

# A Methodological Test to Study the Concrete Workability with the Fractal Model

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**Abstract**—The main parameters affecting the workability are the water content, particle size, and the total surface of the grains, as long as the mixing water begins by wetting the surface of the grains and then fills the voids between the grains to form entrapped water, the quantity of water remaining is called free water.

The aim of this study is to undertake a fractal approach through the relationship between the concrete formulation parameters and workability. To develop this approach a series of concrete taken from the literature was investigated by varying formulation parameters such as G/S, the quantity of cement C and the quantity of water W.

We also call another model as the model of water layer thickness and model of paste layer thickness to judge their relevance, hence the following results: the relevance of the water layer thickness model is considered as a relevant when there is a variation in the water quantity. The model of the paste layer thickness is only applicable if we considered that the paste is made with the grain value  $D_{max} = 2.85$ : value from which we see a stability of the model.

**Keywords**—Concrete, fractal method, paste layer thickness, water layer thickness, workability.

## I. INTRODUCTION

If the water content and the concrete proportions constituents were determined, the workability depends on the size range, particle size, shape and texture [1]; Based on these concepts we study the influence of parameters mentioned above on the workability. However, we have used models such as:

- Model of the water layer thickness [2]: This model is based on the hypothesis that the concrete workability is determined by the water layer thickness surrounding the grains of all granular mixture.
- Model of the paste layer thickness [2]: The hypothesis is that the concrete workability depends on the paste layer thickness (consisting of water and fine aggregate) which surrounds the coarse aggregate that remain.

Precisely, the fractal model allows determining in advance the thicknesses of water and paste layers.

## II. ADOPTION OF FRACTAL MODEL FOR THE CALCULATION OF WATER AND PASTE LAYERS THICKNESSES

To be able to use fractal model, it is required to prove that the studied granular mixture followed a fractal distribution by tracing fractals lines and deduction of fractal dimensions FD. From this moment, we can appeal to the models of water and

paste layers thickness to develop an approach for workability studying.

### A. Hypothesis of Water Layer Thickness

Mixing water starts by wetting the surface of the grains, then fill the voids between the grains to form the trapped water. In this model, the quantity of water that remains called free water will give the workability of the concrete. Hence, the utility of adopting the fractal model to calculate the porosity and the determination of different volumes.

For determining the water layer thickness is essential to know the quantity of free water in the whole granular mixture, defined as the excess water after filling all the pores where the need to calculate the porosity (Por) of the granular mixture. Here, the fractal model can occur for calculating the porosity by an empirical formula shown by [3]:

$$Por(d/D) = por(d) - A_1 \cdot e^{-\frac{(X-X_{opt})^2}{2W^2}} \quad (1)$$

with: Por (d): porosity of the mixture of grains d, X represents  $FD \cdot \log(d/D)$ , FD is the fractal dimension of the mixture.  $X_{opt}$ : represents the value of this parameter to the optimum ( $FD_{opt} \cdot \log(d/D)$ ), W: calculate from  $w \cdot \log(d/D)$ . With w: coefficient given according to (d/D), A1: klemmung.

It will be possible to determine the total volume  $V_T$  and the void volume  $V_V$  by (4) and (5) in which the solid volume  $V_S$  is known from the dosage and the porosity Por being calculated using fractal model.

$$P = \frac{V_V}{V_T} \quad (2)$$

$$V_V = V_T - V_S \rightarrow P = \frac{V_T - V_S}{V_T} \quad (3)$$

$$V_T = \frac{V_S}{(1-P)} \quad (4)$$

$$V_V = V_T - V_S \quad (5)$$

The void volume  $V_V$  is the filling water volume  $V_{WF}$ :

$$V_V = V_{WF} \quad (6)$$

The volume of free water  $V_{WF}$  is the difference between the total volume water  $V_W$  and the filling water volume  $V_{WF}$ :

$$V_{WF} = V_W - V_{WF} \quad (7)$$

Knowing the total grain surface, the volume of free water can be reduced in thickness. For the calculation of the total

grain surface we use the fractal model. Based on the hypothesis of the grains sphericity mixture, the surface  $S_i$  of one grain with a diameter  $D_i$  is:

$$S_i = 4 \cdot \pi \cdot \left(\frac{D_i}{2}\right)^2 \quad (8)$$

Knowing the number of each size range  $N_i$ , taken from the fractal model, we can calculate the area of each particle size range  $S_{Ti}$ , as in (9).

$$S_{Ti} = N_i \cdot S_i \quad (9)$$

Thus, the total surface of the granular mixture  $S_{Tm}$  of "n" class is given by the following relationship:

$$S_{Tm} = \sum_{i=0}^n S_{Ti} \quad (10)$$

Hence, the water layer thickness is equal to:

$$T_{WL} = \frac{V_{WF}}{S_{Tm}} \quad (11)$$

### B. Hypothesis of Paste Layer Thickness

The hypothesis is that the workability of concrete depends on the paste layer thickness (consisting of water and fine aggregate) which surrounds the coarse aggregate. Therefore, it is necessary in this case to define the porosity of the mixture consisting of coarse grains using (1) in order to define the filling paste volume  $V_{Pf}$  which occupies the void between the coarse grains ( $V_{Pf} = V_v$ ). The volume of excess paste  $V_{Pex}$  which acts as a lubricant for this model will be:

$$V_{p,exe} = V_{p,t} - V_{p,f} \quad (12)$$

$V_{PT}$  is the total volume of the paste which is the sum of the volume of water  $V_w$  and the solid of grain fine  $V_{SF}$  (determined from the volume proportions dosage and partial refusal).

$$V_{PT} = V_{SF} + V_w \quad (13)$$

The problem for this model is to define the paste by its maximum grain diameter  $D_{paste}^{max}$ . For this, we calculated the thickness of several pastes with different particle size, and for the  $D_{paste}^{max} = (0.01\text{mm}, 0.1\text{mm}, 1\text{mm}, 2\text{mm}, 2.5\text{mm}, 2.8\text{mm}, 3.15\text{mm}, 5\text{mm}, 6.3\text{mm})$ . Beyond the diameter 6.3mm, we suppose that the surface force acting on the grain is reduced relative to the mass force.

### III. COMPOSITION OF THE STUDIED CONCRETE

The test campaign which was conducted at L CPC, has served as a platform to address the relationship between dosing parameters and workability. The experimental program, formulation, granulometry of the components and results are taken from [4]. The test was done on ordinary concrete; three sub-programs have been established with respect to a control concrete simply called OC (ordinary

concrete)

- Sub-program 1: G/S gravel-sand ratio is variable for OC (G/S = 1.4). Other concretes were formulated with a ratio of 0.5 - 1 - 2, abbreviated as followings: OC0.5, OC1 - OC2.
- Sub-program 2: the quantity of cement is variable (-200kg), (-100kg), (+100kg) and (+200kg) compared to the control concrete OC.
- Sub-program 3: the quantity of water varies with (+5 liter), (+10 liter) and (+15 liter) compared to OC.

TABLE I  
MASS COMPOSITION AND SLUMP VALUES OF THE STUDIED CONCRETE

Concrete	Gravel 12.5-20 Kg/m3	Gravel 5-12.5 Kg/m3	Sand 0-5* Kg/m3	Cement Kg/m3	water Kg/m3	Slump (cm)
OC	709	382	779	356	182	11.7
OC 0.5	387	208	918	349	212	9.7
OC1	596	321	918	349	194	16.2
OC 2	816	349	658	359	177	12.7
OC-200	777	419	854	153	172	12.8
OC-100	744	401	818	255	174	11.5
OC+100	654	352	719	443	207	19.7
OC+200	617	332	678	537	211	16.0
OC+5	704	379	773	353	186	16.7
OC+10	701	377	770	352	190	17.3
OC+15	697	375	766	350	194	19.8

\* The author states that the crushed sand 0-5 contains 8% of the total mass of fine crushing.

TABLE II  
VOLUME COMPOSITION OF THE STUDIED CONCRETE

Concrete	Gravel 12.5-20 liter	Gravel 5-12.5 liter	Sand 0-5* liter	Cement liter	Total liter
OC	265,05	142,86	292,20	112,37	812,48
OC 0,5	144,67	77,79	446,36	107,32	776,14
OC 1	222,80	120,04	344,34	110,16	797,35
OC 2	305,05	164,17	235,56	113,32	818,10
OC-200	290,47	156,69	320,33	48,30	815,79
OC-100	278,13	149,96	306,83	80,49	815,41
OC+100	244,49	131,64	269,69	139,84	785,65
OC+200	230,65	124,16	254,31	169,51	778,63
OC+5	263,18	141,74	289,95	111,43	806,29
OC+10	262,06	140,99	288,82	111,11	802,98
OC+15	260,56	140,24	287,32	110,48	798,60

### IV. METHODOLOGY

For all mixtures of concrete (cement with aggregate):

- We calculate the cumulative number  $N_{cum}$ ;
- We draw the fractal lines  $N_{cum} = f(\text{diam})$  to define the fractal dimension FD of the granular mixture and prove the fractal distribution of the mixtures grains.

Then, to find relationship between formulations parameters and slump (workability indicator), we adopt two hypotheses following the methodology of calculation.

Hypotheses of the water layer thickness:

- Calculation of the total area of granular mixtures.
- Calculation of porosity  $P_{or}$ .
- Calculation of volumes:  $V_v$  and  $V_T$
- Calculating the water layer thickness:  $T_{WL}$ .
- Tracing slump curves according to water layer thickness for each sub-program:  $A = f(T_{WL})$ ,

Hypotheses of paste layer thickness:

- Choice of diameter considered  $D_{i_{paste}}^{max} = (0.01\text{mm}, 0.1\text{mm}, 1\text{mm}, 2\text{mm}, 2.5\text{mm}, 2.8\text{mm}, 3.15\text{mm}, 5\text{mm}, 6.3\text{mm})$
- Determination of the number of coarse grain mixture  $N_{cum}$ , coarse grain (c.g); and plotting the lines fractals to define the fractal dimension FD of the mixture of coarse grain considered.
- Calculation of total areas mixtures composed of coarse grains considered  $S_{Tm,c.g}$ .
- Calculation of porosity  $Por$  mixtures of coarse grains  $Por_{c.g}$ .
- Calculation of volumes:  $V_{V,c.g}$  and  $V_{T,c.g}$  of coarse grains.
- Calculating the volume of the excess paste:  $V_{p,exe}$ .
- Calculating the paste layer thickness  $T_{PL}$ .
- Tracing slump curves according to the paste layer thickness for each diameter  $Di$ :  $A=f(T_{PL})$ .

### V. COMPUTING THE NUMBER OF GRAIN MIXTURES

We calculate the total number of the granular mixture using the proceeding of calculation shown in [5]. This step will allow us to trace the lines fractals granular mixtures for each concrete, to test the hypothesis of fractal distribution of grain concretes studied and to set their fractal dimensions FD as shown in Figs. 1-12.

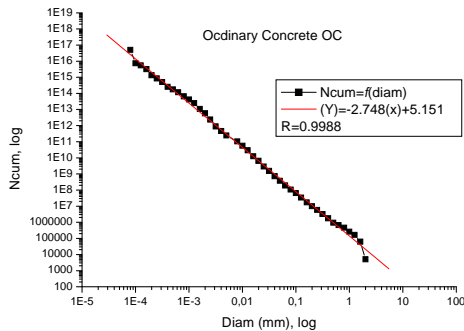


Fig. 1 Fractal line of control concrete OC, FD=2,748

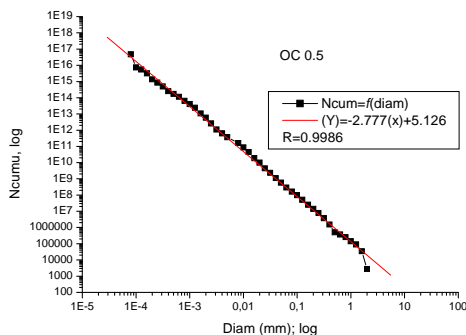


Fig. 2 Fractal line of OC 0,5, FD=2,777

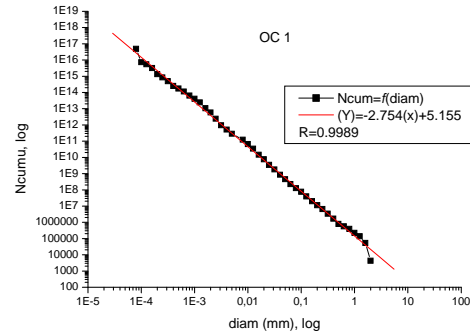


Fig. 3 Fractal line of OC 1, FD=2,754

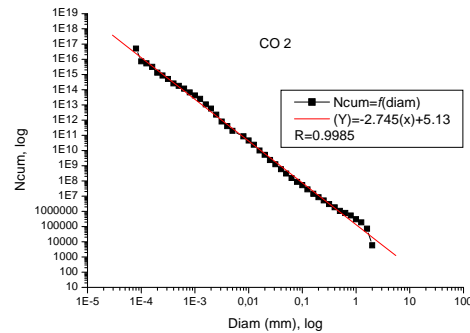


Fig. 4 Fractal line of OC 2, FD=2,754

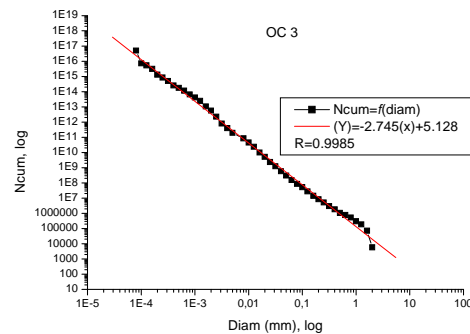


Fig. 5 Fractal line of OC 3, FD=2,754

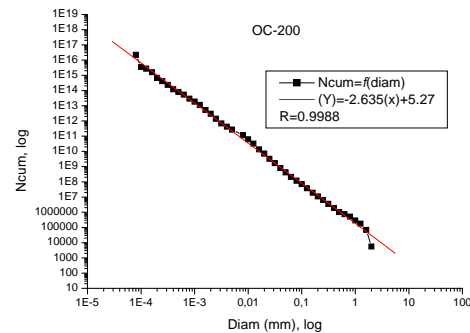


Fig. 6 Fractal line of OC-200, FD=2,635

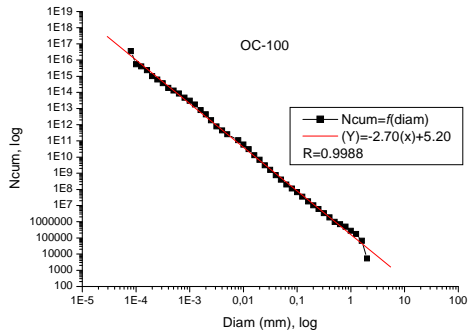


Fig. 7 Fractal line of OC-100, FD=2,70

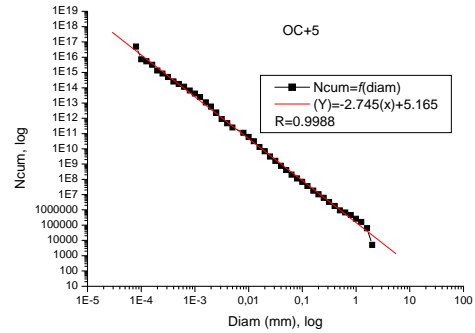


Fig. 10 Fractal line of OC+5, FD=2,745

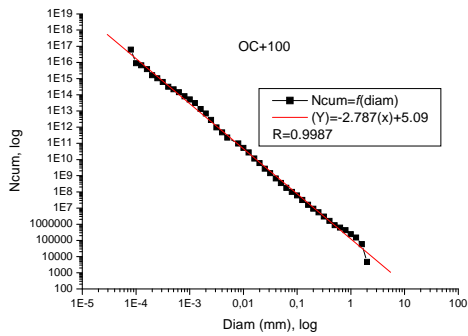


Fig. 8 Fractal line of OC+100, FD=2,787

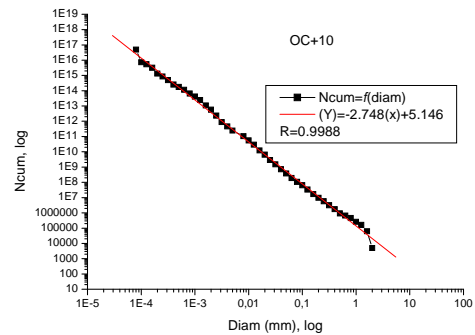


Fig. 11 Fractal line of OC+10, FD=2,748

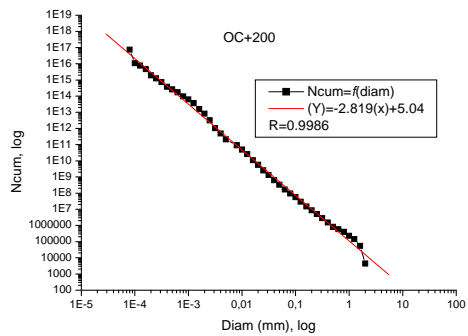


Fig. 9 Fractal line of OC+200, FD=2,819

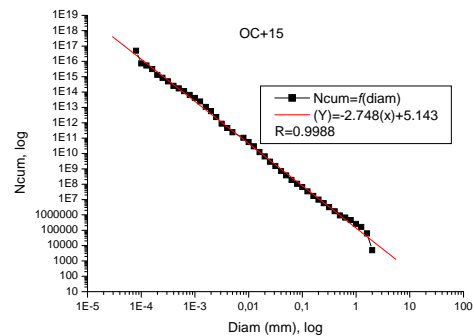


Fig. 12 Fractal line of OC+15, FD=2,748

TABLE III  
 THE WATER LAYER THICKNESS AND SLUMP VALUES

Concrete	d [mm]	D [mm]	FD	Por (d/D)	Vst [l]	V <sub>T</sub> [l]	V <sub>V</sub> [l]	V <sub>w</sub> [l]	V <sub>wf</sub> [l]	S <sub>tm</sub> [cm <sup>2</sup> ]	T <sub>wl</sub> [cm]	Slump (cm)
OC	0,0008	25	2,748	0,080	812,48	883,52	71,04	182	110,96	1,90E+09	0,000058	11.7
OC 0,5	0,0008	25	2,77	0,079	776,14	842,95	66,80	212	145,20	1,89E+09	0,000077	9.7
OC 1	0,0008	25	2,75	0,080	797,35	866,97	69,62	194	124,38	1,89E+09	0,000066	16.2
OC 2	0,0008	25	2,74	0,081	818,10	890,07	71,97	177	105,03	1,89E+09	0,000056	12.7
OC-200	0,0008	25	2,63	0,088	815,79	894,85	79,07	172	92,93	9,06E+08	0,000103	12.8
OC-100	0,0008	25	2,7	0,083	815,41	889,51	74,09	174	99,91	1,40E+09	0,000071	11.5
OC+100	0,0008	25	2,75	0,080	785,65	854,25	68,60	207	138,40	2,32E+09	0,000060	19.7
OC+200	0,0008	25	2,82	0,077	778,63	843,59	64,95	211	146,05	2,78E+09	0,000053	16.0
OC+5	0,0008	25	2,74	0,081	806,29	877,22	70,93	186	115,07	1,89E+09	0,000061	16.7
OC+10	0,0008	25	2,748	0,080	802,98	873,19	70,21	190	119,79	1,88E+09	0,000064	17.3
OC+15	0,0008	25	2,748	0,080	798,60	868,43	69,83	194	124,17	1,87E+09	0,000067	19.8

VI. RELATIONSHIP BETWEEN THE CONCRETE FORMULATION PARAMETERS AND SLUMP

A. Hypothesis of the Water Layer Thickness

Following the methodology previously described, the results found are recapitulated in Table III.

We have studied the evolution of slump for each sub-program: (OC+G/S, OC + cement, OC+ water). The Fig. 13 reflect this evolution.

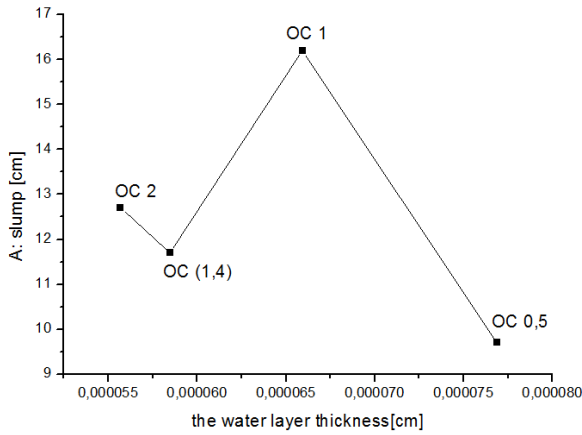


Fig. 13 Sub-program 1: OC+G/S, A=f(T<sub>WL</sub>)

B. Hypothesis of the Paste Layer Thickness:

After choosing  $D_i^{max}_{paste}$ , we followed the methodology described before. For example for  $D_i^{max}_{paste} = 0.01$ , the results are recapitulated in the Table IV.

We do the same for  $D_i^{max}_{paste} = 0.1mm, 1mm, 2mm, 2.5mm, 2.8mm, 3.15mm, 5mm, 6.3mm$ , the results are shown on the Figs. 16-23.

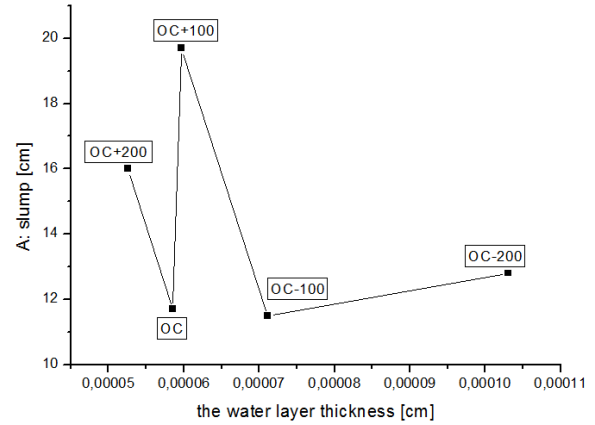


Fig. 14 Sub-program 2: OC+ cement, A=f(T<sub>WL</sub>)

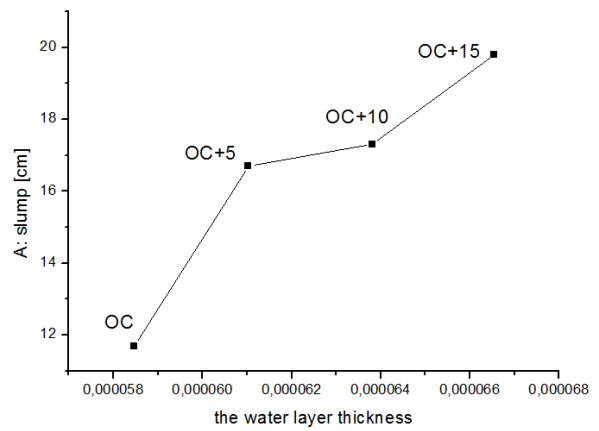


Fig. 15 Sub-program 3: OC+ water, A=f(T<sub>WL</sub>)

TABLE IV  
 THE PASTE LAYER THICKNESS AND SLUMP VALUES FOR  $D_i^{max}_{paste} = 0.01$

Concrete	d [mm]	$D_i^{max}_{paste}$ [mm]	$D_i + 1$ [mm]	D [mm]	Por (Di + 1 /D)	$V_{ST} >0,01$ [l]	$V_T >0,01$ [l]	$V_V >0,01$ [l]	$V_{ST} <0,01$ [l]	$V_W$ [l]	$V_p <0,01$ [l]	$V_{p.exe}$ [l]	$S_{Tm} > 0,01$	$T_{PL}$ [cm]	Slump (cm)
OC	0,0008	0,01	0,0125	25	0,183	756,42	925,73	169,31	56,06	182	238,06	68,75	2,68E+08	0,00026	11.7
OC 0,5	0,0008	0,01	0,0125	25	0,180	721,13	879,65	158,53	55,01	212	267,01	108,49	2,98E+08	0,00036	9.7
OC 1	0,0008	0,01	0,0125	25	0,182	741,87	907,06	165,19	55,47	194	249,47	84,28	2,78E+08	0,00030	16.2
OC 2	0,0008	0,01	0,0125	25	0,185	762,08	934,52	172,44	56,01	177	233,01	60,57	2,56E+08	0,00024	12.7
OC-200	0,0008	0,01	0,0125	25	0,186	789,98	971,07	181,09	25,81	172	197,81	16,72	1,67E+08	0,00010	12.8
OC-100	0,0008	0,01	0,0125	25	0,185	774,40	949,63	175,23	41,01	174	215,01	39,78	2,18E+08	0,00018	11.5
OC+100	0,0008	0,01	0,0125	25	0,182	716,73	876,50	159,76	68,94	207	275,94	116,17	3,10E+08	0,00038	19.7
OC+200	0,0008	0,01	0,0125	25	0,181	696,01	850,22	154,20	82,93	211	293,93	139,72	3,56E+08	0,00039	16.0
OC+5	0,0008	0,01	0,0125	25	0,183	750,71	919,23	168,52	55,59	186	241,59	73,07	2,70E+08	0,00027	16.7
OC+10	0,0008	0,01	0,0125	25	0,183	747,56	915,38	167,82	55,43	190	245,43	77,61	2,66E+08	0,00029	17.3
OC+15	0,0008	0,01	0,0125	25	0,183	743,58	910,51	166,92	55,12	194	249,12	82,19	2,64E+08	0,00031	19.8

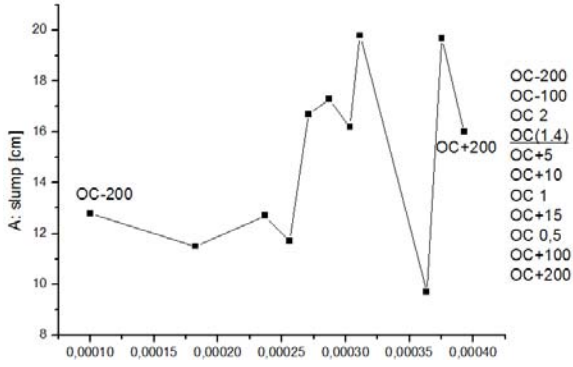


Fig. 16 Paste layer thickness [cm] for  $D_{i_{paste}}^{max}=0.001mm$

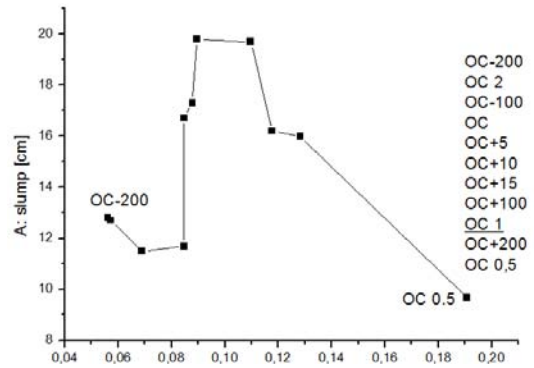


Fig. 20 Paste layer thickness [cm] for  $D_{i_{paste}}^{max}=2.5mm$

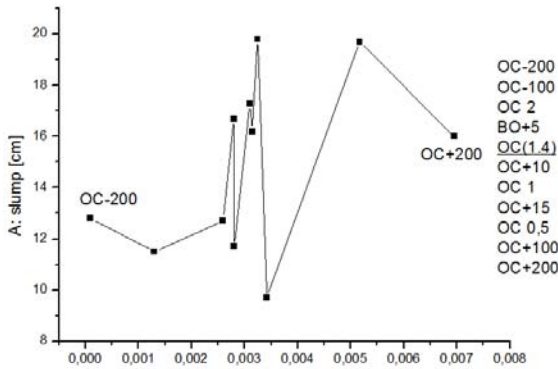


Fig. 17 Paste layer thickness [cm] for  $D_{i_{paste}}^{max}=0.01$

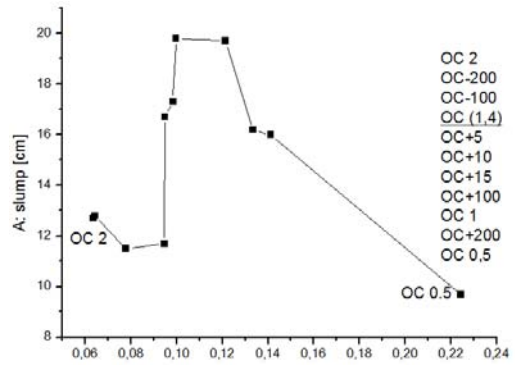


Fig. 21 Paste layer thickness [cm] for  $D_{i_{paste}}^{max}=2.85mm$

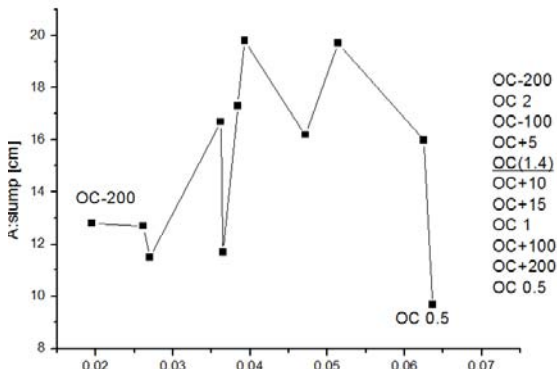


Fig. 18 Paste layer thickness [cm] for  $D_{i_{paste}}^{max}=1mm$

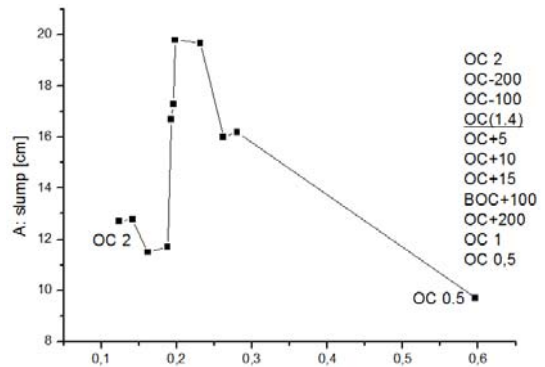


Fig. 22 paste layer thickness [cm] for  $D_{i_{paste}}^{max}=5mm$

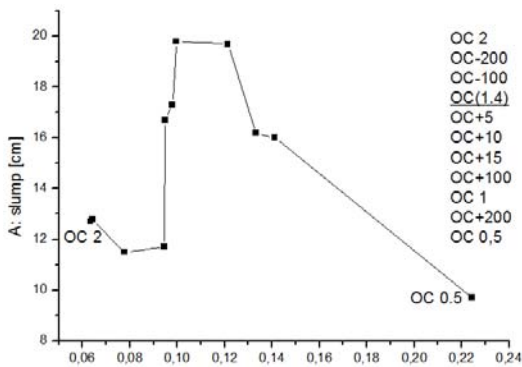


Fig. 19 Paste layer thickness [cm] for  $D_{i_{paste}}^{max}=2mm$

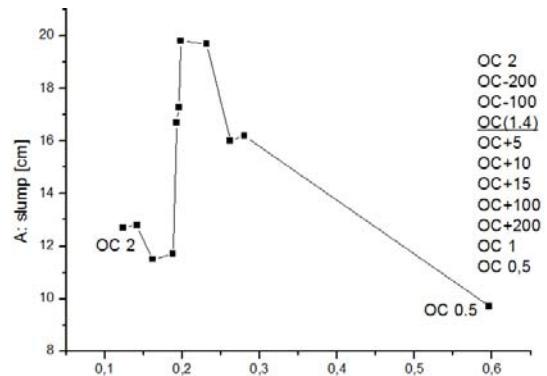


Fig. 23 Paste layer thickness [cm] for  $D_{i_{paste}}^{max}=6.3mm$

## VII. RESULTS AND INTERPRETATIONS

### A. Hypothesis of the Water Layer Thickness

If we see the evolution of the slump in Fig. 12, we see an evolution and a logical succession of concrete in order: OC 2; OC (1.4); OC 1, OC 0.5 in terms of water layer thickness.

Similarly for Fig. 14 with the quantity of water for variant, the evolution is also logical starting with OC, OC + 5, OC+ 10 and OC + 15.

While the evolution of concrete for the sub-program 2, with the quantity of cement as a variable, does not respect the logic evolution: the classification of concretes is disturbed by a permutation on position between OC and OC-100. Regarding the role of the quantity of cement, the influence of the parameter cement does not appear.

### B. Hypothesis of the Paste Layer Thickness:

The appearance of the first three graphs: Figs. 15-17 for  $D_{paste}^{max} = (0.01, 0.1, 1)$  is not similar, and classification of different concretes is not stable.

From Figs. 18-22 for  $D_{paste}^{max} = (2 \text{ to } 6.3\text{mm})$ , the appearance of the graphs is more or less similar and the stability is noticed in different concretes.

For the pastes from  $D_{paste}^{max} > 2$ , concretes permute position and ranking until reaching a stability from  $D = 2.85\text{mm}$  value. From this value concretes remain in their positions and the phenomenon remains same.

## VIII. CONCLUSION

### A. Hypothesis of the Water Layer Thickness

The model of the water layer thickness seems to be pertinent when it is applied to the sub-program 3 (different concretes by the quantity of water) where the curve indicates a non-linear proportionality between slump and the water layer thickness.

This model is less pertinent for the case of sub-program 1 (concrete which differ in the G/S ratio) even if it is noted that the water layer thickness curve increases with the decrease in G/S ratio but slump varies without respect for the increase in G/S ratio.

The model is even less pertinent to the subprogram 3 (concrete which differ in the cement content) because the curve does not indicate any logic.

### B. Hypothesis of the Paste Layer Thickness

It seems that the paste in this model is composed of grains whose value  $D_{paste}^{max} = 2.85\text{mm}$  because from this value a stable classification is obtained.

This result derived from this series of concrete must be confirmed or denied using other series of concrete for we can adopt a paste layer thickness model applicable to all concretes.

More research is needed to find a relationship between workability and formulation parameters, using the fractal model.

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