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Comparative Analysis of Two Modeling Approaches for Optimizing Plate Heat Exchangers

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Abstract**—**In the present paper the design of plate heat exchangers is formulated as an optimization problem considering two mathematical modelling. The number of plates is the objective function to be minimized, considering implicitly some parameters configuration. Screening is the optimization method used to solve the problem. Thermal and hydraulic constraints are verified, not viable solutions are discarded and the method searches for the convergence to the optimum, case it exists. A case study is presented to test the applicability of the developed algorithm. Results show coherency with the literature.

Keywords**—**Plate heat exchanger, optimization, modeling, simulation.

NOMENCLATURE

A_P	Plate effective area, m^2
$\bar{A}$	Eigenvalues and eigenvectors matrix
b	Average thickness channel, m
$\bar{B}$	Binary vector
c_p	Heat capacity, J/kg·K
C_r	Heat capacity ratio
$\bar{C}$	Coefficients vector
E	Exchanger effectiveness, %
IS	Initial set of configurations
k_P	Plate thermal conductivity, W/m·K
L_P	Plate length, m
$\dot{M}$	Mass flow rate, kg/s
$\bar{\bar{M}}$	Tri-diagonal matrix
N	Number of channels per pass
N_C	Number of channels
N_P	Number of plates
NTU	Number of transfer units
OS	Optimal set of configurations
P	Number of passes
RS	Reduced set of configurations
s_i	Binary parameter for flow direction
t_P	Plate thickness, m
U	Global heat transfer coefficient, W/m^2·K
v	Fluid velocity inside channels, m/s
W_P	Plate width, m
Y_h	Binary parameter for hot fluid location
z_i	Eigenvector of the tri-diagonal matrix
Greek symbols	
α	Heat transfer coefficient
β	Angle of inclination of the
ΔP	Pressure drop, Pa
η	Normalized plate length
θ	Dimensionless fluid temperature
λ	Eingevalue of the tri-diagonal matrix
Φ	Enlargement factor of the plate area
ϕ	Parameter for feed connections position
Subscripts	
$cold$	Cold fluid
CC	Countercurrent
hot	Hot fluid
i	Generic element
in	Inlet
j	Generic element
out	Outlet
Superscripts	
I	Odd channels of the heat exchanger
II	Even channels of the heat exchanger
max	Maximum
min	Minimum

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I. INTRODUCTION

THE competitive pressures and the increasing interest in the energy conservation and reduction of the environmental impacts have changed the focus on the industrial processes for the use of heat exchangers with high effectiveness. Although the plate heat exchanger (PHE) is classified at the lower end of compactness, it offers several advantages and unique characteristics when compared with compact heat exchangers. It is due to the flexible thermal design (the plates can simply be added or removed to attend different demand of heat duty and processing), cleaning facilities to maintain extreme hygiene conditions (necessary when food, pharmaceutical or other kind of products are processed), good temperature control (necessary in cryogenic uses) and better performance in heat transfer [1]. Due to its characteristics it became ideal for diary, pharmaceutical, food and drink industries [2].

In the present paper we developed an optimization algorithm based on the screening method to find the optimal set of configurations for a specified plate heat exchanger.

II. MATHEMATICAL MODELING

Before presenting the two modeling approaches we shall introduce five parameters for characterization of the PHE configuration: N_C, P^I, P^{II}, ϕ, Y_h. These parameters were showed in Gut and Pinto [3].

Number of channels (N_C): The space between two adjacent plates is a channel. The end plates are not considered, thus the number of channels of a PHE is the number of plates minus one (Fig. 1). The odd-numbered channels belong to side I, and the even-numbered ones belong to side II. The number of channels in each side is N_C^I and N_C^{II}.

Number of passes (P): It is the number of change direction of determined stream plus one inside de plate pack. P^I and P^{II} are the number of passes in each side.

Hot fluid location (Y_h): It is a binary parameter that assigns the fluids to the PHE sides. If Y_h = 1, the hot fluid occupies side I and if Y_h = 0, the hot fluid occupies side II.

Feed connection (ϕ): The side I feed is arbitrarily set at η = 0 as presented in Fig. 1. Thus the parameter ϕ represents relative position of the side II. Fig. 1 illustrates all possibilities of connection. The η parameter is defined as $\eta = x/L_P$.

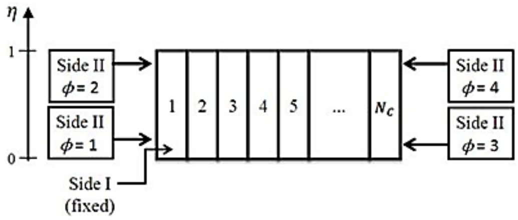

Fig. 1 Feed connection of a PHE

A. Modeling by Differential Equations

The model used was proposed by Gut and Pinto [3]. The following considerations were used:

- Steady state;
- The flow rate is divided equally between the channels of each pass;
- Plug-flow;
- Perfect mixture at the end of each pass;
- Thermal losses are negligible;
- There are no phase changes;
- Heat is exchanged only in the perpendicular direction of flow;
- Physical properties are constant.

The energy balance in the exchanger channel gives the system of differential equations:

$$\frac{d\theta_1}{d\eta} = s_1\alpha^I(\theta_2 - \theta_1) \qquad 1\text{ (a)}$$

$$\frac{d\theta_i}{d\eta} = \begin{cases} s_i\alpha^I(\theta_{i-1} + 2\theta_i + \theta_{i+1}) & \text{if } i \text{ is odd} \\ s_i\alpha^{II}(\theta_{i-1} + 2\theta_i + \theta_{i+1}) & \text{if } i \text{ is even} \end{cases} \qquad 1\text{ (b)}$$

$$\frac{d\theta_i}{d\eta} = \begin{cases} s_{N_C}\alpha^I(\theta_{N_c-1} + \theta_{N_c}) & \text{if } N_C \text{ is odd} \\ s_{N_C}\alpha^{II}(\theta_{N_c-1} + \theta_{N_c}) & \text{if } N_C \text{ is even} \end{cases} \qquad 1\text{ (c)}$$

where s_i is the direction of flow in the channels (s =1 if upward flow and s = - 1 if downward flow), θ is the adimensional temperature and:

$$\alpha^I = \frac{A_P U N^I}{\dot{M}^I c_p{}^I} \qquad \alpha^{II} = \frac{A_P U N^{II}}{\dot{M}^{II} c_p{}^{II}} \qquad (2)$$

This system of linear differential equations can be written in the matrix form:

$$\frac{d\bar{\theta}}{d\eta} = \bar{\bar{M}}.\bar{\theta} \qquad (3)$$

$$\bar{\bar{M}} = \begin{bmatrix} -d_1 & +d_1 & 0 & 0 & \cdots & 0 \\ +d_2 & -2d_2 & +d_2 & 0 & \cdots & 0 \\ 0 & +d_3 & -2d_3 & +d_3 & & \vdots \\ \vdots & & & & & 0 \\ 0 & \cdots & 0 & +d_{N_C-1} & -2d_{N_C-1} & +d_{N_C-1} \\ 0 & \cdots & 0 & 0 & +d_{N_C} & -d_{N_C} \end{bmatrix}$$

$$d_i = \begin{cases} s_i\alpha^I & \text{if } i \text{ is odd} \\ s_i\alpha^{II} & \text{if } i \text{ is even} \end{cases} \qquad i = 1, \ldots, N_C$$

The boundary conditions depend on the PHE configuration and can be divided into three different categories:

1) *Fluid inlet:* the fluid inlet temperature in the channels of the first pass is the fluid feed temperature.

$$\theta_i(\eta) = \theta_{fluid,in} \qquad i \in \text{first pass} \qquad 4\text{ (a)}$$

2) *Change of pass:* the temperature at the beginning of the channels of a determined pass is equal to the arithmetic average of the temperatures in the channels of the previous pass.

$$\theta_i(\eta) = \frac{1}{N}\sum_{\substack{j \in previous \\ pass}}^{N} \theta_j(\eta) \qquad i \in \text{new pass} \qquad 4\text{ (b)}$$

3) *Fluid outlet:* the outlet temperature of the fluid is the arithmetic average of outlet temperatures of the channels of the last pass.

$$\theta_{fluid,out}(\eta) = \frac{1}{N}\sum_{\substack{j \in last \\ pass}}^{N} \theta_j(\eta) \qquad 4\text{ (c)}$$

The analytical solution is:

$$\bar{\theta}(\eta) = \sum_{i=1}^{N_c} c_i \bar{z}_i e^{\lambda_i \eta} \qquad (5)$$

The use of (5) into the boundary conditions for the fluid inlet and change of pass gives a linear system of N_C equations of variables c_i. After the solution of the linear system the outlet temperatures can be determined by using the outlet boundary conditions and, consequently, the thermal effectiveness can be determined.

B. Modeling by Closed Form

Most multi-pass plate heat exchangers can be represented by simple combinations of pure countercurrent and concurrent exchangers, in other words, multi-pass PHE are equivalent to combinations of smaller exchangers of single pass (Fig. 2). Based on this, [4] developed formulas for effectiveness as a function of the ratio between the heat capacities of the fluids, and the number of transfer units, for the arrangements 1-1, 2-1, 2-2, 3-1, 3-2, 3-3, 4-1, 4-2, 4-3, and 4-4.

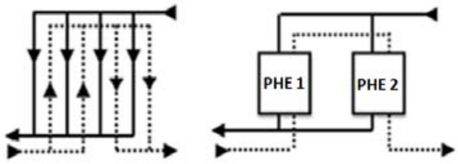

Fig. 2 Equivalent configurations

III. Optimization

The optimization problem is formulated in such a way that the best combination of the parameters of a given PHE minimizes the number of plates. The used optimization method is based on screening, which was studied in [5], in which - for a given type of plate - the number of thermal plates is the objective function that has to be minimized. This is subject to certain inequality constraints and is implicitly given by the heat exchanger configuration parameters.

A. Formulation of the Optimization Problem

Minimize:

$$N_P = f(N_C, P^I, P^{II}, \phi, Y_h) \tag{7}$$

Subject to:

$$N_C{}^{min} \leq N_C \leq N_C{}^{max} \tag{8a}$$

$$\Delta P_{hot} \leq \Delta P_{hot}{}^{max} \tag{8b}$$

$$\Delta P_{cold} \leq \Delta P_{cold}{}^{max} \tag{8c}$$

$$v_{hot} \geq v_{hot}{}^{min} \tag{8d}$$

$$v_{cold} \geq v_{cold}{}^{min} \tag{8e}$$

$$E^{min} \leq E \leq E^{max} \tag{8f}$$

Depending on the equipment model the number of plates can vary between 3 and 700. The first constraint (8a) can be established according to the PHE capacity. Depending on the available power in the pump it can be established the constraints (8b) and (8c). The velocity constraints are imposed generally to avoid dead spaces or air bubbles inside the set of plates. Velocities less than 0.1 m/s are not used in practice [6].

To solve this optimization problem the screening method [5] is used. The constraints are evaluated successively, reducing the number of configurations until the optimal set can be found, if it exists. The screening process begins with the identification of an initial set (IS) of possible configurations considering the limit of channels. By verifying the velocity and pressure drop constraints, a reduced set (RS) is generated. To this RS it is applied in crescent order of the number of channels the constraint of thermal effectiveness. The configurations with the smallest number of channels will form the optimal set. The optimization algorithm is described below.

B. Description of the Optimization Algorithm

In the paper of Gut and Pinto [5] the screening method was developed in C language to obtain the reduced set of configuration (RS) and the simulation of the systems of differential equations of the reduced set of configurations (RS) was performed numerically using the gPROMS software. In this paper, the proposed algorithm is fully developed in MatLab. Thus it is not necessary the use of auxiliary software, like gPROMS. It represents the total independence of additional software to solve the systems of differential equations.

The developed algorithm is presented as follows:

Stage 1. Input data. PHE dimensions, fluids physical proprieties, inlet mass rate and temperature of both streams, constraints.

Stage 2. Generation of the initial set of configurations (IS). The vector $\overline{N}_C$ ($\overline{N}_C = N_C{}^{min}\!:\!N_C{}^{max}$) is generated with all the possible number of channels.

Stage 3. For each element of the vector $\overline{N}_C$, all possible number of passes for the sides I and II are computed, which are integer divisors of the number of channels of the corresponding side.

Stage 4. Verification of the hydraulic constraints of fluid velocity and pressure drop, v andΔP, respectively.

Stage 4.1. Taking$Y_h = 0$, i.e., when the cold fluid is in the side I and the hot fluid is in the side II. In this case, $P^I_{cold} = P^I$ and$P^{II}_{hot} = P^{II}$.

Stage 4.1.1. Cold fluid velocity $v^I{}_{cold}$ is calculated, in a decreasing order of the possible number of passes of a given element of$\overline{\mathrm{N}}_\mathrm{C}$. If $v^I{}_{cold}$ achieves the minimum allowable value, it is not necessary to evaluate configurations with smaller number of passes.

Stage 4.1.2. The cold fluid pressure drop is calculated, $\Delta P^I{}_{cold}$, in a crescent order of the possible number of passes of a given element of $\overline{N}_C$. If $\Delta P^I{}_{cold}$ achieves the maximum allowable value, it is not necessary to evaluate configurations with greater number of passes.

Stage 4.1.3. Verification of the velocity constraint for the stream in side II. Analogous to stage 4.1.1.

Stage 4.1.4. Verification of the pressure drop constraint in side II. Analogous to stage 4.1.2.

Stage 4.2. Taking$Y_h = 1$. Analogous to stage 4.1.

Stage 5. Generation of the reduced set (RS).

Stage 5.1. For $Y_h = 0$, are combined the number of passes selected for the sides I and II of the PHE.

Stage 5.2. Considering $Y_h = 1$, the number of passes selected for the sides I and II of the PHE are combined. With the contributions of Stages 5.1 and 5.2 all the RS configurations are obtained.

Stage 6. Calculate the effectiveness in pure countercurrent flow, E_{CC}. If $E_{CC} < E^{min}$, these configurations can be discarded.

Stage 7. Verification of the thermal effectiveness constraint. The selected configurations in Stage 6 are simulated in a crescent order of the number of channels to find the possible optimum set (OS). The remaining configurations do not need to be simulated.

IV. Results

To test the developed algorithm a case study is considered, presented in [7]. A cold stream of benzene exchanges heat with a toluene hot stream. Table I presents the data used.

TABLE I
EXAMPLE DATA

PLATE CHARACTERISTICS	
$L_P = 53.7$ cm	$\beta = 45$ °
$W_P = 18.8$ cm	$\Phi = 1.24$
$b = 2.85$ mm	$t_P = 0.6$ mm
$D_P = 63.5$ mm	$k_P = 17$ W/m·K
TOLUENE	BENZENE
$T_{in,hot} = 78.0$ °C	$T_{in,cold} = 15.0$ °C
$\dot{M}_{hot} = 0.80$ kg/s	$\dot{M}_{cold} = 1.23$ kg/s
CONSTRAINTS	
$15 \leq N_C \leq 40$	$E^{min} = 85\%$
$0 \leq \Delta P_{hot} \leq 10$ psi	$0 \leq \Delta P_{cold} \leq 10$ psi
${v_{hot}}^{min} = 0.3$ m/s	${v_{cold}}^{min} = 0.3$ m/s

By applying the optimization algorithm until Step 5, the RS is obtained. The optimal set is found applying Step 7. The same optimal set was found for both approaches of modeling, i.e., two heat exchangers' configurations with 30 channels and symmetric arrangement of 3 passes, according to Table II.

TABLE II
THERMAL EFFECTIVENESS OF RS

#	N_C	P^I	P^{II}	Y_h	ϕ	E_{DE}	E_{CF}
1	15	2	1	1	4	0.66	0.67
2	16	1	2	0	4	0.66	0.68
3	16	2	2	0	3	0.78	0.80
4	16	2	1	1	4	0.66	0.68
5	16	2	2	1	3	0.78	0.80
6	17	3	1	1	4	0.70	0.71
7	17	3	2	1	3	0.78	0.79
8	19	2	3	0	3	0.79	0.80
9	20	2	2	0	3	0.80	0.82
10	20	2	2	1	3	0.80	0.82
11	24	2	3	0	3	0.80	0.82
12	24	3	2	1	3	0.80	0.82
13	29	3	2	1	3	0.82	0.83
14	30	3	3	0	4	**0.89**	**0.91**
15	30	3	3	1	4	**0.89**	**0.91**
16	31	2	3	0	3	0.83	0.83
17	31	4	3	1	4	0.88	0.88
18	32	2	4	0	3	0.84	0.85
19	32	4	2	1	3	0.84	0.85

The optimal set is consistent with the values obtained in [5]. The simulated configurations have excellent concordance with the researcher results, which used numerical methods to calculate the thermal effectiveness. E_{DE} is the effectiveness calculated by simulation of the systems of differential equations and E_{CF} is the effectiveness calculated by closed form.

V. Conclusion

The developed algorithm presented excellent performance.

This paper presented an optimization algorithm for the design of plate heat exchangers considering two modeling approaches. It was based on the model based on differential equations [3] and the model based on algebraic equation [4]. Different from [5] the system of differential equations was solved analytically. Also, it is important to highlight that no additional software needs to be used, like gPROMS or other ones. This is the great contribution of the present paper when compared with the work of [5].

As a case study, an example of the literature was used. The optimal set for both modeling was the same. Besides, all values of the effectiveness presented in Table II for the two approaches are in accordance.

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