

# Angle of Arrival Estimation Using Maximum Likelihood Method

H. K. Hwang, Zekeriya Aliyazicioglu, Solomon Wu, Hung Lu, Nick Wilkins, Daniel Kerr

**Abstract**—Multiple-input multiple-output (MIMO) radar has received increasing attention in recent years. MIMO radar has many advantages over conventional phased array radar such as target detection, resolution enhancement, and interference suppression. In this paper, the results are presented from a simulation study of MIMO uniformly-spaced linear array (ULA) antennas. The performance is investigated under varied parameters, including varied array size, pseudo random (PN) sequence length, number of snapshots, and signal to noise ratio (SNR). The results of MIMO are compared to a traditional array antenna.

**Keywords**—Multiple-input multiple-output (MIMO) radar, phased array antenna, target detection, radar signal processing.

## I. INTRODUCTION

THE use of array antennas to estimate a signal's angle of arrival (AOA) has been investigated by many authors [1]-[5]. The angular resolution of antennas is limited by the antenna main lobe beam width (MLBW), where the MLBW is proportional to the signal wavelength and inversely proportional to the antenna aperture size. Good angular resolution requires an antenna with large aperture size. Mobile systems have physical size limitations. Thus, it is difficult to achieve fine angular resolution. Various processing algorithms estimate the AOA using single-input multiple-output (SIMO) array antennas, as have been investigated by the research team at California State Polytechnic University, Pomona [3], [6]-[9].

MIMO [11], [12] antenna systems have shown promising improvements in angular resolution without increasing physical size. With a MIMO antenna system, synthesized virtual arrays can effectively increase the aperture size. This paper presents the results from a simulation study of MIMO ULA antenna. The AOA estimation is based on maximum likelihood estimation (MLE). Performance is investigated under varied parameters, including varied array size, length of PN sequence, number of snapshots, and SNR. The simulation results are compared with the theoretical Cramer-Rao bound (CRB). The results of MIMO are compared to those of a traditional array antenna. Enhanced angular resolution is obtained using the MIMO ULA antenna. This technique can easily be extended to two-dimensional array antennas.

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## II. MIMO ARRAY ANTENNA

A ULA antenna consists of  $M$  uniformly-spaced antenna elements. The inter-element spacing is  $d$ . The ULA with 5 antenna elements is shown in Fig. 1.

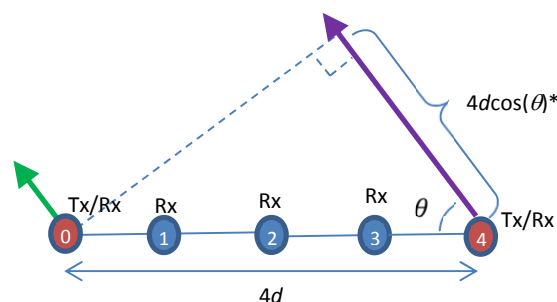


Fig. 1 MIMO Transmitter/Receiver

For radar applications, the MIMO antenna has multiple transmitters which send orthogonal signals and multiple receivers which collect the returned waveforms. The returned waveform is the transmitted waveform reflected from the target plus white noise. In this simulation study, a MIMO antenna is assumed to be a ULA with 5 elements. The 0<sup>th</sup> and 4<sup>th</sup> element transmit narrowband orthogonal waveform and all elements serve as receivers.

Assume the transmitted signals from 0<sup>th</sup> and 4<sup>th</sup> transmitters are  $s_0(n)$  and  $s_4(n)$ , where  $s_0(n) = p_0(n)e^{j2\pi fn}$  and  $s_4(n) = p_4(n)e^{j2\pi fn}$ , where  $p_0(n)$ ,  $p_4(n)$  are orthogonal PN sequences and  $f$  is the carrier frequency.  $x_m(n)$  is the received waveform of the  $m$ <sup>th</sup> receiver. Waveform  $x_m(n)$  consists of the reflected target signal plus the additive white noise. If the signal's AOA is  $\theta$ , then  $x_m(n) = [p_0(n) + p_4(n)e^{-j4\beta}]e^{-jm\beta}e^{j2\pi fn}$ ,  $m = 0, 1, \dots, M-1$  and  $\beta = \frac{2\pi d}{\lambda} \cos \theta$  is the phase factor due to relative propagation delay of adjacent element.

The receiver block diagram is shown in Fig. 2. After each antenna receives reflected waveform, the baseband waveform is recovered by demodulators and matched filters that matches the PN sequences  $p_0(n)$  and  $p_4(n)$ . The matched filter output  $y_{i,j}(n)$  is the output of the  $i$ <sup>th</sup> element due to  $j$ <sup>th</sup> transmitted signal.

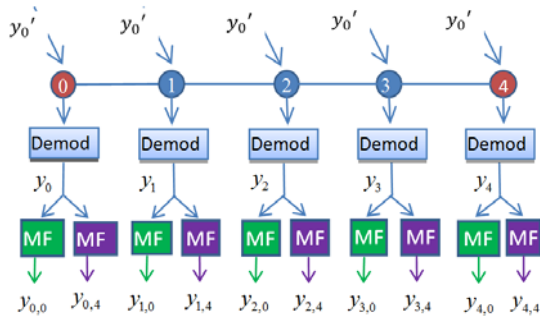


Fig. 2 MIMO Receiver

Using the received RF waveform of the 0<sup>th</sup> receiving antenna  $y_0'(n)$  as the reference, the received RF waveform of the  $m^{\text{th}}$  antenna  $y_m'(n)$  is:

$$y_m'(n) = \left[ p_0(n)e^{-jm\beta} + p_4(n)e^{-j(m+4)\beta} \right] e^{j2\pi fn} + w_m'(n) \quad m = 0, 1, \dots, 4 \quad (1)$$

where  $w_m'(n)$  is the additive white Gaussian noise. The demodulated waveform of the  $m^{\text{th}}$  antenna  $y_m(n)$  is:

$$y_m(n) = p_0(n)e^{-jm\beta} + p_4(n)e^{-j(m+4)\beta} + w_m(n) \quad m = 0, 1, \dots, 4 \quad (2)$$

The output waveform at the first matched filter of each demodulator output  $y_{m,0}(n)$  is:

$$y_{m,0}(n) = \sum_{k=0}^{N-1} p_0^2(k)e^{-jm\beta} + \sum_{k=0}^{N-1} p_0(k)p_4(k)e^{-j(m+4)\beta} + \sum_{k=0}^{N-1} p_0(k)w_m(k) \quad (3)$$

Since  $p_0(n)$  and  $p_4(n)$  are orthogonal sequences, the second term of (3) theoretically is zero. Equation (3) can be rewritten as:

$$y_{m,0}(n) = Ne^{-jm\beta} + w_{m,0} \quad (4)$$

where

$$w_{m,0} = \sum_{k=0}^{N-1} p_0(k)w_m(k)$$

Similarly,

$$y_{m,4}(n) = Ne^{-j(m+4)\beta} + w_{m,4} \quad (5)$$

where

$$w_{m,4} = \sum_{k=0}^{N-1} p_4(k)w_m(k)$$

Equations (4) and (5) show the average signal power is

$N^2\sigma_s^2$  where  $\sigma_s^2$  is the average signal power before the matched filter. The average noise power  $P_w$  is:

$$\begin{aligned} P_w &= E \left[ \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} p_4(k)p_4(l)w_n(k)w_n^*(l) \right] \\ &= \sum_{l=0}^{N-1} \sum_{k=0}^{N-1} p_4(k)p_4(l)E[w_n(k)w_n^*(l)] \\ &= N\sigma_w^2 \end{aligned} \quad (6)$$

The SNR at the output of the matched filter is improved by a factor of  $N$ , where  $N$  is the length of the PN sequence.

The waveforms of virtual array  $v_m(n)$  can be derived from combination of  $y_{m,0}(n)$  and  $y_{m,4}(n)$ . Define the virtual array received baseband waveform  $v_m(n)$  as:

$$v_m(n) = \begin{cases} y_{m,0}(n) & m = 0, 1, 2, 3 \\ \frac{y_{4,0}(n) + y_{0,4}(n)}{2} & m = 4 \\ y_{m-4,4}(n) & m = 5, 6, 7, 8 \end{cases} \quad (7)$$

Received waveforms  $v_m(n)$  for  $m = 0, 1, \dots, 8$  are the equivalent ULA antenna with 9 elements. Thus, the length of ULA effectively increased from  $4d$  to  $8d$ . Estimating the signal's AOA by processing the waveform  $v_m(n)$  should result in a smaller estimation variance.

The MLM has long been used for AOA estimation but new results emerge when it is used with MIMO radar array. The estimated AOA is derived in [1] and given by the following equation:

$$\hat{\theta}_{ML} = \arg \max_{\theta} \{J(\theta)\} \quad (8)$$

where

$$J(\theta) = -\ln(\det(\mathbf{G}(\theta)\hat{\mathbf{R}}\mathbf{G}(\theta) - \sigma_w^2[\mathbf{I}_N - \mathbf{G}(\theta)])) + \frac{1}{\sigma_w^2} \text{Tr}(\mathbf{G}(\theta)\hat{\mathbf{R}}) \quad (9)$$

and

$$\mathbf{G}(\theta) = \mathbf{d}(\theta)[\mathbf{d}^H(\theta)\mathbf{d}(\theta)]^{-1}\mathbf{d}^H(\theta)$$

$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^K \mathbf{v}_k \mathbf{v}_k^H$  is the estimated correlation matrix,  $K$  is the number of snapshot, and  $\mathbf{R} = \mathbf{D}\hat{\mathbf{S}}(\theta)\mathbf{D}^H + \sigma_w^2\mathbf{I}_N$ . Matrix  $\mathbf{D} = [\mathbf{d}_1, \mathbf{d}_2, \dots, \mathbf{d}_i, \dots, \mathbf{d}_M]$   $\mathbf{d}_i$  is the direction vector representing the response of the array from the  $i^{\text{th}}$  signal,  $\sigma_w^2$  is average noise power.

### III. COMPUTER SIMULATION

The ULA used in this simulation study has 5 elements with inter-element spacing equal half of signal's wavelength. The signal's AOA is  $100^\circ$ , the SNR at the input of each antenna is -10 dB, and the length of PN sequence is 15. The received data is derived by averaging over 15 snapshots. Fig. 3 shows the spectrum of MLE and MUSIC methods. The peaks of both spectrums approximately match the signal's AOA ( $100^\circ$ ). The

MUSIC spectrum has a much sharper peak.

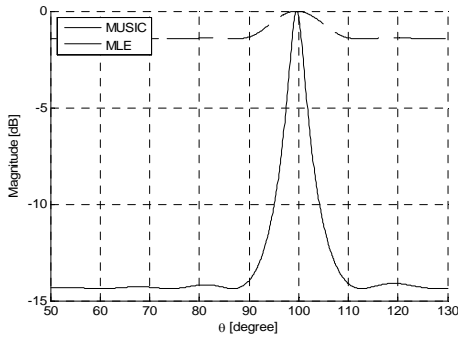


Fig. 3 MLE and MUSIC Spectrum

The histogram of the MLE method is shown in Fig. 4. This histogram is based on 500 independent simulations. The received data vector is averaged over 15 snapshots and the PN sequence length = 15. The input SNR to each array element is -10 dB.

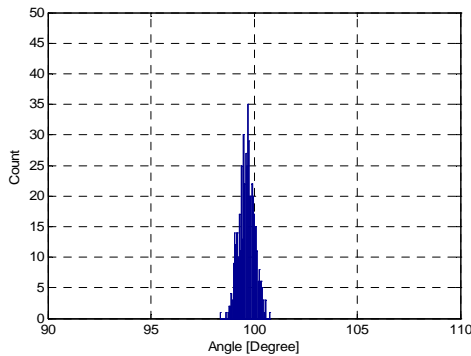


Fig. 4 Histogram Based on 500 Independent Trials

The theoretical AOA estimation variance is given by Cramer-Rao bound (CRB) and is defined by (10) [10].

$$\sigma_{\theta}^2 \geq \frac{6}{(2\pi)^2} \frac{1}{K} \frac{(1 + M\sigma_s^2/\sigma_w^2)}{(M\sigma_s^2/\sigma_w^2)^2} \frac{1}{(M^2 - 1)} \left(\frac{\lambda}{d}\right)^2 \quad (10)$$

where M is the number of element, K is the number of snapshots,  $\sigma_s^2/\sigma_w^2$  is the SNR.

Since the virtual length of 5 element MIMO antenna equivalent to 9 element SIMO antenna, their AOA estimation performance should be very close. By varying the antenna element input SNR, the estimation variance using MLE method for 5 element MIMO antenna and 9 element SIMO antenna are shown in Fig. 5.

The SNR in Fig. 5 is the raw SNR input to each array element. After performing the averaging over 15 snapshots and match filtering with PN length equal 15, the processed SNR is improved by 23.5 dB. For SNR higher than -17 dB, the performance of 5 element MIMO antenna and 9 element SIMO antenna are practically identical and they are very close to the theoretical CRB. This shows the advantage of using

MIMO antenna. Smaller size MIMO antenna achieves the same result of larger size SIMO antenna.

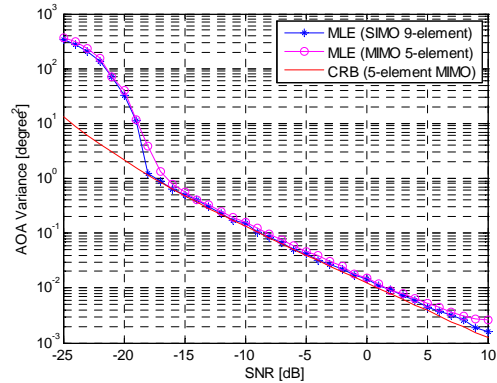


Fig. 5 AOA Estimation Variance

Increasing the length of PN sequence further improves the accuracy of AOA estimation. Fig. 6 shows the AOA estimation variance for length of PN sequence equal 15 and 31. Fig. 6 shows that the additional gain of using longer PN sequence is approximately 3 dB, which is due to roughly doubling the length of the PN sequence. An even smaller estimation variance can be achieved by using a longer PN sequence. However, longer PN sequence corresponds to a shorter chip time. Thus a much wider transmission bandwidth is required for a longer PN sequence.

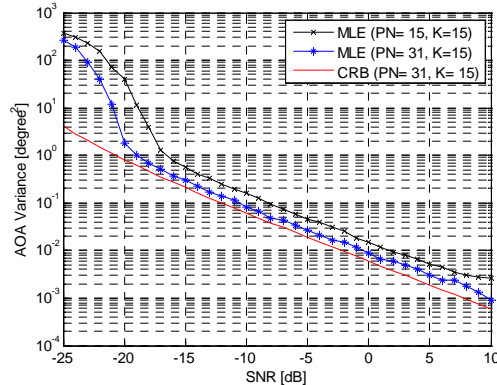


Fig. 6 AOA Estimation Variance using PN Sequence Length = 15 and 31

Increasing the snapshots improves the accuracy of AOA estimation. Fig. 7 shows the AOA estimation variance for snapshots equal 15 and 30. Fig. 7 shows that the additional gain of using more snapshots is approximately 3 dB, which is due to doubling the number of snapshots. An even smaller estimation variance can be achieved by using a more snapshots. However, more snapshots require a longer time to collect data.

The comparison of AOA estimation variance of MUSIC and MLE is shown in Fig. 8. For relative high SNR, they have practically the same performance and they are pretty close to CRB. MLE has slightly lower estimation variance for SNR

less than -17 dB. MLE computation load is higher than MUSIC algorithm due to the fact it has to carry likelihood function computation. In our simulation study, the computation time of MLE algorithm is almost 3 times longer than the MUSIC algorithm.

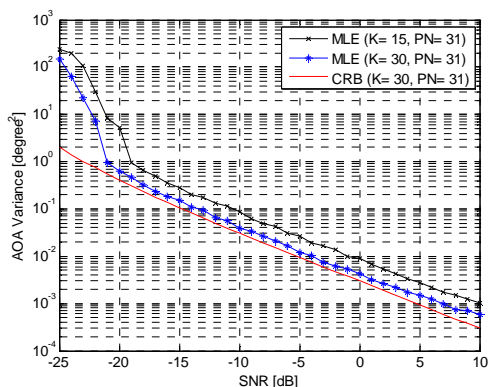


Fig. 7 Estimation Variance for 15 and 30 snapshots

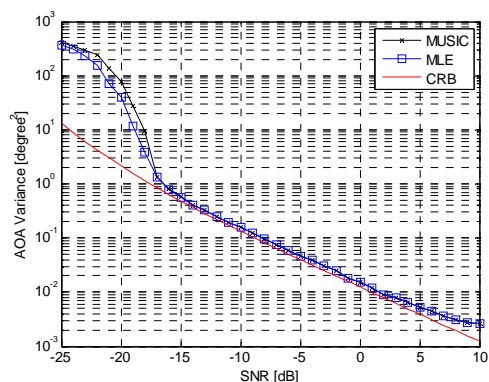


Fig. 8 Estimation Variance of MUSIC and MLE

#### IV. CONCLUSION

Several important conclusions from this simulation study are listed in the following:

- MIMO antennas significantly improve the AOA resolution while maintaining the same physical antenna aperture. Thus, MIMO may be highly desirable for use in mobile systems.
- Estimated AOA is obtained by the peak of the MLE spectrum.
- Estimated angle error decreases exponentially with SNR.
- Signal processing gain can be improved by using longer PN sequence. However, a much wider transmission bandwidth is required for a longer PN sequence.
- SNR can be enhanced by averaging the received data vector over multiple snapshots and performing match filtering to the demodulated waveform.
- Due to the requirement of likelihood function computation, the computation time for MLE is about 3 times longer than the MUSIC algorithm. It provides a slightly lower estimation only at very low SNR environment.

- Although this simulation is based on a one-dimensional array, the algorithms used here can be easily extended for two-dimensional array antennas.

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