Well-Being Inequality Using Superimposing Satisfaction Waves: Heisenberg Uncertainty in Behavioural Economics and Econometrics

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Abstract—In this article, a new method is proposed for the measuring of well-being inequality through a model composed of superimposing satisfaction waves. The displacement of households' satisfactory state (i.e. satisfaction) is defined in a satisfaction string. The duration of the satisfactory state for a given period is measured in order to determine the relationship between utility and total satisfactory time, itself dependent on the density and tension of each satisfaction string. Thus, individual cardinal total satisfaction values are computed by way of a one-dimensional form for scalar sinusoidal (harmonic) moving wave function, using satisfaction waves with varying amplitudes and frequencies which allow us to measure well-being inequality. One advantage to using satisfaction waves is the ability to show that individual utility and consumption amounts would probably not commute; hence, it is impossible to measure or to know simultaneously the values of these observables from the dataset. Thus, we crystallize the problem by using a Heisenberg-type uncertainty resolution for self-adjoint economic operators. We propose to eliminate any estimation bias by correlating the standard deviations of selected economic operators; this is achieved by replacing the aforementioned observed uncertainties with households' perceived uncertainties (i.e. corrected standard deviations) obtained through the logarithmic psychophysical law proposed by Weber and Fechner.

Keywords—Heisenberg Uncertainty Principle, superimposing satisfaction waves, Weber–Fechner law, well-being inequality.

I. INTRODUCTION

ECONOMIC analysis of decision-making builds on the individual direct utility function inferred from the choices made regarding goods and services consumption. The axioms of revealed preference approach in this analysis reveal these decisions, whether rational or not, by deriving individual utilities. The standard utility theory assumes that individuals optimize their well-being, given certain constraints and evaluate their welfare in absolute terms. As an extension of this classic framework of decision-making, individual time use figures for consumption are further supposed to be combined with market goods to transform them into final commodities [1]. These final goods, themselves produced through household activities, are directly represented in the utility function [2]-[5]. Thus, the time-based values of both leisure and work can therefore be estimated by encompassing labour supply and time assignment equations defined within any consumer optimization program [6]. However, two

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problematic areas in this theory exist which are yet to be worked on. The first issue lies in non-separable preferences in time use. In fact, each group of consumption determines a specific satisfactory period owing, for instance, to not feeling hungry, being healthy, not having transportation…etc. [7]. The utilities defined over these satisfactory times are codependent and continuous since they are successively connected to preceding ones. Thus, the time intervals are necessarily superposed since consumers always prefer to consume at least two goods simultaneously [8]. The second issue is about that once the time spent is assumed to be a source of non-monetary cost, agents' true budget domain would be expected to be different from the one observed from the data; hence the violations of rationality axioms [9]. This later raises the problem of figuring the accuracy of an individual's exact amount of consumption from the dataset. In our opinion, these methodological difficulties indicate the existence of a more general measurement problem. In accordance with the first of the aforementioned issues, the violation of the rationality axioms would probably be due to the problem of psychological framing effects existing between the utility and consumption quantities whenever individuals' observed values are different from expected ones. Such a Heisenberg-type uncertainty, would exist in the demand system estimations, a priori implying that one cannot know with perfect accuracy both the utility and amount of consumption at any given point in time since we are not able to know the emotional responses to these un-observed variations. Indeed, it is more reasonable to assume that individuals' actual satisfaction state would instantly determine the true values of these observables. In this respect, this paper has two objectives: (i) to measure well-being inequality through a model of superimposing satisfaction waves which in turn allows (ii) the definition and elimination of Heisenberg-type uncertainty present in household budget surveys.

II. SUPERIMPOSING SATISFACTION WAVES

A. Utility Function

The satisfactory action implies maximizing the utility function depending on the optimal time allocation between working and consuming. Let the utility function, $U(X,T)$ defined over for all consumption quantity of x_i for the market goods i, $x_i \in X$, and working time $T_w \in T$ with consumption times $T_{c_i} \in T$, where $T = T_w + T_{c_i}$, given a Cobb–Douglas form

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[10].

$$U_i = \Omega \prod T_w^{\alpha} \prod_i T_{c_i}^{\beta_i} \prod_i X_i^{\eta_i} \qquad (1)$$

Ω measures individual time storage capacity of satisfactory consumption metabolism [11]; α, β_i, η_i are the exponents of working, consumption activity and amount of consumption goods for commodity i respectively. To isolate good demand as a function of utility and total time use consists of time spent in working and consumption activities, both sides of (1) are divided by $\sqrt[\eta_i]{\Omega \prod T_w^{\alpha} \prod_i T_{c_i}^{\beta_i}}$ which gives

$$X_i = \Omega^{-1/\eta_i} \prod T_w^{\alpha/\eta_i} \prod_i T_{c_i}^{\beta_i/\eta_i} \prod_i U_i^{1/\eta_i} \qquad (2)$$

Hypothesis 1. The goods and services consumed during the time spent in activities determine the satisfaction level of the individuals at time t. Therefore, aggregate time for all activities is equal to T, thus

$$S_i(U_i, T^a) = X_i = \Omega^{-1/\eta_i} \prod T_w^{\alpha/\eta_i} \prod_i T_{c_i}^{\beta_i/\eta_i} \prod_i U_i^{1/\eta_i} \qquad (3)$$

Equation (3) represents the displacement of households' satisfactory state (i.e. *satisfaction*) at time t and at a utility distance u from one end l of the satisfaction string.
Proof 1: Thus, it is expected that the waveform of satisfaction function (3) satisfy the condition

$$S''_T = v^2 S''_{U_i} \quad with \ \forall 0 < U_i < l; T > 0 \qquad (4)$$

Let v be the constant as the duration of the satisfactory state for a given period. This ratio determines the relation between utility and total time depending on density and the tension of the satisfaction string. By subjecting (3), by assuming that $T = \prod T_w^{\alpha} \prod_i T_{c_i}^{\beta_i}$ and $\alpha + \beta = \theta$, to the aforementioned condition in (4) v can be isolated as

$$v = \left[\frac{\left(\frac{\theta^2}{\eta^2} + \frac{\theta}{\eta} \right) \prod_i U_i^{1/\eta_i} \prod_i T_{a_i}^{-\frac{\theta_i}{\eta_i} - 2}}{\left(\frac{1}{\eta^2} + \frac{1}{\eta} \right) \prod_i \left(U_i^{1/\eta_i} \right)^{-2} \prod_i T_{a_i}^{-\frac{\theta_i}{\eta_i}}} \right]^{1/2} \qquad (5)$$

The equality given in (4) could also be reduced to the form

$$U = vT = T \sqrt{(1-\eta)/(\theta(\theta+\eta))} \qquad (6)$$

where v is $\sqrt{1 - \eta/\theta(\theta+\eta)}$. The limit exists for $D = \{(\theta,\eta) \mid \theta+\eta > 0; \theta > 1; 1 > \eta > 0\}$ for the range of $R = \{v \mid v(\eta,\theta) > 0\}$. By using the stochastic variables of corresponding population regression function (6) leaves,

$$\hat{U}_i = T \sqrt{(1-\hat{\eta}_i) \Big/ \left[(\hat{\alpha} + \hat{\beta})_i (\hat{\alpha} + \hat{\beta} + \hat{\eta})_i \right]} \qquad (7)$$

If the exponents of the Cobb-Douglas (1) are homogeneous to degree 1 then $v = 1$. The number of oscillations for satisfaction for a specific consumption activity per unit of time is equal to the number of oscillations. The economic insight of this supposition is that every decision is always made at the end of the utility time period. This period ends up with the moment at which an individual's state of satisfaction falls back again to its initial level at time 0. For the sake of simplicity, aggregate time for all activities is equal to T yields that the degree of increase in satisfactory time must always be equal to the total time degree of increases in working and consumption time; hence $\alpha + \beta = 1$. Thus, the compact form of (7) is

$$\hat{U}_{ih} = T_h \sqrt{(1-\hat{\eta}_{ih})/(1+\hat{\eta}_{ih})} \qquad (8)$$

Equation (8) gives the relation between T and $\hat{U}$ hence the satisfaction functions as $\hat{S}(v, T)$. $\hat{\eta}_{ih}$ determines the satisfaction level individual h; hence the utility. Equation (8) implies that the satisfaction can be defined, *ceteris paribus*, as $D = \{\hat{\eta}_{ih} \mid 0 < \hat{\eta}_{ih} < 1\}$ with

$$R = E(\hat{S}_{ih} = \hat{U}_{ih} \mid \hat{\eta}_{ih}) = \begin{cases} 0 & \lim_{\hat{\eta}_{ih} \to 1} \hat{S}_{ih} \\ 1 & \lim_{\hat{\eta}_{ih} \to 0} \hat{S}_{ih} \end{cases}$$

B. Total Satisfaction

Equation (8) also represents a one-dimensional form for the scalar sinusoidal (harmonic) moving wave function as

$$S(\hat{U}, T) = f(U - T \sqrt{(1-\hat{\eta}_{ih})/(1+\hat{\eta}_{ih})}) \qquad (9)$$

Satisfaction wave function $S(U,T)$ indicates the U coordinate—the transverse position—of any element located at position utility u at any satisfactory leisure time t. Here, v is the speed of a satisfaction wave function equal to $\sqrt{(1-\hat{\eta}_{ih})/(1+\hat{\eta}_{ih})}$.

Different goods and services may be represented in the consumption set as durable or non-durable used over a given period, and hence different utilities are felt from consumptions. Total satisfaction could be obtained by superposition of the satisfaction waves with different frequencies, amplitudes with the phase constant in the short term for these expenditure groups. In this sense, the absolute value of the maximum displacement from a state of equilibrium for a utility of the medium gives the amplitude A for each satisfactory wave measured by $A_{ih} = |\hat{U}_{ih} - \bar{U}_i|$, where $\bar{U}$ is the mean of ith consumption group. In the form of sinusoidal wave motion, A is the ratio between the frequencies over distances travelled by each utility wave. The length of

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these satisfaction waves are the distance from one crest (or troughs) to the next. Consumption decision exists whenever utility falls back to the minimum level of utility for any given wave function. This is the minimum utility point (i.e. trough) within the total satisfaction period, which could also be measured by the curvature function.

Hypothesis 2. The utility has a specific emotional value instantly felt at any point on the satisfaction string.

Proof 2: Equation (8) allows us to see how quickly utility changes direction at any point in time depending on a given satisfaction level. This could be observed through measuring the magnitude of the rate of change of the unit tangent vector with respect to arc length. Thus, it could be argued that the smooth changes on the satisfaction curve could give information about how an individual's emotional state changes at any point in time. Such an instant utility function could be obtained from the curvature equation applied on the two dimensional parabola functions, after taking the square of both sides of (8) as

$$K_{ih}(T_h) = \left[2(1-\hat{\eta}_{ih})(1+\hat{\eta}_{ih})^2 \right] \Big/ \left[T_h 2(1-\hat{\eta}_{ih}) + 1 + \hat{\eta}_{ih} \right]^3 \quad (10)$$

The curvature curve is continuous for $D : \left\{ (\hat{\eta}, T) \in \mathrm{P} : 2x(\hat{\eta}-1) \neq \hat{\eta}+1 \right\}$ with $\hat{\eta} \in (0,1)$. Equation (10) is obtained by tangent points on the satisfaction string and this curve equation allows knowing zero emotional chances (i.e. being neutral for one extra unit of consumption) at a point of time where the consumption stops for given market goods. All maximum and minimum points on that curve satisfy the first order condition for curvature function (10) at

$$\frac{\partial K}{\partial T} = -12(\hat{\eta}^2-1)^2 \Big/ (-2T(\hat{\eta}-1)+\hat{\eta}+1)^4 \quad (11)$$

with $\hat{\eta} \in (0,1)$ and $T > 0$. In order to obtain the solution of (11), let T_λ be the adjacent periodic thought or crest as the wavelength, which is equal to

$$T_\lambda = \left[\sqrt[4]{-12(\hat{\eta}^2-1)^2 / K_{\hat{U}}} - \hat{\eta} - 1 \right] \Big/ \left[-2(\hat{\eta}-1) \right] \quad (12)$$

Once the wavelength is obtained, we get the angular wave number $k = 2\pi / T_\lambda$ and angular frequency $\omega_{an} = 2\pi / T$. The frequency $1/T$ is computed by the period for each consumption group. According to the superposition principle, the net is sum of the individual displacement. More precisely, we superpose all satisfaction waves with different amplitudes and frequencies.

$$S_h(\hat{U}_{ih}, T_{ih}) = \sum_{i=1}^{n} A_i \sin(k_i U_{ih} - \overline{\omega}_i T_{ih} + \mu_{ih}) \quad (13)$$

C. Well-Being Inequality

In the literature, there are the usual income inequality measures such as the Gini-index, Range, Range-ratio, the McLoone-Index, the Coefficient of Variation, the Theil-index, the Pareto-index, and the Atkinson-index or Subjective well-being index [12]. Each index has its own advantages and disadvantages. Most of these indexes suffer from ignoring all but two of the observations; they do not weight observations, are affected by inflation, skewed by outliers, they ignore values above the median, show no intuitive motivating picture, cannot directly compare populations with different sizes or group structures or are comparatively mathematically complex. We choose the Gini Index, which is generally regarded as the gold standard in economic work, which allows incorporating all data and making a direct comparison between units with different size populations.

Let M_k, S_k be the cumulated proportion of the population variable and of the satisfaction variable respectively, for $k=0,...,n$, with $M_0=0$, $S_0=0$, and $M_n=1$, $S_n=1$ where S_k is indexed in non-decreasing order ($S_k > S_{k-1}$). The Gini index (G) is defined as a ratio of the areas on the Lorenz curve. Thus, G is obtained by approximation of Lorenz curve, on each interval as line between consecutive points, the areas can be approximated with trapezoids as

$$G = 1 - \sum_{k=1}^{n} (M_k - M_{k-1})(S_k + S_{k-1}) \quad (14)$$

III. Uncertainty Relation for Self-Adjoint Economic Operators

Let the utility (U) and the consumption quantity(X) are two observables represented by self-adjoint operators on Hilbert space. Utility and consumption operators are additionally positive and have trace 1 since probabilities are positive and normalized to unity: $W^\dagger = W \geq 0$ and $Tr(W) = 1$. States are expressible via state vectors $|\psi\rangle : W = |\psi\rangle\langle\psi|$, hence $\langle [U, X] \rangle = \langle\psi| . |\psi\rangle$ denotes the expectation value in state $|\psi\rangle$.

A. Variation and Correlation

The variation of consumption ΔX in state W is

$$\Delta X := \sqrt{\left\langle \left(X - \langle X \rangle \right)^2 \right\rangle} \quad (15)$$

The correlation coefficient c_{UX} is

$$c_{UX} := \frac{\left\langle \frac{1}{2}(UX + XU) \right\rangle - \langle U \rangle \langle X \rangle}{\Delta U \, \Delta X} \quad (16)$$

Equation (16) is the covariance divided by the product of the variations where $|c_{UX}| \leq 1$.

B. Uncertainty Relation

The Cauchy-Schwarz inequality gives

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$$(\Delta U \Delta X)^2 \geq |\langle UX \rangle|^2 \qquad (17)$$

The left side of (17) is

$$|\langle UX \rangle|^2 = \left| \tfrac{1}{2} \langle UX - XU \rangle + \tfrac{1}{2} \langle UX + XU \rangle \right|^2 \qquad (18)$$

U and X are Hermitian, so the expectation value for $UX + XU$ must be real. However, the expectation value for $UX - XU$ is imaginary since it is anti-hermitian. By using the correlation coefficient in (16) and the commutator $[UX] := UX - XU$, the inequality (17) could be simplified as

$$\begin{aligned} |\langle UX \rangle|^2 &= \tfrac{1}{4} |\langle UX - XU \rangle|^2 + \tfrac{1}{4} |\langle UX + XU \rangle|^2 \\ &= \tfrac{1}{4} |\langle [UX] \rangle|^2 + c_{UX}^2 (\Delta U \Delta X)^2 \end{aligned} \qquad (19)$$

Together with (17) we get

$$\tfrac{1}{4} |\langle [UX] \rangle|^2 \leq (\Delta U \Delta X)^2 (1 - c_{UX}^2) \qquad (20)$$

Further by taking the square root (20) yields

$$\Delta U \Delta X \sqrt{1 - c_{UX}^2} \geq \tfrac{1}{2} |\langle [UX] \rangle| \qquad (21)$$

The inequality (21) is the strong version of the uncertainty relation [13]. This relation provides a lover bound to the product of the variations of U and X. Therefore direct utility functions able to measure cardinal utilities since the expenditure data provided by household budget surveys. In this case, if these two observables commute (i.e. $\hat{U}\hat{X}f(n) = \hat{X}\hat{U}f(n)$), the right hand side is zero and the bound is trivial. Thus, utility and consumption can both be measurable exactly. Furthermore, for 100% (anti)correlated observables, $c_{UX} = \pm 1$, the left hand side of the inequality vanishes. Since the right hand side is clearly nonnegative, the expectation value of the commutator must thus vanish. It would be expected that for the utility and consumption values, which do not satisfy $\langle U \rangle = \langle X \rangle = 0$, the uncertainty relation remains valid. In fact, $U' := U - \langle U \rangle$ and $X' := X - \langle X \rangle$ are centered by construction and the uncertainty relation given in (21) holds for them. However, $\Delta U' = \Delta U$ and $\Delta X' = \Delta X$; also $[U', X'] = [U, X]$ and $c_{U'X'} = c_{UX}$.

C. Not Commuted Observables

In reality, the problem faced by consumers is that uncertainty is pervasive in almost all decision-making and is inevitable whenever an action or decision and its consequence are separated by a period of time [14]. Therefore, a utility function would suffer from two categorical sources of bias:

1. Firstly, the difficulty lies in the fact that individuals' actual utility values may be different from that anticipated.

2. Secondly, observed consumption quantities would be different from the individual expected amounts.
   However, it is not reasonable to assume that the deviations from expected values of these two observables are necessarily correlated. Thus, the correlation value in (21) would be imperfect[1].

In this respect, standard variations ΔU and from the data may be misleading since we are unable to know the actual satisfaction level of each individual. That is to say, perceived values of the same amount of variation in consumption would vary at the outset, depending on the given state of satisfaction for each individual; and, subsequently, the corresponding quantities of the same variations in the cardinal utilities would not be expected to be the same. It thus raises the problem that the utility and amount of consumption do not perfectly commute (i.e. $\hat{U}\hat{X}f(n) \neq \hat{X}\hat{U}f(n)$); hence it is impossible to precisely know the values of U and X.

D. Perceived Uncertainty Relation

We propose to obtain the corrected uncertainty values through the application of the Weber-Fechner Law. This is the logarithmic form of the relationship between the perceived stimuli and the discrimination ratio (expressed as % changes in standard deviation of observables and initial values of observables). In mathematical form that law for our consumption observations is

$$\Delta X^* = k . \ln(\Delta X / X) \qquad (22)$$

where, ΔX^* is individuals' perception of (change in) consumption, ΔX is standard deviation of observed consumption, X is the observed consumption amount from household budget survey. Similarly, for utility this law implies

$$\Delta U^* = k . \ln(\Delta U / U) \qquad (23)$$

where, ΔU^* is individual's perception of (change in) utility, ΔU is standard deviation of cardinal utilities, U is the computed utility values.

The k in both equations is the relationship parameter representing the different units of measurement for magnitude of sensations and stimulus. Such a constant could be known for each individual. The curvature K_{ih} given in (10) corresponds to k constants in (22) and (23) which allow us to take into consideration the emotional states by using individuals' satisfactory time values computed through the estimates of the consumption amounts for each commodity groups. By replacing (10) in to (22) and (23), individual perceived uncertainty values respectively becomes

$$\Delta X_{ih}^* = \frac{2(1 - \hat{\eta}_{ih})(1 + \hat{\eta}_{ih})^2}{[T_h 2(1 - \hat{\eta}_{ih}) + 1 + \hat{\eta}_{ih}]^3} . \ln(\Delta X / X) \qquad (24)$$

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[1] e.g. the Akerlof effect due to information shortages and asymmetries.

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$$\Delta U_{ih}^{*} = \frac{2(1-\hat{\eta}_{ih})(1+\hat{\eta}_{ih})^2}{\left[T_h 2(1-\hat{\eta}_{ih}) + 1 + \hat{\eta}_{ih}\right]^3} . \ln\left(\Delta U / U\right) \qquad (25)$$

The corrected form of the uncertainty relation between utility and consumption amounts given in (21) is

$$\Delta U^{*}\Delta X^{*}\sqrt{1-c_{UX}^{2}} \geq \tfrac{1}{2}\left|\left\langle\left[UX\right]\right\rangle\right| \qquad (26)$$

IV. Measurement Calibration

We cannot measure a system with an accuracy any better than the fundamental quantum uncertainty [15], [16]. Therefore, the corrected standard deviations in (24) and (25) would in turn be expected to increase the correlation coefficient until vanish both sides of (26). In this case, the expected values would satisfy $\langle U \rangle = \langle X \rangle = 0$. Thus, the true values for two observables at individual data would be obtained by replacing with the ones calculated from the corrected deviations given in (24) and (25) respectively as

$$X_i^{*} = \sqrt{-\sum_{i=0}^{n-1}\left[(X_{i+1}-X_i)-\overline{X}\right]^2 + (n-1)\Delta X^{*}} + \overline{X} \qquad (27)$$

$$U_i^{*} = \sqrt{-\sum_{i=0}^{n-1}\left[(U_{i+1}-U_i)-\overline{U}\right]^2 + (n-1)\Delta U^{*}} + \overline{U} \qquad (28)$$

The average changes in mean values for both equalities are supposed to remain the same even after the correction. That is to say, the mean values in (27) and (28) are assumed to be $\overline{X} = \overline{X}^{*}$ and $\overline{U} = \overline{U}^{*}$. Finally, this will allow us to compute (13) and (14) as

$$S_h^{*}(\hat{U}_{ih}^{*}, T_{ih}^{*}) = \sum_{i=1}^{n} A_i \sin(k_i U_{ih}^{*} - \overline{\omega}_i^{*} T_{ih}^{*} + \mu_{ih}) \qquad (29)$$

$$G^{*} = 1 - \sum_{k=1}^{n}\left(M_k - M_{k-1}\right)\left(S_k^{*} + S_{k-1}^{*}\right) \qquad (30)$$

where S_h^{*} and G^{*} corresponds to corrected total satisfaction and Gini coefficient respectively.

V. Conclusion

The theoretical contribution of this paper is threefold: Firstly, the waveform of the households' satisfaction function is introduced for the first time in this literature, which allows the measurement of total utility and overall inequality in well-being. As our second objective, we crystallize the uncertainty relationship that exists in our satisfaction wave between two self-adjoint economic operators such as utility and consumption. Thus, we solve this problem by using the Weber-Fechner Law working as a psychosomatic filter between "observed" and "perceived" uncertainty on the operators depending on individuals' actual satisfaction level. The methodological contribution is drawn from the fact that this analysis allows the integration of the psychological aspects of decision-making into standard household surveys and linking these findings with macro frameworks.

References

[1] G. Becker, "A Theory of the Allocation of Time", The Economic Journal, 75, 1965, pp.493-517.

[2] A. De Serpa, "A Theory of the Economics of Time", The Economic Journal, 81, 1971, pp. 828-846.

[3] A. Evans, "On the Theory of the Valuation and Allocation of Time", Scottish Journal of Political Economy, 19, 1972, pp. 1-17.

[4] R. Gronau, Home Production-A survey. In Handbook of Labour Economics (O. Ashenfelter and R. Layard, eds.), Vol. 1, North Holland, Amsterdam, 1986, pp. 273-304.

[5] S. R. Jara-Díaz, and C. Guevara, "On the Subjective Valuation of Travel Time Savings", European Transport Conference, Proceedings of Seminar F, PTRC, London, 1999, pp. 225-236.

[6] S. R. Jara-Díaz, M.A. Munizaga, P. Greeven, and K.W. Axhausen, "Estimating the Value of Work and Leisure", ETH E-Collection., 2013.

[7] O. Gunes, "Theoretical Appraisal of Satisfactory Decision: Uncertainty, Evolutionary Ideas and Beliefs, Satisfactory Time Use" World Academy of Science, Engineering and Technology Economics and Management Engineering Vol: 2, No: 9, 30199, 2015.

[8] O. Gunes, and A.T Aktuna-Gunes, "Satisfactory Time Use Elasticities of Demand and Measuring Well-Being Inequality through Superposed Utilities", Working Paper, Centre d'Economie de la Sorbonne, Université Paris I, 2015.19.

[9] M-A. Diaye, F. Gardes, and C. Starzec, "GARP Violation, Economic Environment Distortions and Shadow Prices: Evidence from Household Expenditure Panel Data", Annales d'Économie et de Statistique, No. 90, 2008, pp.3-33.

[10] F. Gardes, "Full Price Elasticities and the Opportunity Cost for Time", Documents de Travail du Centre d'Economie de la Sorbonne (CES), 2014.

[11] H.A. Simon, "Rational Choice and the Structure of the Environment", Psychological Review, 63, 1956, pp.129-138.

[12] B.M.S. Van Praag, P. Frijtersb, and A. Ferrer-i-Carbonel, "The Anatomy of Subjective Well-Being", Journal of Economic Behavior & Organization, 51, 2003, pp.29-49.

[13] M. Deserno, "Uncertainty Relation for Self-Adjoint Operators", Max-Planck-Institute for Polymer Research, September, 2014.

[14] A. Deaton, and J. Muellbauer, Economics and Consumer Behavior. Cambridge: Cambridge University Press, 1980.

[15] P. Busch, P. Lahti, and R.F. Werner, "Proof of Heisenberg's Error-Disturbance Relation", Physical Review Letters. 111, 160405, 2013.

[16] T. Miyadera, and H. Imai, "Heisenberg's Uncertainty Principle for Simultaneous Measurement of Positive-Operator-Valued Measures", Physical Review A, 78, 052119, 2008.