with level controlled trigger representation.

We propose an explicit representation based on the concept of time-controlled triggers. This representation has the advantage of satisfying the constraint on the number of pump switches implicitly. In contrast to the binary representation, which encodes the status of a pump during each time interval, the time-controlled triggers representation encodes the time when a pump changes its status. The concept of encoding time has already been proposed in the literature. We can thus limit the maximum number of switches per pump simply by limiting the number of decision variables of each solution.

The median, best and worst values were obtained from the results of 25 runs and presented in Table I.  $C_E$  and  $N_S$  correspond, respectively, to the daily electrical cost and total number of pump switches. Results under ACO<sup>a</sup> were obtained taking into account constraint  $N_S \leq 9$ , while results under ACO<sup>b</sup> were obtained considering constraint  $N_S = 9$ . In Table II, results obtained using Hybrid GA [23] are also presented. The JPSO algorithm produced better results compared to a minimum cost solution and compared well with the median results obtained using Hybrid GA. The statistical parameters of Hybrid GA,

presented in Table II, were calculated based on results from seven random runs. But results of ACO are much better than the others.

TABLE I  
RESULTS OF TEST NETWORK (25 RUNS, 6000 EVALUATIONS,  $N_S = 9$ )

	JPSO		ACO <sup>b</sup>	
	$C_E$	$N_S$	$C_E$	$N_S$
Best	340.49	9	329.1	9
Median	347.66	9	357.9	9
Worst	355.81	9	364.7	9

b: VanZyl et al.[24]

TABLE II  
RESULTS OF TEST NETWORK (25 RUNS, 6000 EVALUATIONS,  $N_S \leq 9$ )

	JPSO		ACO <sup>a</sup>		Hybrid GA <sup>a</sup>	
	$C_E$	$N_S$	$C_E$	$N_S$	$C_E$	$N_S$
Best	341.7	6	326.5	3	344.4	3
Median	344.21	7	349.2	4	348.6	4
Worst	353.11	9	357.6	7	354.8	5

a: van Zyl et al. [24]

Finally, in order to illustrate the structure of schedules obtained, Figs. 2 and 3 show solutions obtained by the JPSO algorithm for the VanZyl network using constraint  $N_S \leq 9$  and  $N_S = 9$ . In both cases, the best solutions take full advantage of the off-peak electricity tariff.

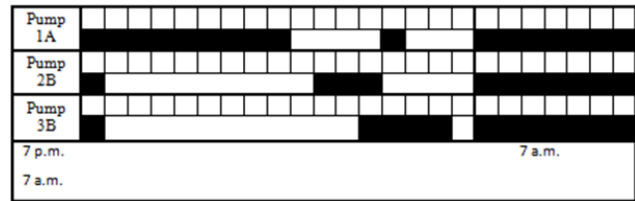


Fig. 2 Pump schedules obtained by JPSO algorithm for the VanZyl network: best solution ( $N_S \leq 9$ )

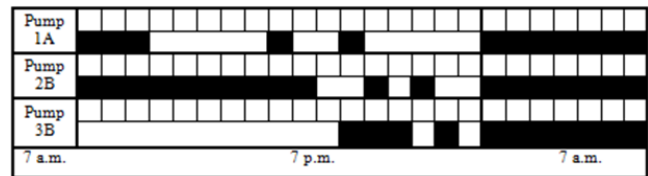


Fig. 3 Pump schedules obtained by JPSO algorithm for the VanZyl network: best solution ( $N_S = 9$ )

## VI. CONCLUSION

The optimization of pump operations in water distribution networks is an important problem in practice since it may result in substantial energy and monetary savings. By means of complex hydraulic simulation models and automatic data collection, a very precise model of a water distribution system can be obtained. However, it is the task of an optimization algorithm to generate candidate pump schedules that minimize energy and maintenance costs while providing a reliable and satisfactory service to customers. In present paper, an evolutionary algorithm namely Jumping Particle Swarm Optimization is used to solve the problem of operation of

pump operations in water distribution networks. The decision variable in this problem is turn pumps on/off at certain moments of the scheduling period. Those optimized schedules will have a predefined maximum number of pump switches, thus limiting the wear and tear of the pumps, and limiting maintenance costs. As a consequence of this, the search space, that is, the number of potential solutions can be significantly reduced.

The optimization algorithm, based on the JPSO, minimizes the electricity cost of pumps while satisfying constraints on minimum pressures and balance between supply and demand from tanks. Results of a VanZyl test network show that the performance of the proposed model with a JPSO algorithm is better than Hybrid GA but the ability of the ant colony algorithm to find the minimum solution than other two algorithms in both cases is higher. Overall, in both cases, a middle answer with the JPSO algorithm is better and the worst answer in the model is less compared to other algorithms.

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