A Stochastic Approach to Extreme Wind Speeds Conditions on a Small Axial Wind Turbine

Nkongho Ayuketang Arreyndip, Ebobenow Joseph

Abstract—In this paper, to model a real life wind turbine, a probabilistic approach is proposed to model the dynamics of the blade elements of a small axial wind turbine under extreme stochastic wind speeds conditions. It was found that the power and the torque probability density functions even-dough decreases at these extreme wind speeds but are not infinite. Moreover, we also fund that it is possible to stabilize the power coefficient (stabilizing the output power)above rated wind speeds by turning some control parameters. This method helps to explain the effect of turbulence on the quality and quantity of the harness power and aerodynamic torque.

Keywords—Probability, Stochastic, Probability density function, Turbulence.

I. INTRODUCTION

In the last century, most of the world's power consumption came from coal and crud oil. But due to the huge consequences of the bi-product of these resources on the environment such as global warming, most worlds researchers, engineers and policy makers have decided to heavily invest and research on alternative, renewable sources of energy such as solar, wind, hydro.

Wind turbine technology has more than doubled in the last three decades with more powerful, efficient wind turbine being produced and marketed [1].
Wind turbines are mechanical devices specifically designed to convert part of the kinetic energy of the wind into useful mechanical energy. Most wind turbines comprise a rotor that turns round propelled by lift or drag forces, which result from its interaction with the wind [2]. Depending on the position of the rotor axis, wind turbines are classified into vertical-axis and horizontal-axis ones [2]. With a vertical axes wind turbine, the generator and transmission devices are located at ground level. Their main advantage is that, they are able to capture the wind from any direction without the need to yaw. However, these advantages are counteracted by a reduced energy capture since the rotor intercepts winds having less energy [2]. Their main disadvantage is that maintenance is not simple since it usually requires rotor removal. In addition, these rotors are supported by guy-ropes taking up large land extensions [2]. In these paper, we will be describing but the horizontal axis wind turbine.

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A. HORIZONTAL AXES WIND TURBINE

Horizontal axis wind turbines are design either in the up-stream mode or the down stream mode. Although some down-wind designs were developed in the past to simplify the yaw mechanism, modern wind turbines have the rotor upstream of the tower. Horizontal-axis wind turbines always have a tower supporting the rotor, transmission and generator. This tower may be tubular in the case of small-scale turbines and are cylindrical in medium ones [2]. Small wind turbines have drown the interest of many researchers because of their ability to capture power at low wind speed conditions because of their aerodynamic structure. main challenging part is controlling the rotational speed of the wind turbine above rated wind speeds. Some existing control mechanisms are: Stalling which is simply increasing the angle at which the relative wind strikes the blades (angle of attack). Furling is done by reducing the angle of attack which reduces the induced drag from the lift of the rotor, as well as the cross-section [3]. Stalling and Furling controls the speed of the turbine at low and high wind speeds respectively. Other existing control mechanisms are: generator torque, yawing, electrical and mechanical breaking systems [3].
The paper is organized as follows : In section II, we will look at the blade element model with fixed wind speeds. Section III will be devoted to studying the stochastic effects of wind speeds on the aerodynamic power and torque and we will draw a general conclusion from our findings in section IV.

II. THE BLADE ELEMENT MODEL WITH FIXED WIND SPEEDS

Here the effect of turbulences are very small and hence can be neglected. The power extracted from a wind by a wind turbine is known to be a function of three main parameters: the wind power, the ability of the machine respond to wind fluctuations and the power curve of the machine. The mathematical expression of the aerodynamic power captured by the wind turbine is given by the non-linear equation [1]

$$P_a = \frac{1}{2}\rho\pi R^2 C_p(\lambda)v^3 \qquad (1)$$

where

$$\lambda = \frac{\omega R}{v} \qquad (2)$$

is what is generally called the tip-speed ratio that is the ratio between the linear blade tip speed and the wind speed v. R is the blade radius [4]. Substituting for v is the above equation, we have that

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$$P_a = \frac{1}{2}\rho\pi R^5 \frac{C_p(\lambda)}{\lambda^3}\omega_r^3 \qquad (3)$$

where ω is the turbine's rotational speed.

The actual efficiency of the wind turbine that is the maximum energy it can convert to useful work is called the power coefficient C_p. Theoretically, the power coefficient is calculated as the ratio of actual to ideal extracted power. It depends non-linearly on the stip-speed ratio given by the expression

$$C(\beta,\lambda) = C_1\left(\frac{C_2}{\lambda} - C_3\beta - C_4\beta^x - C_5\right)e^{-\frac{C_6}{\lambda}} \qquad (4)$$

with

$$\frac{1}{\lambda} = \frac{1}{\lambda + 0.08\beta} - \frac{0.035}{1+\beta^3} \qquad (5)$$

We see that this power coefficient C_p can adjust by controlling the angle of attack, β, and the tip speed ratio λ [4].

Now, using the relation

$$P_a = \omega_r T_a \qquad (6)$$

where T_a is the aerodynamic torque given by

$$T_a = \frac{1}{2}\rho\pi R^3 C_q(\lambda)v^2 \qquad (7)$$

with $C_q(\lambda)$ the torque coefficient, we have that

$$C_q(\lambda) = \frac{C_p(\lambda)}{\lambda} \qquad (8)$$

The coefficients C_P and C_q are specific for each wind turbine and are usually provided by the manufacturer [4].

Considering the fixed pitch, variable speed type wind turbine, we studied the aerodynamic characteristics through computer simulations shown below.
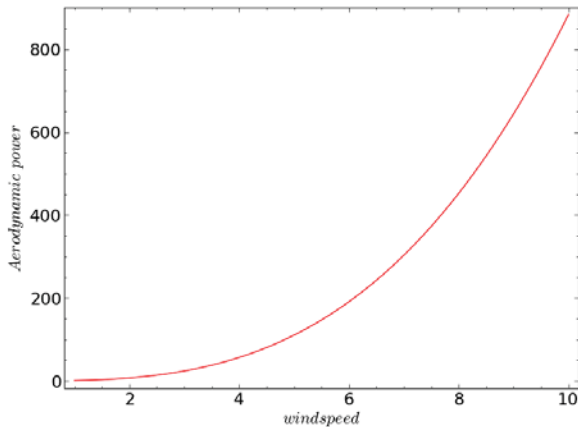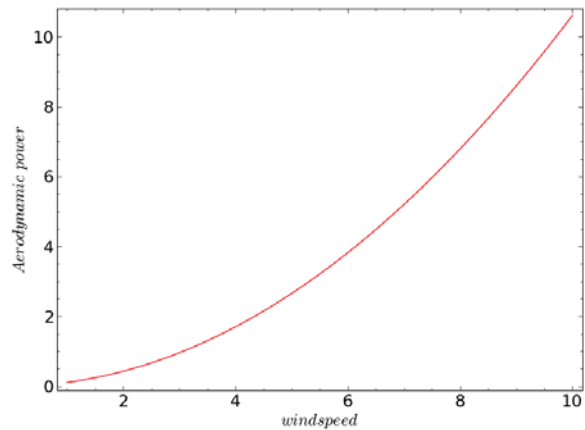
Fig2: Torque vs wind speed for $\rho = 1.125$, $R = 1$, $C_q = 0.06$

Fig3: P_a vs λ for $C_1 = 0.5$, $C_2 = 98$, $C_3 = C_4 = 0$, $C_5 = 5.0$, $C_6 = 16.6$.

Fig1: output power vs wind speed for $\rho = 1.125$, $R = 1$, $C_p = 0.5$

Fig4: T_a vs λ for $C_1 = 0.5$, $C_2 = 98$, $C_3 = C_4 = 0$, $C_5 = 5.0$, $C_6 = 16.6$.

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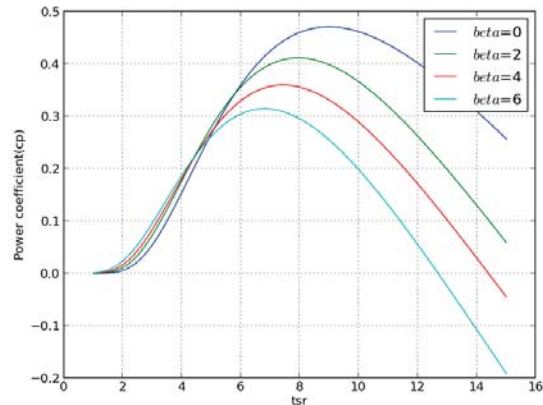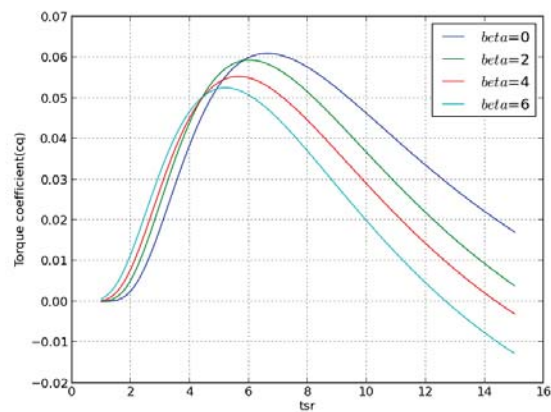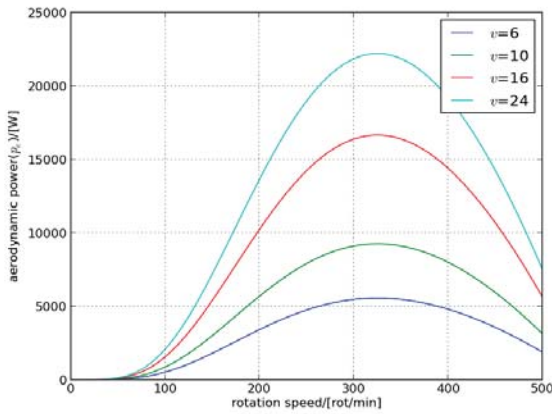International Journal of Mechanical and Mechatronics Engineering
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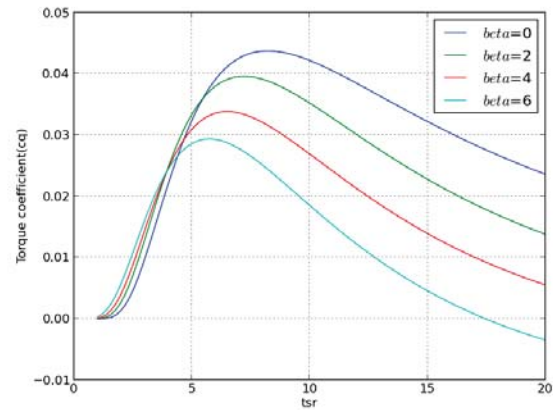
fig5: P_a vs ω for $C_1 = 0.5$, $C_2 = 98$, $C_3 = C_4 = 0$, $C_5 = 9.0$, $C_6 = 16.6$.

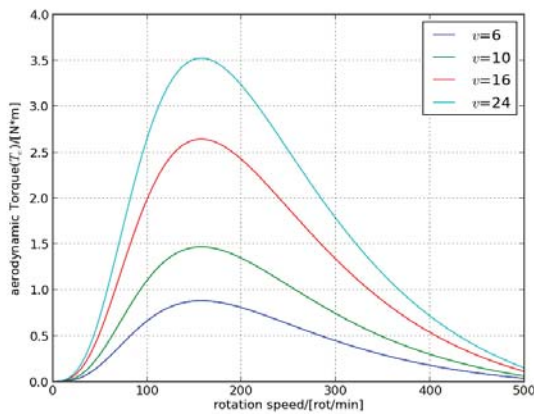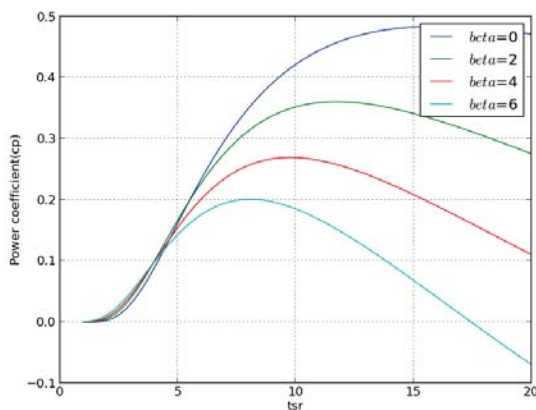Fig8: T_a vs ω for $C_1 = 0.5$, $C_2 = 45$, $C_3 = C_4 = 0$, $C_5 = 0.1$, $C_6 = 16.6$.

A. DISCUSSION

- In Fig1 and Fig2, we see that the harness power from the wind increases as the cube of the wind velocity and torque as the square of the wind velocity which does not truly represent the performance of a real wind turbine. This issue will be addressed in the next section.
- In Fig3 and Fig4, we studied the effect of varying the tip-speed ratio at different pitch angle on the power and torque coefficients and we see that the turbine best operate under normal conditions i.e with $\beta = 0$.
- Fig5 and Fig6 are for the case where we study the effect of changing the wind speed on the power capture and torque which tells us that as the wind speed decreases, the output power drops.
- In Fig7 and Fig8, we tried to stabilize the power coefficient above rated wind speeds by turning the control parameter.

Fig6: T_a vs ω for $C_1 = 0.5$, $C_2 = 98$, $C_3 = C_4 = 0$, $C_5 = 9.0$, $C_6 = 16.6$.

III. STOCHASTIC WIND SPEED MODEL

In real life situations, the wind speed is not well defined and highly fluctuates over time. Within a very short period of time, we can have different values of the wind speed. Therefore what is mostly considered in modelling is the average wind speed or the group velocity of wind flow. But in this section, we are going to consider stochastic variations in extreme wind speeds on the blades of a small axial wind turbine. For stochasticity our wind speed can now be seen as a kind of a Gaussian white noise with a non-zero mean which is rightly skewed. This special Gaussian white noise is called the generalized extreme value (GEV) distribution. Like the extreme value distribution [5], the generalized extreme value distribution is often used to model the smallest or largest value among a large set of independent, identically distributed random values representing measurements or observations. The generalized extreme value combines three simpler distributions into a single form, allowing a continuous range of possible shapes that includes all three of the simpler distributions. They are often referred to as the Types I, II, and III. Types I, II, and III are sometimes also referred to

Fig7: T_a vs ω for $C_1 = 0.5$, $C_2 = 45$, $C_3 = C_4 = 0$, $C_5 = 0.1$, $C_6 = 16.6$.

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as the Gumbel, Frechet, and Weibull types, with the shape parameter differentiating them.

Now we will consider our wind speed no-longer as v but as a probability density function (GEV distribution) given by

$$F(v) = \begin{cases} \exp\left\{ -\left[1 + \xi\left(\frac{v-\mu}{\sigma}\right)\right]^{-1/\xi} \right\} & \xi \neq 0 \\ \exp\left(-exp\left[-\left(\frac{v-\mu}{\sigma}\right)\right]\right) & \xi = 0 \end{cases} \quad (9)$$

Define for $\left\{v : 1 + \xi\frac{(v-\mu)}{\sigma} > 0\right\}$, $-\infty < \mu < \infty$, $\sigma > 0$, and $-\infty < \xi < \infty$ where μ is the location parameter or the mean speed, ξ is the shape parameter and σ is the scale parameter. The Location parameter specifies the center of the GEV distribution. Scale parameter, determines the size of deviations of μ. And Shape parameter which shows how rapidly the upper tail decays [5]-[6].
Here positive ξ implies a heavy tail while negative ξ implies a bounded tail, and the $\lim \xi \to 0$ implies an exponential tail.

The out put power can also be describe by a probability density function which is proportional to the cube of the velocity density function and the aerodynamic torque as the square of the velocity density function given by

$$P(v) = \frac{1}{2}\rho\pi R^2 C_p(\lambda)\left(\exp\left\{-\left[1 + \xi\left(\frac{v-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}\right)^3 \quad (10)$$

$$T(v) = \frac{1}{2}\rho\pi R^3 C_q(\lambda)\left(\exp\left\{-\left[1 + \xi\left(\frac{v-\mu}{\sigma}\right)\right]^{-1/\xi}\right\}\right)^2 \quad (11)$$

The consequence of stochastic variation of wind speed is the induction of a turbulence effect on the vanes of the wind turbine which have a grate effect on the aerodynamic power capture and aerodynamic torque. It has earlier been found that turbulence effects decreases with height. Hence to solve this problem, wind mills should be belt with a very high tower. During turbulence, the rotational frequency becomes much higher than the turbulence bandwidth, which might cause the wind field to be frozen. Then, a rotating blade element will experience a cyclic wind fluctuation. So, the corresponding power spectrum will consist of impulses at integers of frequency 1P. The amplitude of this fluctuation will be larger for blade elements near the tip and lower for blade elements near the hub [2].
In real wind turbines, the rotational frequency may be several times higher than the turbulence bandwidth. So, the wind experienced by the blades will have high energy concentration which might generate harmonics. This can now be considered as external cyclic perturbations. these cyclic wind perturbations are higher in the outer part of the rotor,

which precisely more contributes to the aerodynamic torque.

In our simulation, we considered a wind turbine operating at a defined C_p value of 0.5 and a C_q value of 0.06. The power spectrum and the aerodynamic torque spectrum are shown below.
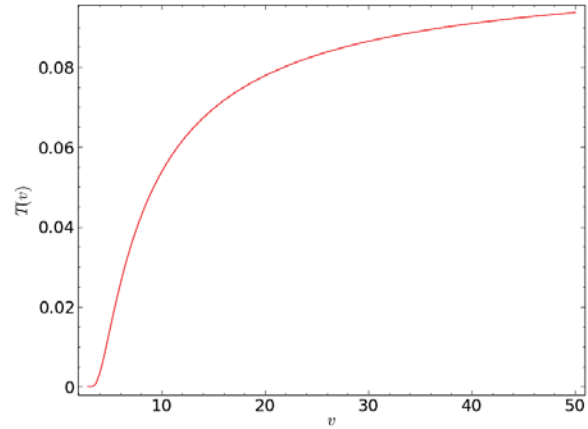

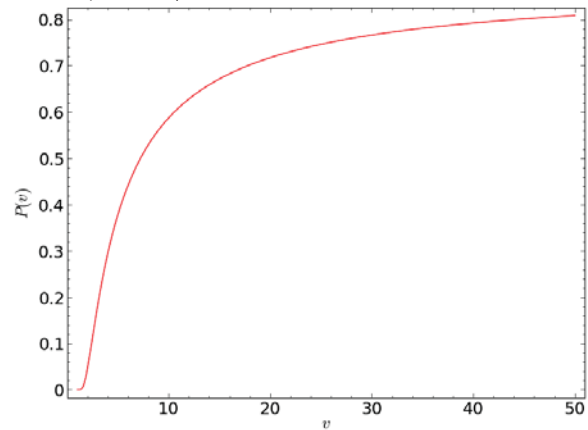Fig9: Torque spectrum for $C_q = 0.06$, $\rho = 1.125$, $R = 1$, $\xi = 1.02$, $\mu = 5m/s$, $\sigma = 2.2$

Fig10: Power spectrum for $C_p = 0.06$, $\rho = 1.125$, $R = 1$, $\xi = 1.02$, $\mu = 5m/s$, $\sigma = 2.2$

A. DISCUSSION

From the above figures, we have seen that, considering the stochastic effect of the wind speed on the blade elements and the choice of the appropriate parameters, the power and torgue spectrum have an upper bound which gives us a true picture of a real wind turbine. Therefore from this analysis, v which is the wind speed should be denoted as $< v >$ the average or mean wind speed when used in our modelling. The power and torque can now be written as

$$P_a = \frac{1}{2}\rho\pi R^2 C_p(\lambda) < v >^3 \quad (12)$$

$$T_a = \frac{1}{2}\rho\pi R^3 C_q(\lambda) < v >^2 \quad (13)$$

But when it comes to distributions that have a large deviation from the median like the GEV distribution, it is more convenient to use the root-mean-square speed (v_{rms}) than the mean speed because most of the wind particles under extreme

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drift velocity are likely to have these speed. Hence our power and torque factors now becomes

$$P_a = \frac{1}{2}\rho\pi R^2 C_p(\lambda)v_{rms}^3 \qquad (14)$$

$$T_a = \frac{1}{2}\rho\pi R^3 C_q(\lambda)v_{rms}^2 \qquad (15)$$

IV. CONCLUSION

To conclude, we have studied the wind turbine blade element characteristics at fixed wind speed conditions and at fluctuating wind speed conditions where we have seen that to model a real wind turbine, it is advisable to use probabilistic theory because of randomness in the dynamics of the system. Moreover, power control methods can also be achieved numerically by turning some control parameters. The constant quest for more efficient and reliable source of energy has open a broad field of research with scientist and engineers searching for the most appropriate way to improve the efficiency of a modern day wind turbine. All of the existing theories and those that are to be develop will be geared towards addressing real life issues.

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