World Academy of Science, Engineering and Technology
International Journal of Economics and Management Engineering
Vol:9, No:6, 2015

Operations Research Applications in Audit Planning and Scheduling

Abdel-Aziz M. Mohamed

Open Science Index, Economics and Management Engineering Vol:9, No:6, 2015 publications.waset.org/10002029/pdf

***Abstract*—**This paper presents a state-of-the-art survey of the operations research models developed for internal audit planning. Two alternative approaches have been followed in the literature for audit planning: (1) identifying the optimal audit frequency; and (2) determining the optimal audit resource allocation. The first approach identifies the elapsed time between two successive audits, which can be presented as the optimal number of audits in a given planning horizon, or the optimal number of transactions after which an audit should be performed. It also includes the optimal audit schedule. The second approach determines the optimal allocation of audit frequency among all auditable units in the firm. In our review, we discuss both the deterministic and probabilistic models developed for audit planning. In addition, game theory models are reviewed to find the optimal auditing strategy based on the interactions between the auditors and the clients.

***Keywords*—**Operations research applications, audit frequency, audit planning, audit-staff scheduling.

I. Introduction

THE importance of a well-designed and effective internal audit plan cannot be over-emphasized. A review of the models for evaluating the reliability of internal control structure is presented in [1]. However there has not been any comprehensive review article about the available optimization (normative) models for audit planning and scheduling.

The main objective of internal auditing is to reduce the losses that occur due to certain undesirable events such as bookkeeping errors, fraud, waste, etc. [2]. The word *risk* is used to describe situations involving potential losses and in any audit planning a systematic risk assessment is highly recommended. Auditors can decrease losses by either reducing the probability of its occurrence or by decreasing its magnitude. Assessment of the probability of undesirable events should be made to obtain the degree of relative risk of each division (unit) in the organization. The internal audit manager should design a plan that fits the organization and helps to achieve the audit goals in an effective way. Each unit should be assigned a relative risk score according to the expected losses in that unit caused by fraud, waste and error. A risk score indicates the rate at which losses are expected to accrue in the absence of auditing. The desired audit plan should prescribe a more frequent of risky unit, or alternatively more audit resources should be allocated to such units. Two alternative approaches are followed in the literature to design an audit plan: (1) determining the internal audit frequency for each auditable unit, and (2) allocating audit resources among all auditable units. For each auditable unit in the company, the first approach determines the optimal elapsed time (or the optimal number of transactions) after which an audit should be performed. The second approach, however, determines the optimal audit resource allocation among auditable units. This paper presents a comprehensive survey of the optimization models developed for audit planning. To our knowledge such a complete survey has yet to be reported in the literature and, thus, the motivation behind this article.

Reference [3] presented a comparison between periodic audit and continuous audit. He defied the periodic audit to be the one conducted once at the close of financial or trading period while the continuous audit is the system of audit conducted continuously throughout the year at regular intervals. Therefore the optimization models presented in this research paper can identify the optimal continuous audit.

A. M. Mohamed is an Associate Dean of Graduate Studies at the Arab Academy for Science, Technology, and Maritime Transport, Cairo, Egypt (phone: 202-26-90723; fax: 202-26-87403; e-mail: amohamed55@ yahoo.com).

II. Internal Audit Timing

The creation of internal audit department is necessary to monitor the performance and compliance of different divisions (units) in a company. The importance of audit staff scheduling is critical to any business organization. Reference [4] reported that an efficient audit staff schedule reduces the overall audit cost while [5] reported that it reduces the rate of turnover among auditors.

Reference [6] highlighted the importance of the audit department establishing a formal method to develop a comprehensive set of integrated plans. One of the main suggestions in its proposed list was to plan an effective schedule for the activities of the internal audit department. In this section, we present mathematical models designed to determine the optimal audit schedule. Some of the proposed models are deterministic, which implies model parameters are known with certainty. On the other hand, if models include probabilistic terms, they are referred to as probabilistic (stochastic) models

A. Deterministic Models

Two categories of optimization models are presented in this section: (1) non-constrained models which result in closed form solutions, and (2) constrained models to be solved by appropriate optimization software. A closed form solution refers to a mathematical expression (formula) that directly provides the optimal solution in terms of the model parameters. Such a formula is typically obtained by taking the

Open Science Index, Economics and Management Engineering Vol:9, No:6, 2015 publications.waset.org/10002029/pdf

derivatives of the objective function of non-constrained model and setting the result equal to zero.

a) Closed Form Solutions

Reference [7] was pioneer in developing a deterministic non-constrained model to find the optimal audit interval. Its model aimed to minimize the sum of the audit costs and the expected losses that accrue in the absence of auditing. The authors assumed audit costs were incurred at a uniform rate and were obtained from multiplying the number of days required for auditing (*w*) by the standard cost per day (*c*). The authors noted that overheads such as travel expenses should be included in the audit costs; however, expenses that are incurred regardless of whether an audit is conducted or not, should not be considered. The losses that accrue in the absence of auditing are losses due to fraud, waste, error, etc. It assumed such losses would rise exponentially to an asymptotic value E^* dollars per year as the time elapsed science previous audit (t) increases. It proposed the following expression to compute the expected losses:

$$E^*(1 - e^{-bt}) \quad (1)$$

b is the growth rate of the loss function that depends mainly on the management quality. If management quality is high, the growth rate will be low. The model assumed, once the audit is completed and recommendations go into effect, the expected losses will drop to zero and then start rising again. The objective function of the model was formulated as the sum of the above-mentioned audit costs and expected losses. They adapted the machine replacement model from the field of operations research to develop their closed form solution for finding the optimal audit interval. They employed $e^x \cong 1 + x$ approximation and derived the following:

$$t_a \cong \frac{1}{b}\sqrt{\frac{cw(r+b)}{E^*}} \quad (2)$$

t_a is the optimal audit interval, and r is the annual interest rate. The main advantage of this approach is the simplicity of their closed form solution. However, the major drawback is the assumption that audit cost is constant regardless of when the last audit was conducted. In general, audits become more costly as the time interval between audits increases. The other disadvantages are the difficulties in estimating the model parameters b and E^* and the approximation used for the exponential function.

In a follow up study, [8] developed a closed form expression to find the optimal internal audit timing for an auditable unit. The objective function of their model was similar to the one presented in [7] with the goal of minimizing the sum of the audit costs, C, and the expected losses, $L(t)$. They cited the finding of [9] that the compliance with controls within auditable units deteriorates over time unless appropriate actions are taken to restore compliance to its proper level. Reference [8] assumed that the expected losses which accrue if a unit remains unaudited rise exponentially with time:

$$L(t) = M\left(1 - e^{-\beta t}\right) \quad (3)$$

The total assets of the auditable unit could be used as an estimate for M. Parameter β is the growth rate coefficient, the value of which could be set equal to the Audit Unit Priority Index (AUPI). The AUPI is a number that expresses the relative riskiness of the auditable unit. Several methods can be used to assess the value of AUPI based on either subjective or objective approaches [2], [10]-[14].

Reference [8] assumed that the audit costs were incurred at the end of audit cycles and the expected losses were accrued continuously during the entire cycle. Both the audit costs and the expected losses were discounted to their present values at the start of the corresponding audit cycles. They assumed the interest rate is *r* per year, compounding continuously and they arrived at the following equation, the solution of which provides the optimal audit timing, t_a.

$$r\left[\frac{1}{\beta+r} - \frac{C}{M}\right] = e^{-\beta t_a}\left[1 - \frac{\beta}{\beta+r}e^{-rt_a}\right] \quad (4)$$

This equation can be solved either graphically or by using software based on a numerical method such as the Golden Section search technique [15]. The "Goal Seek" module of Excel Software can also be used to find t_a. The main advantage of the approach in [8] is its simplicity. The above closed form expression provides a simple method to find the optimal audit timing t_a without a need to develop an optimization model. However, there are some disadvantages to this approach including the difficulties in estimating the model parameters as well as the assumption that the audit cost remains the same regardless of how frequently the audit is performed.

Reference [16] extended the work of [8] and developed a more realistic model for audit scheduling. In their model, they considered the audit cost as a function of time elapsed since the last audit. They applied their model to obtain the optimal schedule for the external audit of the financial statements of 53 private companies. Their goal was to minimize the sum of the present values of audit cost and expected losses due to the absence of auditing. They used the same loss function presented in [8]; however, they assumed the cost of audit would increase when less frequent audits are conducted. The audit cost for auditable unit *j* is:

$$C_j = F_j\left(\frac{t_j}{A}\right)^{\alpha} \quad (5)$$

C_j is the cost of auditing unit j if the time elapsed since the previous audit is t_j months; F_j is the audit cost if unit j is audited once a year; A is a constant that is set equal to 12 to prevent any adjustment to audit cost if audits were conducted each 12 months; and α is a parameter that shows the growth rate of audit cost with time. Following the same approach as suggested by [8], the authors identified the revised TDC(t_a) function, which lead to the following closed form expression for the optimal audit timing:

$$\frac{r}{r+\beta_j} - \frac{F_j\left[\frac{t_a}{A}\right]^{\propto}\left[r-\frac{\propto}{t_a}\right]}{M_j} = e^{-\beta_j t_a}\left[1 - \frac{\beta_j}{r+\beta_j}e^{-rt_a}\right] \quad (6)$$

The advantage of the above approach is that it takes into account the fact that the audit costs increase with less frequent auditing. The disadvantages include the need to estimate the values of the growth rate parameters $\propto_j$ and β_j.

b) Optimization Models

Reference [17] presented a mathematical programming model to determine the optimal number of times each unit should be audited given the risks and costs involved in the audit process. The model consists of an objective function and $I + 1$ constraints, where I is the number of auditable units in the firm. The objective function is the sum of audit costs and expected losses as discussed earlier. The cost of auditing unit i consists of two components: (1) fixed cost $C_{Fi,}$ and (2) variable cost $C_{vi}t_i$, where t_i is the time elapsed since the previous audit, C_{vi} is the variable audit cost per unit of time, and T is the planning horizon. The authors assumed the expected losses in the absence of auditing unit i can be expressed as:

$$L(t_i) = 0.5 b_i t_i^2 \quad (7)$$

Parameter b_i represents the rate at which losses for unit i accrue per unit of time. An appropriate value of b_i should be assign to each auditable unit such that higher values are given to units with higher risk, and lower values for less risky units. The model objective function is expressed as:

$$\text{Min T} \sum_{i=1}^{I}[(C_{Fi}/\, t_i) + \, 0.5\, b_i\, t_i] + k_1 \quad (8)$$

where $k_1 = T\sum_{i=1}^{I} C_{vi}$. If C is the total monetary resources available for audit during the planning horizon, the following constraint keeps the total audit cost less than the total available resources:

$$\sum_{i=1}^{I}\left[(C_{Fi} + C_{Vi}t_i)\; int\; \left(\frac{T}{t_i}\right)\right] \leq C \quad (9)$$

If the integer restriction is removed, the constraint is reduced to:

$$\sum_{i=1}^{I}\left[\frac{C_{Fi}T}{t_i}\right] \leq k_2, \text{ where } k_2 = C - T\sum_{i=1}^{I} C_{Vi} \leq k_2 \quad (10)$$

For each auditable unit, the time between any two consecutive audits must be less than T and more than t^*, where t^* is the shortest practical audit interval set by the management. Therefore, the following I constraints are also included in the model:

$$t^* \leq t_i \leq T \text{ for each } i,\ i = 1, 2, \ldots, I$$

To demonstrate the application of their model, the authors considered a case of eight auditable units and assumed t^* to be one month and the planning horizon T to be 48 months. This means that a unit could not be audited more than once each month and at least once during the planning horizon. They used Miller's separable programming technique [18] to obtain the optimal audit intervals. They showed that the results are relatively less sensitive to risk estimation error.

B. Stochastic Models

1. Closed Form Solutions

Reference [19] developed stochastic closed form solution methods to determine the optimal audit timing for individual accounts such as sales, inventory, accounts receivable, etc. The authors considered an individual account because error distribution tends to differ by the type of account [20]. Reference [19] showed that error rate increases as the number of transactions increases; also, the size and variability of errors in a period are related to the number of transactions that occur in that period. Furthermore, to initiate an audit, they proposed to use either the total number of transactions occurring since the previous audit, or the time elapsed since the last audit. They developed closed form solutions for both cases. The goal was to identify an audit plan that would ensure a high probability that the *error* in the account balance is less than a certain limit (τ). Error was defined as the difference between the recorded account balance (as shown in the company's books) and the actual account balance (as shown in the bank statements). They considered the cases of *perfect audits*, where audits help detect and correct all errors in the account balance, and *imperfect audits*, where audits may not detect all errors. Furthermore, they analysed two cases: (1) account balance overstatement discrepancy, and (2) account balance two-sided (over- and under- statement) discrepancy. They considered the problem of determining the optimal number of transactions between two audits that would ensure a significantly high probability (1 - α) that overstatement of the account balance would be less than a pre-specified tolerance (τ). The level of significance, $\propto$, is usually set in the range of 0.01 to 0.1. If the random variable representing the error in transaction $i$is $z_i$which has a mean of 0 and variance of σ_z^2, then the optimal number of transactions between successive audits (t_1^*) is:

$$t_1^*(\tau, \alpha) = \left[\frac{\tau}{\emptyset^{-1}(1-\alpha/2)\sigma_z}\right]^2 \quad (11)$$

$\emptyset^{-1}(1-\alpha/2)$ is the inverse distribution function corresponding to a confidence level of $(1-\alpha/2)$ which alternatively can be presented by $z_{\alpha/2}$. In other words, [19] showed that if the audit frequency is set at every t_1^* transaction, the difference between the recorded balance in the company books and the true balance will be less than a pre-specified level τ with probability of at least 1-α. The above formula can also be used to find the optimal time between two consecutive audits. If the mean number of transactions per time period is b, then the optimal number of periods between two successive audits is obtained by dividing t_1^* by b. The advantages of the above solution technique include: (1) error distribution does not need to be normal; (2) this technique can assess the audit costs and manpower impacts of varying accuracy and precision goals; and (3) the optimal solution is easily obtained because of the

Open Science Index, Economics and Management Engineering Vol:9, No:6, 2015 publications.waset.org/10002029/pdf

Open Science Index, Economics and Management Engineering Vol:9, No:6, 2015 publications.waset.org/10002029/pdf

closed form solution. However, the model has some drawbacks such as the assumption that the mean of the error term has to be zero; in other words, errors are assumed to be random and not intentional (misappropriation). Another drawback is the need to estimate the standard deviation of errorσ_z. More importantly, the above model is developed based on only one account, whereas in practice the audit timing should depend on all accounts an auditable unit is in charge of.

2. Optimization Models

The model presented in the previous section assumed that the state of an auditable unit gradually changes to the state of being out of compliance resulting in increased losses. Such losses could be reduced to a tolerable limit by conducting an audit. Reference [21] used the Markov process approach to model the optimal audit timing as a sequential decision problem. He then employed the dynamic programming technique to find the optimal solution for the model. The author assumed that each auditable unit can be in one of *n* discrete levels of internal control effectiveness. As an example, when *n* = 2, the two states would be: in-compliance and out of compliance. He developed an optimization model for this case in which the objective function was to minimize the expected total discounted costs and losses. Examples of such costs and losses include the cost of performing audits, the cost of correcting errors, the cost of corrective actions to improve internal control effectiveness, and losses due to employees not adhering to internal control procedures. A disadvantage of this approach is its high level of mathematical complexity which limits its applications. Another drawback is the underlying assumption that Markov transitional probabilities remain constant over time, which is not a valid assumption in many situations. In addition, estimation of the transitional probability figures is difficult.

Reference [22] proposes a stochastic programming model with mixed integer linear programming (MILP) and constraint programming (CP) for the audit timing problem. The constraint programming is the result of research in artificial intelligence in the area of logic control and constraint satisfaction [23], [24]. Their research problem considers a planning horizon consisting of N time periods. There are M units to be audited. Random losses may occur over time for each one of the auditable unit. Losses that may accrue in each auditable unit m at any point of time t is assumed to be normally distributed with mean $\mu_t^m$and standard deviation σ_t^m. Coefficient of variation which is the ratio of standard deviation and mean is assumed to be constant. Losses at different time periods are assumed to be independent. In addition, the audit team is given a strict deadline to complete the audit. Time to perform an audit is T time periods; i.e. after T periods the loss accrued will drop to zero. The proposed stochastic programming model balances both the audit cost with the cost of losses accrued in the absence of auditing. The decision variables are the audit timing for each auditable unit. The objective function is the sum of the expected value of the audit cost and the expected value of the losses that are expected to accrue in the absence of auditing. Some constraints state that if an internal audit starts at the beginning of period t, the loss level at the beginning of period $t + T$ must be equal to zero. Other constraints identify the maximum number of audit teams that can work simultaneously. The disadvantage of this model is that it assumes that the audit time is constant which is supposed to be a random variable. Reference [25] presented a dynamic programming model to optimize the audit interval in an inventory system. The research assumes a single item periodic review. Demand is assumed to be probabilistic. Ordering cost is proportional to order size and shortages are backordered.

Banks are subject to internal and external auditing of items such as assets, liabilities, and capitals. An audit evaluates the bank system internal control, the degree of complaisance with regulations and procedures, and the efficiency of the banking operations. Random audits (external audit) are the most performed in the banking industry. The report prepared by the internal auditors is always made available to the external auditors and may influence the outcome of the external audit.

Reference [26] developed an optimization model with emphasis on auditing of reserves, assets and capital in random or non-random framework. In case of random audit process, the regular audit of the bank asset value, A, is assumed to follow a Poisson probability distribution function with mean number of audits per time unit. They developed an optimization model to determine audit timing. Reference [27] used multi-objective linear programming to find the optimal credit capital allocation in financial institutions. The author compared several budget allocation techniques and showed the benefits of using the operations research approach methodology.

III. Internal Audit Scheduling

The audit scheduling problem determines the optimal assignment of a set of audit tasks to a group of assigned auditors. Since auditors have different levels of expertise and audit tasks have different levels of difficulty, it is required to assign the audit tasks to appropriate auditors. One of the objective functions proposed in the literature is to minimize the sum of the mismatch between the level of auditor expertise and level of task difficulty. Other proposed objective functions are cost minimization and to minimize late completion of audit engagements. Constraints proposed include auditor time availability, precedence relationship between audit tasks, and the required due date of each audit engagement.

A. Optimization Models

Reference [28] developed a linear programming model that assigns audit tasks to audit staff. The decision variables are number of hours the i^{th} auditor should work on the j^{th} audit activity. Constraints make sure that an auditor is not assigned for more time than his/her available time. The drawback of this model is that it does not take the precedence relationship between activities into considerations. Reference [29] developed a zero one integer linear programming model for the audit staff assignment problem. The problem assigns audit

Open Science Index, Economics and Management Engineering Vol:9, No:6, 2015 publications.waset.org/10002029/pdf

tasks to auditors. The decision variables are binary; a variable assumes a value of 1 when a task is assigned to a specific auditor and 0 otherwise. They used constraints to impose proper utilization of auditors. Other constraints make sure that each audit task is completed by one auditor. Their objective function minimizes the total mismatch of the audit engagement. Each auditor is ranked from 1 to 5 (example 1 = average, 2 = good, 3=very good, 4 = excellent, and 5 = outstanding). Each activity in the audit engagement is also ranked based on its difficulty level. The mismatch factor coefficient c_{ij} when audit task j is assigned to auditor i is the difference between the auditor rank m_i and activity level of difficulty n_j. The disadvantage of this model is that it does not also take into considerations the precedence relationships between audit activities, audit due dates, penalty costs and resource leveling.

Reference [30] has extended the mathematical model presented in [29] to include the precedence relationships between activities. The extended model is also a zero-one integer linear programming model. Two sets of decision variables were proposed. The first set of decision variables has binary variable taking values of 1 if auditor i completes task j at the end of period k. The other set has binary variables assuming values of 1 if all tasks of engagement g are completed by period k. Their objective function minimizes the sum of the mismatch described before and the penalty cost of completing an engagement after its assigned deadline. Their constraints consider: auditor availability, each audit task is done by one auditor, precedence relationship between activities.

Reference [31] considers the case when the audit scheduling problem involves auditors with different efficiencies; several audit engagements at different locations and different engagement due dates. The problem was formulated as a zero-one integer linear programming problem. The objective function of the model is the sum of mismatching cost; penalty cost when an engagement is completed after due date as well as the regular audit cost. The constraints are chosen to make sure that each auditor work time is in a specified range; each auditor task is audited by one auditor; an audit task can start when its predecessors are done; an auditor cannot process more than one task at a time, audit set up cost is incurred whenever an auditor switch from one task to another one. Due to the large number of decision variables and constraints, finding an exact solution is difficult. They proposed a heuristic procedure based on the Tabu search method to solve the problem. Reference [32] developed a decision support system for audit staff scheduling problem when there is multiple and large scaled engagements. The main objective is audit cost minimization. The constraints are to best utilize available resources. The advantages of developing a decision support system is the ability for the user to change the constraints as required in each situation.

B. PERT/CPM Techniques

Due to the precedence relationships that exist between audit activities, some authors employed the BERT/CPM techniques and used the project management Network to model the audit scheduling problem. Reference [33] extended the PERT/CPM method to be used in internal audit planning. Reference [34] used the project management network analysis in audit planning. Reference [35] extended the idea of project cost analysis to include in audit planning. Reference [30] extended the integer linear program presented in [29] as described earlier in the article to include the activity presentence relationships as in PERT/CPM networks. References [36], [37] considered the case in which an activity does not have to wait until all its predecessors are done. This overlap is required to speed up the auditing process, help making good use of the audit staff which leads to the increase of the audit efficiency. There are four types of lead/lag relationships between audit activities exist in project management literature [38], [39]. If we assume that activity j follows activity i. Finish–to-start relationship (FS_{ij}) is the one identifies the minimum number of time units activity j can start after the completion of activity i. When $FS_{ij} = 0$, this case is known by zero lag as specified in [40]. This case is similar to the one used in PERT/CPM. Start-to-Start (SS_{ij}) identifies the minimum number of time units activity i must complete before the start of activity j. Finish-to-finish FF_{ij} identifies the minimum number of time units that must remain to be completed on j after the completion of i. Start-to-Finish SF_{ij} is the minimum number of time units that must pass from the start of i to the completion of j. Reference [36], [37] have considered both FS_{ij} and SS_{ij}. If any other relationship is required, they propose to transfer it to one of the first two types. They proposed one of two ways to analyze the project networks. The first approach is followed if splitting activities are allowed; each overlapping activity will be broken into two activities. The other approach is followed when splitting activities are not allowed and they proposed to follow the methodology presented in [41]. Following either of the two approaches, early as well as late activity completion time is determined for each activity in the network of all audit engagements. This research also considers the travel time and cost involved when an auditor switches from an audit task to another one in a different engagement. Reference [42] modelled each audit engagement problem as a PERT/CPM network. He was able to combine all firm engagements into one single network. Their model can allow zero lags between tasks or overlap tasks. The problem considered is to identify the schedule of each auditor taken into consideration the auditor's preferences as much as possible. The model also finds each audit engagement schedule taken into consideration the client objectives.

IV. Internal Audit Resource Allocation

This section presents an alternative modelling and solution approach for the audit planning problem. This approach determines the optimal amount of audit resources (audit budget or audit time) to allocate to each auditable unit as discussed in the following sections. Reference [43] used both the factor analysis and regression analysis techniques to study

Open Science Index, Economics and Management Engineering Vol:9, No:6, 2015 publications.waset.org/10002029/pdf

the factors that impact the resource allocation decisions in auditing. Their results revealed that the total assets, total revenue, client status (public/private) are the primary factors in audit resource allocations.

A. Single Objective Optimization Models

1. Allocation of Audit Budget

Reference [2] presented a constrained optimization model to determine the optimal allocation of audit budget among divisions or units such that the firm's expected losses due to intentional and unintentional errors are minimized. They suggested dividing the firm into a number of auditable units. The risk factor of each unit was obtained as the weighted sum of the individual risk factors within the unit (factors such as quality of internal control, integrity of management, economic conditions, etc.). The objective of the model was to minimize the total losses for the firm; and, the decision variables were the percentages of audit budget allocated to audit units.

References [44], [45] provided a list of factors which can be used to assess the risk of each auditable unit in any particular organization. Reference [46] provided a list of 53 factors to help assess the management fraud risk. The information extracted from the list can be used to guide internal auditors in the allocation of audit resources. Reference [47] showed that the likelihood of detecting errors and fraud will be higher if audit resources are directed to high risk units. Reference [48] developed an optimization model to allocate internal audit resources with the objective of minimizing the detection risk associated with employee theft. The main reason they focused on employee theft is that seventy percent of retail fraud is due to employee theft [49]-[51]. They considered numerous asset types from which theft may occur. In conducting an audit, the auditors have the following choices: (1) sample from each asset type that is controlled by a specific employee, (2) randomly sample across asset types, or (3) focus on a few asset types based on the asset value and its auditing cost. Ref. [48] assumed a single (dishonest) employee controls J asset types, and that employee may steal a fraction of asset j which has a value of v_j. The model determined the optimal audit budget to be allocated to asset j as:

$$\frac{k}{c} v_j a_j$$

where a_j is the cost of auditing one unit of asset j, k is the total audit budget, and c is the required budget if all assets are audited.

2. Allocation of Audit Time

Reference [52] employed the audit planning model in [2] to determine the optimal audit time allocation in Janseen Pharmaceutical group of companies. They divided Janseen into fifty-seven independent auditable units and used the Delphi method [53] to collect data and reach consensus about the relative risk factors of various auditable units. The Delphi method involved the formation of a panel of internal auditors, who were asked individually to assess the risk factor of each unit in the firm. Their responses were summarized and redistributed to the internal auditors for a possible modification of their previous inputs. This process was repeated until all auditors agreed on the final results, which were used as the relative risk scores.

Reference [52] presented an optimization model to determine the optimal allocation of audit time to each unit in the firm. Three levels of audit times were considered for each unit. The first level was based on a *limited review* involving interviews with management, but no detailed testing. The second level was based on an *intensive review* and the third level entailed a *complete review* of all cycles. For instance, auditable unit 1 in Janseen required 170 man-hours for limited review; 378 man-hours for intensive review; and 912 man-hours for complete review. A zero-one integer linear programming model was developed to identify the optimal audit level for various units with the objective of minimizing the total risk, subject to the available total audit time. Results revealed that out of 57 units, 44 needed to be audited and the level of audit for each unit was identified. The difficulty of this approach was embedded in the estimation of risk reduction values needed in the objective function of the model.

Reference [54] divided the labour involved in audit activities into grades such as partner, manager, etc. They defined the internal audit time to be the time of each grade of labour charged to audit activities. Reference [55] developed a linear programming model to optimize the allocation of internal audit time among several audit projects (auditable units). The model was employed to select the best subset of projects for audit in a multinational Midwest- based financial service company. The audit staff was comprised of an associate general auditor, an audit director, two senior auditors, and six staff auditors. The company had fourteen types of audit projects. Since the company had four expertise levels and fourteen types of audit projects, the model included fifty-six integer variables. Variables represented the number of audit-hours that an auditor of a specific level should allocate to a particular type project. Fifty-one constraints were included to impose technological limitations, resource restrictions, and policy mandates. The objective function was formulated to maximize the total benefits provided by the internal audit department. Coefficients of the objective function represented the benefits of assigning a particular audit staff at a given level to a particular audit project. The final results showed the majority of the audit time allocated to financial operation audits should be conducted by staff auditors. Adequate hours were assigned to each of the three more experienced levels to ensure high audit quality.

B. Multiple Objective Decision Making Models

Several authors developed multiple criteria linear programming models for the resource allocation problem in auditing. Reference [56] developed a goal programming model to identify the optimal audit staff assignment problem considering a one year planning horizon. The model identifies the optimal number of audit partners, audit managers, audit

Open Science Index, Economics and Management Engineering Vol:9, No:6, 2015 publications.waset.org/10002029/pdf

seniors and audit staffs required to perform an audit job. It also identifies the new optimal hourly billing rate for each audit partner, manager, senior, and audit staff. The model focus only on the audit function and does not take into considerations the other functions performed by the firm such as tax and management services. Reference [57] developed an interactive multi-objective model for the CPA firm audit staff planning problem. They proposed several objective functions such as maximize profit, minimize underutilization of high skills audit staff, and achieve professional development goals. They assumed that the firm audit staff consists of 6 levels: Partner A who has the highest level, Partner B, Manager, staff auditor A, Staff auditor B, and Staff auditor C. They formulated the objects into goals.

Reference [58] developed a multicriteria decision making approach to solve the audit workload problem. The model gives the decision maker (DM) the chance to participate during the computer solution process in an interactive way to articulate his/her preferences until reaching the best compromise solution. Reference [59] developed a multicriteria decision making model with objective functions such as: profit, late work completion, declined work, staff reduction, and work underutilization. Reference [60] proposed a general resource allocation method which can be employed in audit planning. The method was developed to allocate resources in an organization with a centralized decision making environment, where the objective is to maximize the total outputs of all units. In audit planning, the output can be considered as the benefits (savings) due to auditing each unit, as proposed by [55]. Reference [60] employed the multi-criteria decision making approach to incorporate the decision maker's preference information concerning the relative importance of the outputs of different units. They presented a multi-objective linear programming (MOLP) model to find the most preferred resource allocation plan. The solution to this model can be obtained using an appropriate MOLP algorithm such as the one suggested by [61].

Reference [62] considered the problem of determining the optimal allocation of internal auditing time among competing projects. They proposed a multi-criteria decision making model with qualitative and quantitative factors. The analytic Hierarchy process (AHP) was employed to deal with qualitative risk assessment and the Goal programming model was used to distribute available auditing time among the projects in a way that leads to minimum risk. The proposed decision variables are the number of hours of audit time needed to be allocated to each project as well as some binary variables to help select the projects to be audited if the available number of auditing hours is not enough to include all projects. The objective function of the Goal programming model is the weighted sum of the deviation variables that should be minimized.

C. Models Based on the Interactions between the Auditors and Clients

Reference [63] presented a model to the internal audit planning problem using a game theoretic framework that incorporated the interactions between the auditors and clients. In their research, they viewed the audit planning problem as a conflict between two strategic players: the auditor and the client. Due to the anticipatory effect which occurs if the client cannot anticipate the time when his or her unit will be audited, auditors may influence the behaviour of clients, which discourages the abuse of policy, fraud and other intentional errors. The authors presented a table to demonstrate the auditor-client game in strategic form. In the table, the rows referred to all possible auditor strategies and the columns referred to client strategies. Each cell in the table contained the payoffs (auditor payoff as well as client payoff) resulting from different strategies. Then they reduced the table by eliminating the rows corresponding to the auditor strategies which were dominated by other strategies. They used this table as a guideline to identify the preferred strategy. The main difficulty with this approach was the estimation of probability coefficients as well as cost figures required to calculate the payoffs in the table. According to [63] game theory applications require considerably more development.

Reference [64] proposed a resource allocation model in which strategic interactions between the auditor and client were taken into consideration. They employed a game theoretic approach to determine the optimal amount of audit resources to allocate to n identical units. Their objective was to minimize the total misappropriation in the organization. The auditor was assigned a fixed budget (B dollars) that had to be completely spent during a given period. In traditional audit resource allocation models where n identical units were involved, an equal amount of resources would be allocated to audit each unit. However, when interactions between the auditor and client are taken into account, different strategies may be considered to audit identical units. For example, the entire audit budget may be allocated to intensively audit one randomly selected unit, or the budget could be dispersed to audit a number of units at different audit levels. In the model, the authors assumed that a client would misappropriate either nothing, a little, or a lot. At each unit, the auditor has the choice of conducting one of three possible audit plans: (1) an intensive audit, (2) a moderate audit, or (3) no audit. The auditor will choose one of these plans and will not disclose it to the client. It was assumed that the probability of detecting the misappropriation would be high if a large amount of asset was misappropriated, or an intensive audit plan was selected. If the misappropriated asset was detected, the client must return the misappropriated asset as well as pay a penalty. The authors highlighted the fact that the result of their model should not be looked at as a unique prescriptive solution of "the game" and emphasized that the values of their modelling approach is to clarify the relationship between optimal audit plans and audit environment.

V. Conclusion

Among major challenges faced by internal auditors are the decisions on how frequently to audit a division, how to schedule audit staff and how much audit resources to allocate to each auditable unit. Over the past five decades several

mathematical models have been developed to address these issues and identify an optimal audit plan. This paper presents a comprehensive survey of such models as well as the assumptions behind them and their strengths and drawbacks. The information in this paper should assist audit managers to devise an optimal audit plan regarding internal audit timing, the allocation of audit resources and the audit-staff scheduling.

REFERENCES

[1] A. Mohamed, M .Qureshi and A. Behnezhad, "Reliability Evaluation and Design of AICS: A Survey of Models and Experiments," *Review of Accounting and Finance*, vol. 4 no.2, pp. 59-85, 2005.

[2] J. Patton , J. Evans, B. Lewis, "A Framework for Evaluating Internal Audit Risk," *Institute of Internal Auditors, Research Report number 25*, Altamonte Springs, Fl, 1982.

[3] K. Prasad, "Continuous and Periodic Audit," Dynamic Tutorials and Services. www.dynamic tutorials. blogspot.com, 2014

[4] P. Bernstein, "Competition Comes to Accounting," *Fortune*, pp. 88-96, July 17, 1978.

[5] E. Blocher, "Performance Effects of Different Audit Staff Assignment Strategies," *The Accounting Review*, pp. 125 -140, January 1982.

[6] J. Boritz, "Planning for the Internal Audit Function," *Institute of Internal Auditors Research foundation*, 1983.

[7] D. Wilson, and R. Ranson "Internal Audit Scheduling – A Mathematical Model," *The Internal Auditor*, pp. 42-50, July-August 1971.

[8] J. Boritz, and D. Broca,"Scheduling Internal Audit Activities," *Auditing: A Journal of Practice* & *Theory*, vol. 6, no. 1, pp.1-19, 1986.

[9] R. Barefield, "The Impact of Audit Frequency on the Quality of Internal Control," SAR # 1Sarasota, FL: American Accounting Association), 1975.

[10] J. Barzilai, D. Cook, and M. Kress, "A Generalized Network Formulation of the Pairwise Comparison Consensus Ranking Model," *Management Science*, pp. 1007-1014, August,1986

[11] H. David, *The Method of Paired Comparisons*, New York: Oxford University Press, 1988.

[12] S. Ramamootri, A. Bailey Jr, R. Traver (1999) "Risk Assessment in Internal Auditing: A Natural Network Approach," *International Journal of Intelligent Systems in Accounting*, vol. 8, pp. 159-180, 1999.

[13] T. Saaty, "A Scaling Method for Priorities in Hierarchical Structures," *Journal of Mathematical Psychology*, vol. 15, pp. 234-281, 1977.

[14] T. Shibano, "Assessing Audit Risk from Errors and Irregularities," *Journal of Accounting Research*, vol. 28- Supplement, pp. 110-140, 1990

[15] W. Winston, *Operations Research Applications and Algorithms*, 4th edition, Duxbury/Thomson, 2004.

[16] P. Carey and R. Guest, "Determining the Optimal External Audit Interval for Private (and Family Controlled) Companies," *Journal of Accounting, Auditing & Finance*, vo. 15, no. 4, pp. 439-458, 2000.

[17] W. Knechel and H. Benson, "An Optimization Approach for Scheduling Internal Audits of Divisions *Decision Sciences*, vol. 38, no. 9, pp. 1245-1263, 1991.

[18] C. Miller, "The Simplex Method for Local Separable Programming. *Recent Advances in Mathematical Programming*, New York, McGraw Hills, 1963.

[19] R. Morey and D. Dittman, "Optimal Timing of Account Audits in Internal Control," *Management Science*, vol. 32, no. 3, pp. 272-282, 1986.

[20] L. Johnson, R. Leitch, and J. Neter, "Characteristics of Errors in Accounts Receivable and Inventory Audits," *The Accounting Review*, vol. 56, no. 2, pp. 270-293, 1981.

[21] J. Hughes, "Optimal Internal Audit Timing," *The Accounting Review*, vol. LII, no. 1, pp. 56-68, 1977.

[22] R. Rossi, S. Tarim, B. Hinch, S. Prestwich, and S. Karacaer, "Scheduling Internal Audit Activities: A Stochastic Combinatorial Optimization Problem," *Journal of Combinatorial Optimization*, vol. 19, pp.325-346, 2010.

[23] E. Tsang, "Formulation of Constraint Satisfaction," *Academic Press*, London/San Diego, 1993.

[24] P. Van Hentenryck, *Constraint Satisfaction in Logic Programming*, MIT Press, Cambridge, 1989.

[25] J. Flynn, S. Garstka, "A Dynamic Inventory Model with Periodic Auditing," *Oper. Res.*, vol. 38, no. 6, pp. 1089-1103, 1990

[26] T. Bosch, J. Mukuddem-Petersen, M. Petersen, and I. Schoeman, "Optimal Auditing in the Banking Industry," *Optim. Control Appl. and Meth*. DOI: 10.1002/oca.828, 2007.

[27] K. Mizgier and J. Pasia,"Multiobjective Optimization of Credit Capital Allocation in Financial Institutions," *Central European Journal of Operations Research*, March 2015.

[28] E. Summer, "The Audit Staff Assignment Problem: A Linear Programming Analysis," *The Accounting Review*, vol. 47, no. 3, pp. 443-453, 1972.

[29] B. Balachandran and A. Zoltners, "An Interactive Audit-Staff Scheduling Decision Support System. *The Accounting Review*, vol. 56, no. 4, pp. 801-812, 1981.

[30] K. Chan and B. Dodin, "A Decision Support System for Audit Staff Scheduling with Precedence Constraints and Due Dates," *The Accounting Review*, pp. 125-140, January 1986.

[31] B. Dodin, A. Elimam and E. Rolland, "Tabu Search in Audit Scheduling," *Eur. J. Oper. Res.*, vol. 106, pp. 373-392, 1998.

[32] C. Chang, "A Decision Support System for Audit Staff Scheduling of Multiple and Large-Scaled Engagements," *The Review of Business Information Systems*, vol. 6, no. 1, pp. 27-32, 2002.

[33] P. Burgher, "PERT and the Auditor," *The Accounting Review*, pp. 103-120, January 1964.

[34] A. Cirtin, "Network Analysis and Internal Auditing," .*Internal Auditor*, vl. 34, no.1, pp. 27-31, 1977.

[35] C. Rackie, J. Krogstad, G. Grudnitski and D. Bryant, "PERT and PERT/Cost for Audit Planning and Control," *Internal Auditor*, vol. 36, no. 4, pp. 57-68, 1977.

[36] B. Dodin and A. Elimam, "Audit Scheduling with Overlapping Activities and Sequence Dependent Setup Costs," .*Eur. J. Oper. Res.*, vol. 97, pp. 22-33, 1997.

[37] B. Dodin, A. Elimam, "Erratum: Audit Scheduling with Overlapping Activities and Sequence Dependent Setup Costs," .*Eur. J. Oper. Res.*, vol. 104, pp.262-264, 1998.

[38] S. Elmaghraby, and J. Kamburowski, "The Analysis of Activity Networks Under Generalized Precedence Relations (GPR)," *Management Science*, vol. 38, no. 9, pp. 1245-1263, 1992.

[39] J. Moder, C. Phillips, and E. Davis, *Project Management with CPM, PERT and Precedence Diagrams*, Van Nostrand Reinhold, New York, 1983.

[40] S. Elmaghraby, *Activity Networks: Project Planning and Control by Network Models*. Wiley, New York, 1977.

[41] B. Dodin and H. Chan, "Application of Production Scheduling Methods to External and Internal Audit Scheduling," .*Eur. J. Oper. Res.*, vol. 52, pp. 267-279, 1991

[42] B. Dodin, "Project Management in Audit Staff Scheduling," .*International Series in Operations Research and Management Science*, vol. 14, pp. 509-527, 1999.

[43] H. Fukukawa, T. Mock, and A. Wright, "Client Risk Factors and Audit Resource Allocation Decisions," *ABACUS: A Journal of Accounting, Finance and Business Studies*, vol. 47, no. 1, DOI: 10.1111/j.1467-6281.2011. 00332.x, 2011,

[44] L. Noxon, "Taking the Close Look at Your Internal Controls," *The Internal Auditor*, pp. 19-25, June 1980.

[45] H. Siers and J. Blyskal, "Risk Management of the Internal Audit Function," *Management Accounting*, vol. 29-35, February 1987.

[46] B. Green and T. Calderon, "Information Privity and Internal Auditor's Assessment of Fraud Risk Factors," *Internal Auditing*, pp. 4-15, Spring 1996.

[47] B. Lacativo, "A Risk-Based Approach to Auditing for Fraud.," *Internal Auditing*, pp. 41-44, summer 1995.

[48] P. Newman, J. Park and R. Smith, "Allocating Internal Audit Resources to Minimize Detection Risk Due to Theft," *Auditing: A Journal of Practice &Theory*, vol. 17, no. 1, pp. 69-82, 1998

[49] W. Albrecht, G. Wernz, and T. Williams, *Fraud: Bringing Light to the Dark Side of Business*, Richard D. Irwin, Inc., 1995.

[50] W. Albrecht, "Employee Fraud", *Internal Auditor*, pp. 26-37, October 1996.

[51] J. Hayes, "Employee Theft is no. 1 Problem Facing Today's Busy Retailers", *Discount Store News*, vol. 108, September 17, 1990.

[52] D. Miltz, G. Calomme and M. Willekens M (19991) A Risk Based Allocation of Internal Audit Time: A Case Study. *Auditing: A Journal of Practice* &*Theory* 10: 49-61.

[53] T. Spinelli, "The Delphi Decision Making Process. Journal of Psychology," pp. 73-80, January 1983.

Open Science Index, Economics and Management Engineering Vol:9, No:6, 2015 publications.waset.org/10002029/pdf

[54] K. Hackenbrack and W. Knechel, "Resource Allocation Decision in Audit Engagements," *Contemporary Accounting Research*, vol. 14, no. 3, Fall 1997
[55] D. Gotlob, J. Moore, and K. Moore, "Optimizing Internal Audit Resources: A Linear Programming Perspective," *Internal Auditing*, pp. 20-30, Fall 1997.
[56] L. Killough and T. Souders, "A Goal Programming Model for Public Accounting Firms," *The Accounting Review*, vol. XL VIII, no. 2, pp. 268-279, 1973.
[57] K. Balachandran and R. Steuer, "An Interactive Model for the CPA Firm Audit Staff Planning Problem with Multiple Objectives," *The Accounting Review*, vol. 57, no. 1, pp. 125-140, 1982.
[58] J. Gardner, R. Huefner and V. Lotfi, "An Interactive Multi-Criteria Model for Audit Workload Planning," *Lecture Notes in Economics and Mathematical Systems*, vol. 335, pp. 234-243, 1989.
[59] J. Gardner, R. Huefner, and V. Lotfi, "A Multi-Period Audit-Staff Planning Model Using Multiple Objectives: Development and Evaluation," *Decision Sciences*, vol. 21, pp. 154-170, 1990.
[60] P. Korhonen and M. Syrjanen, "Resource Allocation Based on Efficiency Analysis," *Management Science*, vol. 50, no. 8, pp. 1134-1144, 2004.
[61] R. Steuer, *Multiple Criteria Optimization: Theory, Computation, and Application*, Wiley, New York, 1986.
[62] H. Krüger and J. Hattingh, "A Combined AHP-GP model to Allocate Internal Auditing Time to Projects," ORiON, vol. 22, no. 1, pp. 59-76, 2006.
[63] L. Fellingham, D. Newman, "Strategic Considerations in Auditing," *The Accounting Review*, vol. *LX, no.* 4, pp. 634 -650, 1985.
[64] U. Anderson and R. Young, "Internal Audit Planning in an Interactive Environment," *Auditing: A Journal of Practice &Theory*, vol. 8, no. 1, pp. 23-42, 1988.

Abdel-Aziz M. Mohamed (Date of Birth: April 22, 1955) is an Associate Dean of Graduate Studies, College of International Transport and Logistics, Arab Academy for Science, Technology and Maritime Transport. He completed his M.Sc. and Ph.D. degrees in Industrial Engineering from the University of Oklahoma, USA in 1985 and 1990. He worked as an assistant and associate professor at the California State University- Northridge, USA for over 15 years.

He served on several positions (General Secretary, financial officer, and president) in the Southern California Chapter of INFORMS for several years. His publications are in the journals of Reliability Engineering and System Safety, International Journal of Supply Chain and Operations Management, International Journal of Operations and Quantitative Management, and Review of Accounting and Finance. His research interests are in the areas of reliability modeling, supply chain modeling, and Multi Criteria Decision Making.

Open Science Index, Economics and Management Engineering Vol:9, No:6, 2015 publications.waset.org/10002029/pdf