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Nonlinear Slow Shear Alfven Waves in Electron-Positron-Ion Plasma Including Full Ion Dynamics

B. Ghosh, H. Sahoo, K. K. Mondal

***Abstract*—**Propagation of arbitrary amplitude nonlinear Alfven waves has been investigated in low but finite β electron-positron-ion plasma including full ion dynamics. Using Sagdeev pseudopotential method an energy integral equation has been derived. The Sagdeev potential has been calculated for different plasma parameters and it has been shown that inclusion of ion parallel motion along the magnetic field changes the nature of slow shear Alfven wave solitons from dip type to hump type. The effects of positron concentration, plasma-β and obliqueness of the wave propagation on the solitary wave structure have also been examined.

***Keywords*—**Alfven waves, Sagdeev potential, Solitary waves.

I. Introduction

THE study of nonlinear Alfven waves in electron-positron-ion plasmas has become of interest in plasma research because Alfven wave activity can be observed in a wide variety of astrophysical plasmas containing electrons, positrons and ions. Large amplitude Alfven wave research attracted renewed attention [1], [2] because of the observation of the Alfven wave activity in the auroral regions by the Freja satellite [3], [4]. The nonlinear solitary structure of kinetic Alfven wave was first analytically studied by [5] in electron-ion plasma. They showed that density humps can be associated with nonlinear Alfven wave dynamics. Thereafter a number of authors have reconsidered the problem by including various factors influencing the solitary wave structure [6]-[10]. Nonlinear dynamics of slow shear Alfven wave was also studied by [11] in low- β plasma. It was found that density dips can be associated with this wave in the nonlinear regions. The importance of slow shear Alfven wave was emphasized long ago by Lysak and Carlson [12] in connection with the magnetospheric-ionospheric coupling. All these works are restricted to simple electron-ion plasmas. Most of the astrophysical plasmas contain electrons, ions as well as positrons. The properties of wave motion in e-p-i plasmas are expected to be different from those in two components electron-ion or electron-positron plasmas [13], [14]. For example, kinetic Alfven wave in electron-positron-ion plasma can form density dips while in electron-ion plasma they form density humps [5]. Recently nonlinear Alfven waves in electron-positron-ion plasmas have been studied by a few authors [15], [16].

Saleem and Mahmood [17] have studied nonlinear kinetic Alfven wave in electron-positron-ion plasma ignoring ion parallel motion. On the other hand, [18] considered solitary kinetic Alfven waves in electron-positron-ion plasma including parallel ion motion and current. Mahmood and Saleem [19] investigated nonlinear solitary structures of arbitrary amplitude slow shear Alfven wave in electron-positron-ion plasma ignoring ion parallel motion. They have shown that electron density dips of SSAW are formed in super Alfvenic region. As ion inertia plays an important role in Alfven wave dynamics due to ion polarization drift, it would be interesting to study nonlinear slow shear Alfven wave in electron-positron-ion plasma including full ion dynamics.

In this paper we present a detailed study of nonlinear slow shear Alfven wave in electron-positron-ion plasma with $\beta <<(m_e/m_i) <<1$ in which ion inertia and current along the magnetic field are taken into consideration. We show that unlike the previous work [19] which ignored ion parallel motion and predicted dip type density soliton, here hump type solitons for slow shear Alfven wave may be excited.

The paper is organized as follows: Basic equations are presented in Section II. In Section III, we derive the energy integral equation and Sagdeev Potential. In Section IV, we present the numerical analysis graphically and discuss the results.

II. Basic Equation

We consider cold homogeneous electron-positron-ion plasma in presence of a stationary ambient magnetic field along z-axis i.e., $\vec{B}_0 = B_0\hat{e}_z$. The equations governing the dynamics of the nonlinear slow shear Alfven waves in x-z plane in a low- $\beta(=8\pi n_0 T / B_0^2)$ electron-positron-ion plasma in the low frequency limit are:

For electrons:

$$\frac{\partial n_e}{\partial t}+\frac{\partial(n_e v_{ez})}{\partial z}=0 \tag{1}$$

$$\frac{\partial v_{ez}}{\partial t}+v_{ez}\frac{\partial v_{ez}}{\partial z}=\frac{\beta}{2Q}\frac{\partial\psi}{\partial z} \tag{2}$$

where ψ denotes the electromagnetic potential associated with the parallel electric field E_z; $Q=m_e/m_i$ is the electron to ion mass ratio and V_{ez} is the parallel component of electron velocity with respect to the external magnetic field. In writing

Basudev Ghosh is with the Department of Physics, Jadavpur University, Kolkata-700032, India (e-mail: bsdvghosh@gmail.com).
Himangshu Sahoo is with the Madanmohanpur R.C.S. Vidyamondir, kulpi, South 24Pgs, India (e-mail: himangshusahoo@gmail.com).
K.K. Mondal is with the Sovarani Memorial College Jagatballavpur, Howrah-711408, India(e-mail: monkalyan@gmail.com)

these equations we have assumed that electron inertia dominates over electron pressure.

For positrons:

$$\frac{\partial n_p}{\partial t}+\frac{\partial (n_p v_{pz})}{\partial z}=0 \tag{3}$$

$$\frac{\partial v_{pz}}{\partial t}+v_{pz}\frac{\partial v_{pz}}{\partial z}=-\frac{\beta}{2Q}\frac{\partial \psi}{\partial z} \tag{4}$$

Here, $Q=m_p/m_i$ is the positron to ion mass ratio and v_{pz} is the parallel component positron velocity.

For ions:

$$\frac{\partial n_i}{\partial t}+\frac{\partial (n_i v_{ix})}{\partial x}+\frac{\partial (n_i v_{iz})}{\partial z}=0 \tag{5}$$

$$\frac{\partial v_{iz}}{\partial t}+v_{ix}\frac{\partial v_{iz}}{\partial x}+v_{iz}\frac{\partial v_{iz}}{\partial z}=-\frac{\beta}{2}\frac{\partial \psi}{\partial z} \tag{6}$$

$$v_{ix}=-\frac{\beta}{2}\frac{\partial^2 \phi}{\partial x \partial t} \tag{7}$$

Here v_{ix} and v_{iz} are respectively the perpendicular and parallel component of the velocity of ions with respect to the external magnetic field; ϕ is the electrostatic potential associated with the perpendicular component E_x of the electric field. The above equations have been written in dimensionless form by normalizing number density (n_a) by unperturbed number density (n_{a0}) (a=e, p, i), time (t) by $1/\Omega_i$, velocity by Alfven speed (v_A) and potentials (ϕ,ψ) by (k_BT_e/e). Here Ω_i and k_B are the ion gyro-frequency and Boltzmann constant respectively.

This ion velocity is taken to be solely due to polarization drift which is necessary for an electron-positron-ion plasma to maintain the quasineutrality condition. It may be noted here that the electron and positron polarization drifts can be ignored for such plasmas. The expression for the parallel current density is

$$j_z = n_i v_{iz} + n_p v_{pz} - n_e v_{ez} \tag{8}$$

Ampere's law for j_z gives

$$\nabla_\perp^2 \frac{\partial(\phi-\psi)}{\partial z}=\frac{\partial j_z}{\partial t} \tag{9}$$

The quasineutrality condition is prescribed by

$$n_p + n_i = n_e \tag{10}$$

III. Derivation of Sagdeev Potential Equation

We seek stationary localized planar solution of the nonlinear equations given by (1)-(10). So we transform these equations in a moving frame η defined by

$$\eta = k_\perp x + k_\parallel z - Ut \tag{11}$$

Here U is the velocity of the nonlinear structure in the moving frame; $k_\perp$ and $k_\parallel$ denote the direction cosines in the perpendicular direction i.e., x direction and the parallel direction i.e., the z-direction respectively such that

$$k_\perp^2 + k_\parallel^2 = 1\cdot \tag{12}$$

Now the equation for electron continuity (1) and the equation for positron continuity (3) with the use of the transformation given by (11) and appropriate boundary conditions i.e.,

as $|\eta|\to\infty$; $n_e, n_p \to 1$; $v_{ez}, v_{pz} \to 0$, yield

$$v_{ez}=\frac{U}{k_\parallel}\left(1-\frac{1}{n_e}\right)$$
$$v_{pz}=\frac{U}{k_\parallel}\left(1-\frac{1}{n_p}\right) \tag{13}$$

Here n_e and n_p are the electron density and positron density normalized to n_{e0} and n_{p0} respectively. Using (12) and (13) in the equations for electron and positron momentum balance equations (2) and (4) we obtain

$$\frac{U^2}{k_\parallel^2}\cdot\frac{1}{n_e^2}=\frac{U^2}{k_\parallel^2}+\frac{\beta}{Q}\psi \tag{14}$$

$$\frac{U^2}{k_\parallel^2}\cdot\frac{1}{n_p^2}=\frac{U^2}{k_\parallel^2}-\frac{\beta}{Q}\psi \tag{15}$$

From the equation for ion continuity (5) we get

$$k_\perp v_{ix}+k_\parallel v_{iz}=U\left(1-\frac{1}{n_i}\right) \tag{16}$$

where $n_i = n_i/n_{i0}$; n_{i0} being the unperturbed ion density. In deriving (16), the transformation given by (11) and boundary conditions

$n_i \to 1$, $v_{ix}, v_{iz} \to 0$ as $|\eta|\to\infty$

has been used.

Combining (14) and (15) one obtains

$$n_p = \frac{1}{\sqrt{2-\frac{1}{n_e^2}}} \tag{17}$$

Using the transformation (11) and the relation (16) we obtain the following equation from the equation for ion momentum (6):

$$\frac{\partial v_{iz}}{\partial \eta} = \frac{\beta k_{\parallel}}{2U} n_i \frac{\partial \psi}{\partial \eta} \tag{18}$$

Now the quasi-neutrality condition (10) expressed in terms of the normalized number densities of the species takes the form

$$n_i = \frac{n_e - p n_p}{1-p} \tag{19}$$

where $p=n_{p0}/n_{e0}$ is the ratio of the unperturbed positron and electron densities. Using (17) and (19) we obtain from (18)

$$\frac{\partial v_{iz}}{\partial \eta} = \frac{\beta k_{\parallel}}{2U(1-p)}[n_e - \frac{p}{\sqrt{2-\frac{1}{n_e^2}}}]\frac{\partial \psi}{\partial \eta} \tag{20}$$

On combining (14) and (20) one gets

$$v_{iz} = \frac{UQ}{k_{\parallel}(1-p)n_e} + \frac{pUQ}{k_{\parallel}(1-p)}\sqrt{2-\frac{1}{n_e^2}} \tag{21}$$

Now from (16) we obtain the transverse component velocity as:

$$v_{ix} = U[1 - \frac{(1-p)\sqrt{2-\frac{1}{n_e^2}}}{n_e\sqrt{2-\frac{1}{n_e^2}} - p}] - \frac{UQ}{1-p}\frac{1}{n_e} - (\frac{p}{1-p})UQ\sqrt{2-\frac{1}{n_e^2}} \tag{22}$$

Again (7) under the transformation (11) turns out to be

$$v_{ix} = \frac{1}{2}Uk_{\perp}\beta\frac{\partial^2 \phi}{\partial \eta^2} \tag{23}$$

Comparing (23) and (22) one readily obtains

$$\frac{\partial^2 \phi}{\partial \eta^2} = \frac{2}{Qk_{\perp}}[1-(1-p)\sqrt{2-\frac{1}{n_e^2}} / (n_e\sqrt{2-\frac{1}{n_e^2}} - p)] - \frac{2}{n_e k_{\perp}}\frac{1}{1-p} - \frac{p}{1-p}\frac{2}{k_{\perp}}\sqrt{2-\frac{1}{n_e^2}} \tag{24}$$

Differentiating (14) twice with respect to η we readily obtain

$$\frac{\partial^2 \psi}{\partial \eta^2} = -\frac{2U^2 Q}{k_{\parallel}^2 \beta}[\frac{-3}{n_e^4}(\frac{\partial n_e}{\partial \eta})^2 + \frac{1}{n_e^3}\frac{\partial^2 n_e}{\partial \eta^2}] \tag{25}$$

Equation (9) in the moving frame can be written as

$$k_{\perp}^2 k_{\parallel}\left(\frac{\partial^2 \phi}{\partial \eta^2} - \frac{\partial^2 \psi}{\partial \eta^2}\right) = -UJ_z \tag{26}$$

Finally using (8), (13), (21), (24) and (25) we obtain from (26), the desired "energy law" in the following form

$$\frac{1}{2}(\frac{dn_e}{d\eta})^2 + V(n_e) = 0 \tag{27}$$

where V (n_e) is the effective Sagdeev potential given by

$$V(n_e) = \frac{n_e^6}{1-k_{\parallel}^2}[\frac{\beta}{2(1-p)^2}A + \frac{p\beta}{2(1-p)^2}B + \frac{\beta}{2Q}\left(\sqrt{2-\frac{1}{n_e^2}} + \frac{1}{n_e} - 2\right) + \frac{k_{\parallel}^2}{U^2 Q}C - \frac{k_{\parallel}^2}{U^2(1-p)}\left(\frac{1}{2n_e^2} - \frac{1}{3n_e^3} - \frac{1}{6}\right) - \frac{k_{\parallel}^2 p}{U^2(1-p)}D] \tag{28}$$

Equation (28) contains several constants *A, B, C* and *D* which depend on different plasma parameters such as positron concentration, plasma β and obliqueness of wave propagation. These constants are given by

$$A = \frac{1}{n_e} - \frac{1}{2}\left(1+\frac{1}{n_e^2}\right) - p\sqrt{2-\frac{1}{n_e^2}}\left(\frac{1}{2n_e} - 1\right) - p\left[sin^{-1}\sqrt{1-\frac{1}{2n_e^2}}\right] + p\frac{\pi}{4} - \frac{p}{2}$$

$$B = sin^{-1}\sqrt{1-\frac{1}{2n_e^2}} - \frac{1}{2n_e}\sqrt{2-\frac{1}{n_e^2}} - \frac{\pi}{4} + \frac{1}{2} + \left(\frac{1}{n_e}-1\right) + \frac{p}{2}\left(\frac{1}{n_e^2}-1\right) + p\left(\sqrt{2-\frac{1}{n_e^2}}-1\right)$$

$$C = \frac{1}{2} - \frac{1}{2n_e^2} - (1-p)\int_1^{n_e} \frac{dn_e}{n_e^4\left(1-\frac{p}{\sqrt{2n_e^2-1}}\right)}$$

and

$$D = \frac{1}{3}\left(2-\frac{1}{n_e^2}\right)^{\frac{3}{2}} - \frac{5}{6} + \frac{1}{2n_e^2} \quad (29)$$

IV. Results and Discussion

Equation (27) may be identified as the energy equation of an oscillating quasi-particle of unit mass, velocity $\frac{dn_e}{d\eta}$ and position n_e in potential V (n_e). For solitary wave solutions, particle motion represented by (27) must be confined between two points n_e=1 and $n_e = n_{em}$. The conditions for the existence of solitary wave solutions are [20, 21]:

(i) $V(n_e)=0$ at n_e=1 and $n_e = n_{em}$

(ii) $\left.\frac{dV}{dn_e}\right|_{n_e=1} = 0$ and $\left.\frac{d^2V}{dn_e^2}\right|_{n_e=1} \langle\ 0$

(iii) $V(n_e) < 0$ for $1 < n_e < n_{em}$ (hump soliton)

For $n_{em} < n_e < 1$ dip soliton is formed. The particle is reflected back at $n_e = n_{em}$ so that $V(n_e) > 0$ for $n_e > n_{em}$ (or $n_e < n_{em}$) if $n_{em} > 1$ (or $n_{em} < 1$).

As the nature of the Sagdeev potential $V(n_e)$ determines the conditions for existence and nature of the solitary wave, it is important to study the nature of variation of $V(n_e)$ for different plasma parameters such as positron concentration, plasma β and obliqueness of wave propagation. For numerical analysis we have chosen parameter ranges keeping in mind experimental as well as space and astrophysical situations.

To show the region of existence of solitary wave solution $V(n_e)$ is drawn against n_e for different values of positron concentration (Fig. 1). It indicates hump type soliton solution whose amplitude decreases with increase in positron concentration. For $p < 0.070747$, the soliton solution does not exist.

In Fig. 2 we show the plot of $V(n_e)$ versus n_e for different values of β it again shows excitation of hump type soliton. Amplitude of this soliton increases with increase in β. However there is an upper limit of β (=0.150043) beyond which hump solitons cannot exist.

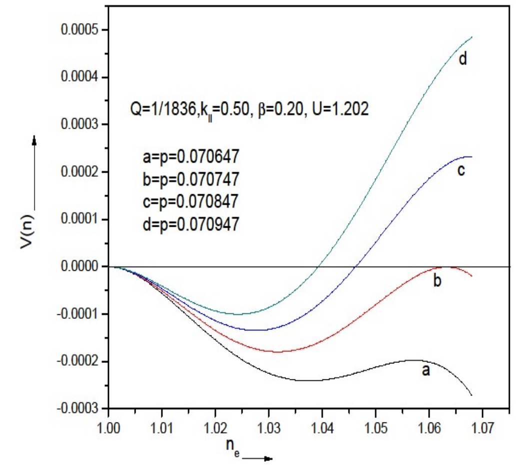

Fig. 1 Sagdeev profile for different values of positron concentration. The curves labeled *a, b, c* and *d* correspond respectively to p= 0.070647, 0.070747, 0.070847 and 0.070947

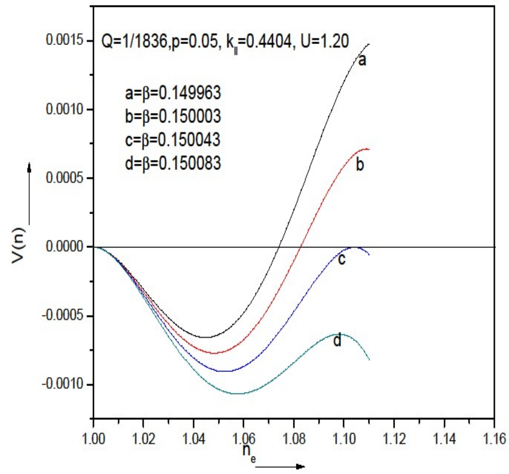

Fig. 2 Sagdeev profile for different values of plasma β. The curves labeled *a, b, c* and *d* correspond respectively to β= 0.149963, 0.150003, 0.150043 and 0.15083

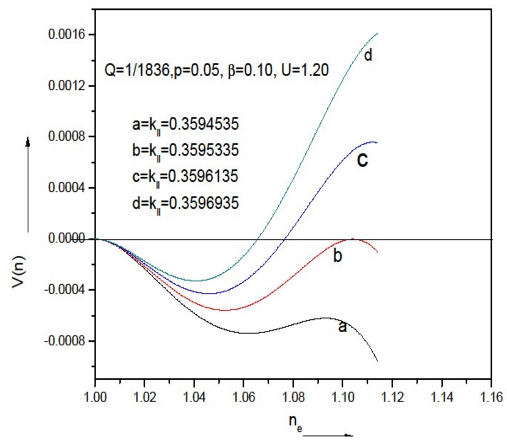

Fig. 3 Sagdeev profile for various values of the obliquity factor $k_{\|}$. The curves labeled *a, b, c* and *d* correspond respectively to $k_{\|}$ = 0.3594635, 0.3595335, 0.3596135 and 0.3596935

Fig. 3 shows the plot of $V(n_e)$ for various values of the obliquity factor $k_{\|}$. Obviously the amplitude of these solitons depends sensibly on the propagation angle with respect to the external magnetic field. However, there is a lower limit to the value of $k_{\|}$ (i.e., upper limit to the angle of propagation) beyond which hump solitons cannot exist.

To summarize we have investigated in detail nonlinear slow shear Alfven wave in electron-positron-ion plasma including ion motion parallel to the external magnetic field. We have examined the effects of positron concentration, strength of magnetic field and propagation angle with respect to the external magnetic field on the conditions of existence and nature of the slow shear Alfven waves in electron-positron-ion plasmas. We have also examined the parametric regions of existence of nonlinear slow shear Alfven solitary waves. There exists a lower limit of positron concentration (p = 0.070747) below which soliton solution does not exist. There exists an upper limit to the value of β (=0.150043) beyond which hump solitons cannot exist. It has also been shown that the amplitude of slow shear Alfven wave soliton depends sensibly on the propagation angle with respect to the external magnetic field and there is a lower limit to the value of $k_{\|}$ (i.e., upper limit to the angle of propagation) beyond which hump solitons cannot exist. Earlier authors [16] studied the same problem ignoring ion parallel motion and predicted dip type soliton. Here we find that inclusion of ion parallel motion plays an important role and changes the nature of soliton from dip type to hump type. So, one must include full ion dynamics in order to study nonlinear slow shear Alfven wave.

Finally, we would like to point out that the present investigation may be useful in many astrophysical and space environments where slow shear Alfven wave may be excited in electron-positron-ion plasmas .Our investigation has been restricted to cold, homogeneous and collisionless plasma. The analysis can be generalized to include the effects of temperature, inhomogeneity and collision which are beyond the scope of the present work.

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Basudev Ghosh is an associate professor with the Department of Physics, Jadavpur University. Previously he was a reader in Ramakrishna Mission Vidyamandira, Belur Math, affiliated to the University of Calcutta. He graduated from University of Burdwan, India (1977), and did his post graduation from the same university (1979) and topped at both levels. He did his PhD from University of Calcutta (1989).He became a life member of the Indian Physical society and the Plasma Science Society of India (PSSI). His field of interest includes nonlinear waves in plasma. He has published 18 books and about 60 research papers. He is actively involved in teaching for more than 30 years. Very recently he has published a text book on plasma physics titled 'Basic Plasma Physics'-Alpha Science International (2014), Cambridge, UK & Narosa Publishing House, New Delhi, India.

Himangshu sahoo is Assistant Teacher in the Madanmohanpur R.C.S. Vidyamondir, Kulpi, South 24Pgs, India. He graduated from University of Calcutta and did post graduation from the G.G.U. Currently he is doing his Ph D work on theoretical plasma physics under the supervision of Prof. B Ghosh, Department of Physics, Jadavpur University, Kolkata, India. His area of research is the nonlinear propagation of various wave modes in classical, quantum and relativistic plasmas. He is a life member of the Indian Physical society and the Plasma Science Society of India (PSSI).

Kalyan Kumar Mondal is a principal of Sovarani Memorial College Jagatballavpur, Howrah-711408, India. He graduated from University of Calcutta and did post graduation from the same University. He did his PhD from Jadavpur University on plasma physics. He is a life member of the Indian Physical society and the Plasma Science Society of India (PSSI). He is actively engaged in doing theoretical research on nonlinear wave propagation in classical, quantum and relativistic plasmas.