Influence of Flexural Reinforcement on the Shear Strength of RC Beams without Stirrups

Guray Arslan, Riza S. O. Keskin

***Abstract*—**Numerical investigations were conducted to study the influence of flexural reinforcement ratio on the diagonal cracking strength and ultimate shear strength of reinforced concrete (RC) beams without stirrups. Three-dimensional nonlinear finite element analyses (FEAs) of the beams with flexural reinforcement ratios ranging from 0.58% to 2.20% subjected to a mid-span concentrated load were carried out. It is observed that the load-deflection and load-strain curves obtained from the numerical analyses agree with those obtained from the experiments. It is concluded that flexural reinforcement ratio has a significant effect on the shear strength and deflection capacity of RC beams without stirrups. The predictions of diagonal cracking strength and ultimate shear strength of beams obtained by using the equations defined by a number of codes and researchers are compared with each other and with the experimental values.

***Keywords*—**Finite element, flexural reinforcement, reinforced concrete beam, shear strength.

I. Introduction

IN the last four decades, many equations have been proposed to estimate the shear strength of reinforced concrete (RC) beams. However, in terms of accuracy and uniformity of the prediction, there is considerable diversity between the existing test results, the requirements of concrete design codes and the predictions provided by various researchers [1]. Despite the influence of the flexural reinforcement ratio ρ on the shear strength is significant, it is neglected in some of the design equations, such as the ones given by [2]-[5] and the simplified equation of ACI318 (11)-(3) [6]. On the other hand, it is considered by a number of equations [1], [7]-[17] and the detailed equation of ACI318 (11)-(5) [6]. Based on the experimental results, [18] suggests that concrete shear strength is proportional to $\rho^{0.31}$, whereas [11], [12], Eurocode 2 [8] and [16] suggest a proportion of $\rho^{1/3}$. It is observed from the predictions of artificial neural network developed by [17] that the flexural reinforcement has a greater influence ($\rho^{0.5}$) on the shear strength.

Rodrigues et al. [19] investigated the influence of shear on the rotation capacity of RC members without stirrups for various values of shear span and for two types of flexural reinforcement, and proposed an analytical expression to estimate the rotation capacity of one-way members without stirrups. Lee and Kim [20] studied the effects of flexural reinforcement ratio and shear span-to-depth ratio on the minimum amount of stirrups required in RC beams. According to the General Method of CSA-A23.3-04 [21], an increase in the flexural reinforcement ratio reduces the longitudinal strain in the reinforcement, resulting in a larger contribution of concrete to the shear strength. Omeman et al. [22] studied the shear behavior of RC beams without stirrups experimentally, and observed that the strains in the flexural reinforcement decreases with the increasing effective depth of beam and increasing flexural reinforcement ratio for a constant load level. Lubell et al. [23] examined the validity of using flexural reinforcement ratio and/or the corresponding reinforcement strains for predicting shear capacities of RC members without stirrups. According to [23], shear capacity models must take into account the influences of size effect and flexural reinforcement.

In this study, the influence of flexural reinforcement ratio on the diagonal cracking strength and ultimate shear strength of RC beams without stirrups was investigated numerically. Three-dimensional nonlinear finite element analyses (FEAs) of the beams with flexural reinforcement ratios ranging from 0.58% to 2.20% and a shear span-to-depth ratio of 2.5 subjected to a mid-span concentrated load by [24] were conducted. A good agreement between load-deflection and load-strain (strain in the reinforcement at the mid-span) curves obtained from the experiments and the numerical analyses is observed. A significant effect of flexural reinforcement ratio on the shear strength and deflection capacity of RC beams without stirrups is observed. Load-strain curves according to CSA-A23.3-04 [21] and SIA 262 [25] were constructed by using the strains obtained from the FEAs. The predictions of diagonal cracking strength and ultimate shear strength of the beams obtained by using the equations defined by various codes [5]-[9], [21], [25], and researchers [1], [3], [4], [11]-[16] are compared with each other and with the experimental values.

II. Finite Element Model

Five RC beams tested by [24] were analyzed numerically by using the commercial finite element software ANSYS v12.1. Three-dimensional nonlinear FEA was undertaken for each beam. The properties and geometric characteristics of the beams in the nonlinear finite element models are the same as those of the actual beams. The simply supported beams, with a span length of 1150 mm between the supports, are 150 mm wide and 260 mm deep. The shear span-to-depth ratio is 2.5 for all beams. The flexural reinforcement ratios range from 0.58% to 2.20%. The beam designation includes a

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combination of letters and numbers: H to indicate the series; 1 or 2 to indicate the number of bars; 16, 22, and 26 to designate the diameter of bars. For example, a beam of series H having two bars with a diameter of 16mm is designated as 2H16. The yield strength and tensile strength of reinforcing bars are 420 MPa and 550 MPa, respectively. The uniaxial tensile strength and compressive strength of concrete are 1.55 MPa and 25.0 MPa, respectively. The beams were tested under a mid-span concentrated load and all of them failed in shear.

Only the half of each beam was modeled by exploiting the symmetry of the loading and geometry. A load-controlled analysis was performed by increasing the load at the tip of the half-beam incrementally. The analysis was carried out using the Newton-Raphson technique. Reinforcing bars were modeled discretely by using Link8 element by assuming a perfect bond between concrete and reinforcing bars. Solid45 elements were used at the supports and at the loading regions to prevent stress concentrations at those regions. Concrete was modeled by using Solid65 eight-node brick element, which is capable of simulating the cracking and crushing behavior of brittle materials. The Solid65 element requires linear isotropic and multi-axial isotropic material properties to model the concrete properly. An optimum mesh size was chosen to avoid the mesh dependence problem. Based on [26], and [27], mesh size was determined as two or three times the maximum aggregate size. The modulus of elasticity E_c is $4730 f_c^{0.5}$, where f_c is the compressive strength of concrete [6].

III. Results and Discussion

A. Comparison of Load–Deflection Curves of Beams

The load–deflection curves obtained from FEAs are plotted in Figs. 1-3 together with the experimental curves provided by [24]. A good agreement exists between the experimental and numerical values in terms of the first flexural crack loads and ultimate loads of beams.

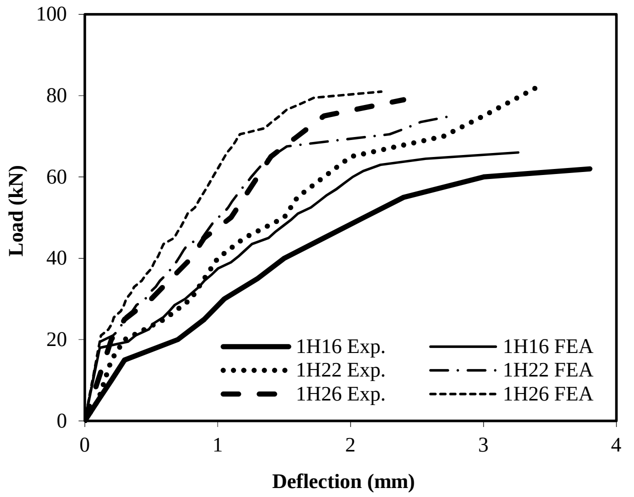

Fig. 1 Load-deflection curves (1H16, 1H22 and 1H26)

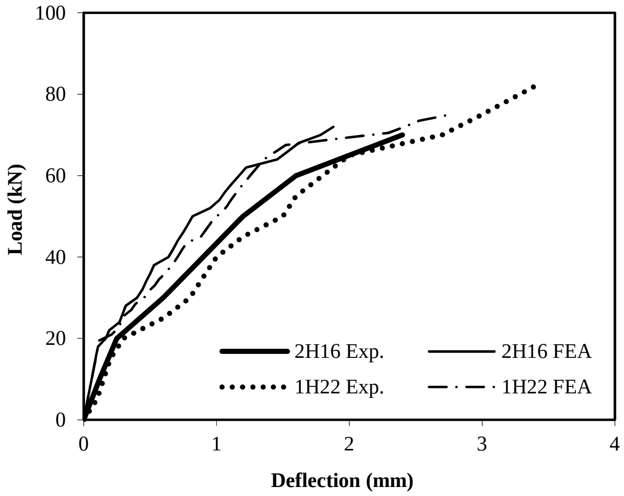

Fig. 2 Load-deflection curves (2H16 and 1H22)

For beams having similar flexural reinforcement ratios (2H16 and 1H22, Fig. 2), the load-carrying capacity and the deflection capacity of the beam having less reinforcing bars are larger. It is observed through the experimental and numerical results that the load-carrying capacity and the deflection capacity of 2H16 are less than those of 1H22 because the total surface area of reinforcement closer to the beam surface is larger in 2H16, compared to that in 1H22.

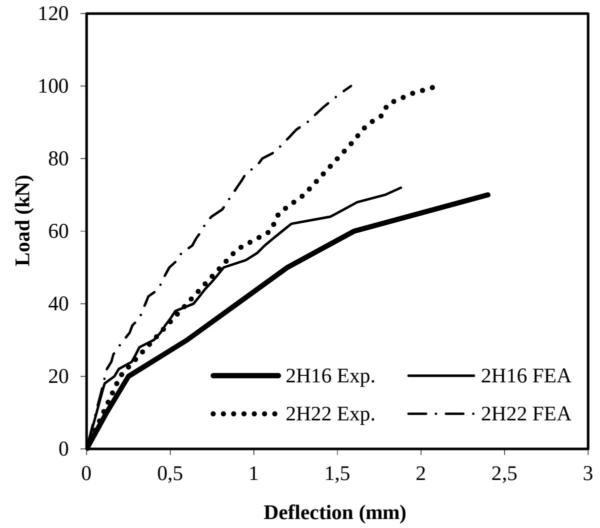

Fig. 3 Load-deflection curves (2H16 and 2H22)

Table I presents the experimental and numerical loads at first flexural/diagonal cracking and failure loads of all beams. Comparing the first flexural crack loads, the FEAs deliver 0.98, 1.05, 1.20, 0.75 and 0.80 times the loads obtained experimentally for 1H16, 1H22, 2H16, 1H26 and 2H22, respectively. Comparing the maximum loads, the loads calculated through FEAs are 1.06, 1.00, 1.03, 1.03 and 1.00 times the experimental results for 1H16, 1H22, 2H16, 1H26 and 2H22, respectively. However, the deflection capacities of the beams obtained from FEAs are smaller than the corresponding experimental values for the same load level.

TABLE I
FLEXURAL/DIAGONAL CRACKING AND FAILURE LOADS OF BEAMS

Beam	ρ (%)	Exp. P_{fl}[a] (kN)	Exp. P_{cr}[b] (kN)	Exp. P_u (kN)	Exp. P_{fl}/P_u	Exp. P_{cr}/P_u	FEA P_{fl} (kN)	FEA P_u (kN)	Pred. / Exp. Load	
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(6)/(1)	(7)/(3)
1H16	0.58	20	45	62	0.32	0.73	19.5	66	0.98	1.06
1H22	1.10	20	50	75	0.27	0.67	21.0	75	1.05	1.00
2H16	1.17	20	60	70	0.29	0.86	24.0	72	1.20	1.03
1H26	1.54	30	70	79	0.38	0.89	22.5	81	0.75	1.03
2H22	2.20	55	65	100	0.55	0.65	44.0	100	0.80	1.00

[a] Flexural cracks extended up to mid-height of the beam.
[b] Diagonal cracks extended up to mid-height of the beam.

B. Comparison of Load–Strain Curves of Beams

The load-strain curves according to CSA-A23.3-04 [21] and SIA 262 [25], which predict the shear strength of RC beams based on the strain in the flexural reinforcement, were constructed by using the strains obtained from FEAs. According to CSA-A23.3-04 [21], the shear resistance V_u of RC beams without stirrups can be obtained as

$$V_u = \beta \sqrt{f_c}\, b_w d_v , \tag{1}$$

where b_w is the beam web width, d_v is the effective shear depth which can be taken as the greater of 0.9d or 0.72h (d is the effective depth of beam and h is the height of beam.), and β can be calculated as

$$\beta = \frac{520}{(1+1500\,\varepsilon_x)(1000 + S_{ze})} , \tag{2}$$

where

$$S_{ze} = \frac{35 S_z}{15 + d_{ag}} \geq 0.85 S_z \tag{3}$$

and ε_x is the longitudinal strain at the mid-depth of member due to the factored loads which can be derived as

$$\varepsilon_x = \frac{M_f / d_v + V_f}{2 E_s A_s} \geq 0.85 S_z , \tag{4}$$

where M_f and V_f is the moment and the shear force due to the factored loads, respectively, E_s is the modulus of elasticity of non-prestressed reinforcement, A_s is the area of flexural reinforcement, $S_z(=d_v)$ is the crack spacing parameter dependent on crack control characteristics of flexural reinforcement and d_{ag} is the maximum aggregate size.

According to [28], the hypothesis of the dependence of the shear strength on the width w and roughness characterized by the maximum aggregate size of the critical shear crack can be written as

$$\frac{V_u}{b_w d \sqrt{f_c}} = f\left(w, d_{ag}\right). \tag{5}$$

For beams failing in shear without development of plastic strains in the flexural reinforcement, the expression for the crack width is given as

$$w \propto \varepsilon d \tag{6}$$

where ε is a reference strain in the beam [28]. SIA 262 [25] defines the shear strength of a RC beam without stirrups by using the flexural strain as

$$\frac{V_u}{b_w d \sqrt{f_c}} = \frac{0.3}{1 + 68 \dfrac{d}{16 + d_{ag}} \dfrac{f_y}{E_s}} \tag{7}$$

where f_y is the yield strength of flexural reinforcement.

The first flexural cracking loads, visually observed in the experiments [24], are compared with the value where the slopes of the numerical load-strain curves change. The formation of first diagonal cracks, also visually observed in the experiments [24], is verified most of the time by a sudden jump in the strain (Figs. 4-8).

Table II presents the ultimate shear loads obtained from the experiments and the code equations. The performances of code equations (1) [21] and (7) [25] in predicting the ultimate shear load P_u resisted by the beams having flexural reinforcement ratios ranging from 0.58% and 2.20% are compared in Figs. 4-8. (1) and (7) underestimate the ultimate load resisted by the beams regardless of flexural reinforcement ratio, as the ratio of the experimental result to the prediction of (1) ranges between 1.435 and 2.227 with a mean value of 1.722 and the ratio of the experimental result to the prediction of (7) ranges between 1.048 and 1.512 with a mean value of 1.168 (Table II). In general, (1) delivers more conservative predictions than (7) do, and it can be used safely for predicting the load-carrying capacities of RC beams without stirrups as confirmed in this study. The predictions of the load-carrying capacities of 2H16 and 1H22, which have flexural reinforcement ratios of around 1%, according to SIA 262 [25] are in good agreement with the experimental results. The predictions according to SIA 262 [25] differ from the experimental results as the flexural reinforcement ratio gets farther away from 1%. In other words, SIA 262 [25] becomes more and more conservative with the increasing flexural reinforcement ratio.

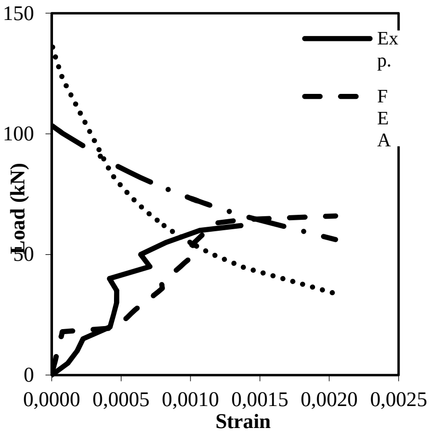

Fig. 4 Load-strain curves of 1H16

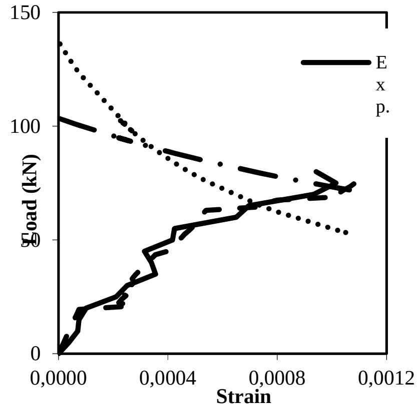

Fig. 5 Load-strain curves of 1H22

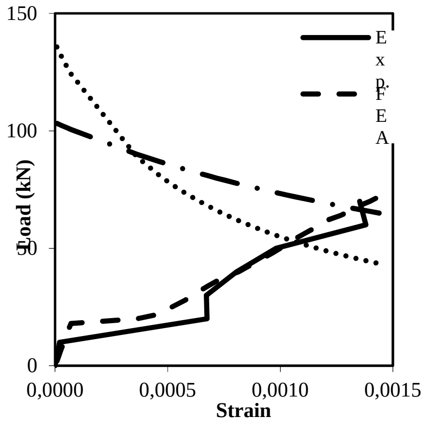

Fig. 6 Load-strain curves of 2H16

C. Comparison of Equations by Codes and Researchers

A number of equations defined by codes [5]-[9], [21], [25] and researchers [1], [3], [4], [11]-[16] are considered. All the equations considered within the scope of this study are summarized in Table III. The diagonal cracking strength and ultimate shear strength of beams obtained from the equations given in Table III are compared with the experimental results.

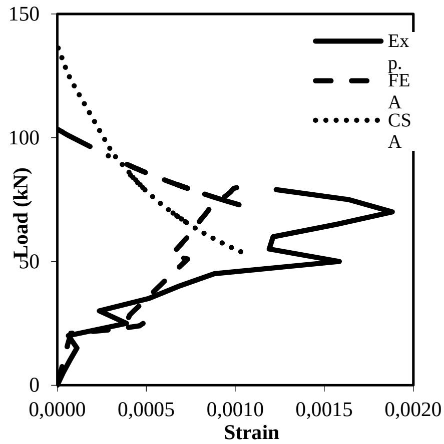

Fig. 7 Load-strain curves of 1H26

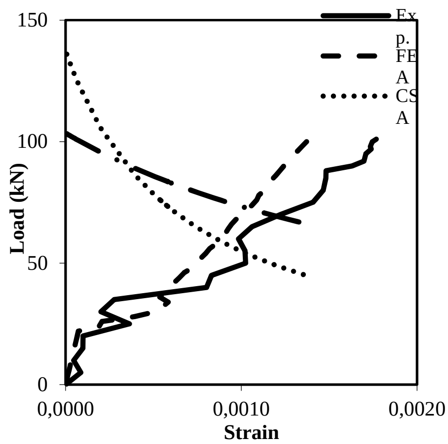

Fig. 8 Load-strain curves of 2H22

TABLE II
ULTIMATE LOADS OF BEAMS FROM EXPERIMENTS AND CODE EQUATIONS

Beam	P_u (kN) Exp. (1)	CSA [21] (2)	SIA 262 [25] (3)	Exp. / Pred. (1)/(2)	(1)/(3)
1H16	62	33.75	56.19	1.837	1.103
1H22	75	52.26	71.55	1.435	1.048
2H16	70	43.47	65.00	1.610	1.077
1H26	79	52.57	71.77	1.503	1.101
2H22	100	44.91	66.15	2.227	1.512
Mean Value (MV)				1.722	1.168
Standard Deviation (SD)				0.321	0.193
Coefficient of Variation (COV)				0.186	0.165

Table IV summarizes the comparisons of the predictions obtained from the considered equations with the experimental values. It is observed from Table IV that the ratios of the experimental values to the predictions obtained from the equations of Eurocode 2 [8], Zsutty [11], Okamura and Higai [12], Kim and Park [14], Rebeiz [15], Khuntia and Stojadinovic, Zararis and Papadakis [4], and Arslan [1] have lower coefficients of variations and provide better predictions compared to the rest of the equations. It is also observed that predicting the shear strength of the beams tested by Garip [24] by using strains in the flexural reinforcement according to CSA-A23.3-04 [21] or SIA 262 [25] does not provide any better predictions, however this cannot be generalized since the number of beams is limited.

TABLE III
SHEAR STRENGTH MODELS

Code/Researcher(s)	Shear strength model
ACI 318 [6]	$v_c = 0.16\sqrt{f_c} + 17\rho(V_u d / M_u) \le 0.29\sqrt{f_c}$ or $v_c = 0.17\sqrt{f_c}$, f_c in MPa.
TS 500 [5]	$v_c = 0.2275\sqrt{f_c}$, f_c in MPa.
NZS 3101 [7]	$v_c = \min\{(0.07 + 10\rho)\sqrt{f_c}, 0.2\sqrt{f_c}\} \ge 0.08\sqrt{f_c}$, f_c in MPa.
Eurocode 2 [8]	$v_{rd,c} = 0.18k(100\rho f_c)^{1/3} \ge 0.035k^{3/2}\sqrt{f_c}$, f_c in MPa. $k = 1 + \sqrt{200/d} \le 2.0$, $\rho = A_s/(b_w d) \le 0.02$, d in mm.
CEB-FIP10 [9]	$v_{Rd,c} = k_v\sqrt{f_c}(z/d)$, f_c in MPa and z is the internal moment arm which can be taken as 0.9d. $k_v = \dfrac{0.4}{(1+1500\varepsilon_X)}\dfrac{1300}{(1000 + k_{dg}z)}$, $k_{dg} = \dfrac{32}{16 + d_a} \ge 0.75$, z and d_a in mm.
Zsutty [11]	$v_u = 2.2(\rho f_c d/a)^{1/3}$, $a/d \ge 2.5$, f_c in MPa.
Okamura and Higai [12]	$v_c = 0.2(100\rho f_c)^{1/3} d^{1/4}(0.75 + 1.40d/a)$, f_c in MPa and d in m.
Bazant and Sun [13]	$v_u = 0.54\sqrt[3]{\rho}\left(\sqrt{f_c} + 249\sqrt{\dfrac{\rho}{(a/d)^5}}\right)\left(\dfrac{1+\sqrt{5.08/d_a}}{\sqrt{1+d/(25d_a)}}\right)$, f_c in MPa and d_a in mm.
Kim and Park [14]	$v_u = 3.5 f_c^{\alpha/3}\rho^{3/8}(0.4 + d/a)(1/\sqrt{1+0.008d} + 0.18)$, f_c in MPa and d in mm. $\alpha = 2 - (a/d)/3$ for $1.0 \le a/d < 3.0$, $\alpha = 1$ for $a/d \ge 3.0$
Collins and Kuchma [3]	$v_c = \dfrac{245}{1275 + [25S_X/(d_a + 16)]}\sqrt{f_c}$, $S_X \approx 0.9d$, f_c in MPa, d and d_a in mm.
Rebeiz [15]	$v_c = 0.4 + \sqrt{f_c\rho d/a}(2.7 - 0.4A_d)$, $v_u = 0.4 + \sqrt{f_c\rho d/a}(10 - 3A_d)$, $A_d = 2.5$ for $a/d \ge 2.5$, f_c in MPa.
Khuntia and Stojadinovic [16]	$v_c = 0.54\sqrt[3]{\rho(f_c V_u d/M_u)^{0.5}}$, $M_u/(V_u d) = a/d - 1$, f_c in MPa.
Zararis and Papadakis [4]	$v_u = (1.2 - 0.2a)(c/d)f_{ct}$, $f_{ct} = 0.3f_c^{2/3}$, $(1.2 - 0.2a) \ge 0.65$, f_c in MPa, c is depth of compression zone.
Arslan [1]	$v_{cr} = 0.2\left[(c/d)\sqrt{f_c} + \sqrt{\rho f_c}\right](300/d)^{0.28}$, f_c in MPa and d in mm.

v_c is the shear strength of RC members without stirrups, v_{cr} is the diagonal cracking strength, v_u is the ultimate shear strength.
v_c and v_u are considered to be equal to v_{cr} in calculating the shear strength.

TABLE IV
COMPARISON OF SHEAR STRENGTH PREDICTIONS WITH EXPERIMENTAL VALUES

Exp. Value/Prediction by	Diagonal cracking strength			Ultimate shear strength		
	MV	SD	COV	MV	SD	COV
ACI 318 [6]	0.942	0.139	0.147	1.252	0.172	0.137
TS 500 [5]	0.739	0.132	0.179	0.984	0.181	0.184
CSA-A23.3-04 [21]	1.290	0.192	0.149	1.722	0.321	0.186
SIA 262 [25]	0.876	0.123	0.140	1.168	0.193	0.165
NZS 3101 [7]	0.943	0.087	0.092	1.265	0.171	0.135
Eurocode 2 [8]	0.770	0.071	0.093	1.023	0.060	0.059
CEB-FIP10 [9]	1.113	0.348	0.313	1.451	0.355	0.245
Zsutty [11]	0.767	0.071	0.093	1.019	0.060	0.059
Okamura and Higai [12]	0.708	0.066	0.093	0.940	0.055	0.059
Bazant and Sun [13]	0.696	0.088	0.126	0.924	0.095	0.102
Kim and Park [14]	0.577	0.055	0.095	0.767	0.047	0.061
Collins and Kuchma [3]	0.968	0.173	0.179	1.289	0.238	0.184
Rebeiz [15]	0.837	0.081	0.097	0.868	0.047	0.054
Khuntia and Stojadinovic [16]	0.907	0.084	0.093	1.205	0.071	0.059
Zararis and Papadakis [4]	0.730	0.068	0.093	0.971	0.065	0.066
Arslan [1]	0.992	0.096	0.097	1.319	0.092	0.070

IV. CONCLUSION

Based on the results presented in this paper, the following conclusions are drawn:

- For beams having similar flexural reinforcement ratios (1H22 and 2H16), the load-carrying capacity and the deflection capacity of the beam having less reinforcement bars are larger. It is observed through the experimental and numerical results that the load-carrying capacity and the deflection capacity of 2H16 are less than those of 1H22 because the amount of surface area of reinforcement closer to beam surface is larger in 2H16, compared to that in 1H22.
- It can be deduced from Table I that the percentage ratio of the first diagonal cracking load to the ultimate load increases from 67% to 89% with the increase in the flexural reinforcement ratio from 1.10% (1H22) to 1.54% (1H26). This indicates that the beam with a lower flexural reinforcement ratio has lower post-diagonal cracking shear strength.
- The equations of Eurocode 2 [8], [11], [12], [14], [15], [4], and [1] provide better predictions compared to the equations of CSA-A23.3-04 [21] and SIA 262 [25], which predict shear strength by using the strains in the flexural reinforcement. However, this should be verified with more data.
- The load-carrying capacities of beams with flexural reinforcement ratios of around 1% are predicted well according to SIA 262 [25]. The agreement between the experimental results and the predictions gets worse as the flexural reinforcement ratio gets farther away from 1%.

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