

The Optimal Public Debt Ceiling in Taiwan: A Simulation Approach

Ho Yuan-Hong, Hunag Chiung-Ju

Abstract—This study conducts simulation analyses to find the optimal debt ceiling of Taiwan, while factoring in welfare maximization under a dynamic stochastic general equilibrium framework. The simulation is based on Taiwan's 2001 to 2011 economic data and shows that welfare is maximized at a debt/GDP ratio of 0.2, increases in the debt/GDP ratio leads to increases in both tax and interest rates and decreases in the consumption ratio and working hours. The study results indicate that the optimal debt ceiling of Taiwan is 20% of GDP, where if the debt/GDP ratio is greater than 40%, the welfare will be negative and result in welfare loss.

Keywords—Debt sustainability, optimal debt ceiling, dynamic stochastic general equilibrium, welfare maximization.

I. INTRODUCTION

ECONOMIC growth and the significant advancement of democracy in the late 20th century has lead several nations down the path of increased spending in hopes to sustain and satisfy growing infrastructure and public needs of the general public. Meanwhile, government revenues have not met the increased spending, resulting in significant budgetary deficits. A number of countries have turned to public loan as a measure to sustain government expenditures, further adding weight to their fiscal deficit. Recently due to the proliferation of the global financial crisis and the debt problems suffered by an increasing number of euro area economies have shown the urgency for new fiscal frameworks to prevent unsustainable fiscal policies reoccurring.

Debt sustainability refers to a status where significant change of fiscal policy is not necessary in order for the economy to achieve a dynamic equilibrium. The concept of sustainability relies on the premise that a government needs enough resources to ensure their ability to carry out their functions, that is, if the government revenue can meet its spending and pay off its debt so that the government can be functioning normally, the public debt will be sustainable. If the public debt level is sustainable, a country can service debt at this level with tax revenue and the public debt-to-GDP ratio can be maintained. Analyzing sustainability helps determine whether a current fiscal policy can be maintained in the long run given a government's ongoing ability to generate fiscal resources. A non-sustainable fiscal policy potentially creates a risk of rising future interest rates that leads to a slowdown in economic growth.

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Empirically Public debt sustainability is measured by whether the intertemporal solvency condition is satisfied. A necessary and sufficient condition for the intertemporal solvency condition to hold is a stationary discounted stock of public debt. Another necessary condition for the sustainability of public debt is the cointegration between government expenditure and tax revenue. References [1]-[4] focus on the stationarity of government debt or deficit and adopt unit root tests to examine whether government debt or deficit is stationary. Others such as [5]-[7] have concentrates instead on the long run relationship between government expenditure and tax revenue and employs cointegration tests to examine whether the observed data are consistent with this requirement. The stationary tests by using traditional unit root tests assume public debt behaves under a continuous and constant speed adjustment process. However, facing the possibility of public debt, adjustments may very well be asymmetric. References [7]-[11] have used a nonlinear model to examine the sustainability of public debt given that traditional unit root tests are inadequate when public debt exhibits a threshold behavior. References [12]-[14] and others have identified nonlinear relationships between public debt and economic growth. Such relationships imply an optimal level of debt, in the sense of growth maximizing.

Aiyagari [15], on the other hand, incorporates a different role for government debt to capture different trade-offs between the benefits and costs of varying the quantity of debt, and then calculates the optimum quantity of risk free public debt and the welfare costs of being at levels other than the optimum. Aiyagari [15] finds that the optimum quantity of debt for the United States over the post-second world war period is 2/3. Moon [16] estimated Korean optimal public debt ratio by weighing benefits against costs of public debt. The optimal public debt is defined as the level that maximizes social welfare for the entire economy. Moon's [16] calculation showed that projected public debt-to-GDP ratio of Korea is likely to stand at 52.8 percent in 2020, and it will likely rise to 67.8 percent in 2030 and further increase to 113.3 percent in 2050. Röhrs and Christoph [17] compute the welfare-maximizing level of government debt of an economy in which households are subject to uninsurable income shocks. Their computation shows that the level of government debt that maximizes aggregate steady state welfare is significantly negative. This implies that the high debt levels that are currently observable in most developed countries will decrease welfare in the long-run. Since these studies are purely empirical, we have little idea what the optimal level of debt depends on. Hence, empirical

studies with different optimal criteria and fiscal regimes are still needed before getting a conclusive result.

In the past ten years, the debt of Taiwan has accumulated rapidly and the nation's financial status has worsened daily. Taiwan's government had experienced deficits each year with the exception of having a slight government surplus in 1998. Therefore, the general government debt as a percentage of GDP dramatically increased from 25.35% in 1997 to 40.03% in 2014 (in Fig. 1). According to the Global Competitiveness Report 2013-2014 published by the World Economic Forum, although Taiwan ranks 12th overall out of 148 economies, Taiwan's government budget deficit is ranked at 91 while its government debt is ranked at 69. Therefore, the aim of this study is to investigate whether Taiwan's public debt is sustainable under a certain debt ceiling.

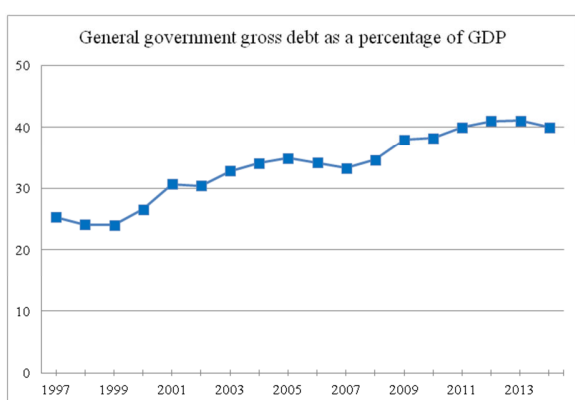


Fig. 1 Taiwan's public debt ratio

In the light of public debt, the issue of debt sustainability has increasingly attracted attention. Taiwan has recently adjusted her debt ceiling from 48% of debt to GDP to 50% of debt to GDP. Debates on how to set up a suitable debt ceiling are launched in Taiwan. Theoretically there is no certain way to determine how to set up a specific debt ratio for a specific country, however empirical work might be helpful to do the job. In this study we investigate debt sustainability by using a dynamic stochastic general equilibrium framework. Here, sustainable debt means repayable debt. The optimal level of public debt is the sustainable level of debt that can maximize the social welfare of the entire economy. We then use MATLAB software to conduct simulation analyses to find the optimal debt ceiling, that is, the largest value of public debt of Taiwan under the welfare maximization. This paper is organized as follows. Section two describes the methodology used in this study, section three discusses the simulation results, and section four presents our conclusions.

II. METHODOLOGY

The analytical model used for the simulation is an augmented version of the model in [18]—augmented to permit growth and to include government debt, proportional income tax, and government consumption. This model incorporates a different role for government debt than standard, deterministic growth models do and captures different trade-offs between the

benefits and costs of varying levels of debt. While permits intergenerational transfers to be negative as well as positive, this mode does not permit the family as a whole to carry negative financial assets from one period to the next. Most aspects of this model can be parameterized in the same way that the representative agent growth model has been parameterized for quantitative analysis of growth. The only aspect different from the representative agent growth model is the stochastic process that governs the idiosyncratic labor productivity shocks. This study parameterized Aiyagari model using data for the Taiwan economy, and then compute the welfare-maximizing level of government debt. The rest of this section is organized as follows. First we describe the analytical model used in this study. We then describe the parameter values for the analytical model.

A. The Model

The final output is produced with a constant return to scale technology uses physical capital and labor as inputs. The technology is assumed to take the Cobb-Douglas form:

$$Y_t = AK_t^\alpha L_t^{1-\alpha} \quad (1)$$

Y_t is the per capita output, K_t is per capita capital, L_t is per capita labor input, and A is the rate of technological progress and is assumed to be constant. α , $1 - \alpha$ are positive fractions of inputs devoted to the production of final outputs. The final output sector, the labor market, and the physical capital market are perfectly competitive. Given the wage rate and the price of production inputs, final output producers choose how much labor and production inputs to employ. The wage rate w_t and interest rate r_t are then given by

$$w_t = (1 - \alpha)AK_t^\alpha L_t^{-\alpha} \quad (2)$$

$$r_t = \alpha AK_t^{\alpha-1} L_t^{1-\alpha} - \delta \quad (3)$$

δ is the depreciation rate of capital. Along the balanced growth path, all the per capita variables will be growing at constant rate g , which is the annual growth rate of per capita gross domestic product. Whereas the interest rate will be constant (i.e., $r_t = r$ for all t).

The government finances public expenditure using taxation on labor income and interest income. With these distorting taxes there is a role for government debt as a means of smoothing tax distortions over time. Assumes the proportional tax rate is τ_y , and let $\tilde{w} = \frac{w_t}{Y_t}$, $\tilde{r} = \frac{r_t}{Y_t}$, we have

$$\tilde{w} = (1 - \tau_y)(1 - \alpha)AK_t^\alpha L_t^{-\alpha} = (1 - \tau_y)\tilde{w} \quad (4)$$

$$\tilde{r} = (1 - \tau_y)\alpha AK_t^{\alpha-1} L_t^{1-\alpha} - \delta = (1 - \tau_y)r_t \quad (5)$$

Assume that the economy consists of a large number of infinitely lived agents who receive idiosyncratic shocks to their labor productivities and supply labor elastically. Denotes e_t as an individual's labor productivity and which is i.i.d. across agents and follows some Markov process over time. For

simplicity, normalize per capita labor productivity to unity so that $E(e_t) = 1$. When financial markets are incomplete, household's saving is influenced by precautionary motives and borrowing constraints. Let C_t , a_t and TR_t denote an individual's consumption, asset and government transfer payments in period t respectively. Individual's utility is given by $c_t^{1-\nu}/(1-\nu)$, where $\nu > 0$ is the relative risk aversion coefficient. The consumer's optimum choice with discount factor β is

$$\max_{\{c_t, a_{t+1}\}} E \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\nu}}{1-\nu} \mid a_0, e_0 \right] \quad (6)$$

$$\text{s.t. } (1 + \hat{r})a_t + \hat{w}e_t + TR_t \geq C_t + a_{t+1} \quad (7)$$

$$C_t \geq 0, a_t \geq 0, t \geq 0$$

With $a_t \geq 0$, the model rules out the possibility of borrowing. In a balanced growth economy, per capita consumption, per capita asset holding and per capita government transfer payments will be growing at the rate g . For analytical convenience, transform variables into a stationary form. Let $\tilde{c}_t = \frac{C_t}{Y_t}$, $\tilde{a}_t = \frac{a_t}{Y_t}$, $\mathcal{X} = \frac{TR_t}{Y_t}$, and rewrite the consumer's optimum problem as:

$$\max_{\{\tilde{c}_t, \tilde{a}_{t+1}\}} E \left[Y_0^{1-\nu} \sum_{t=0}^{\infty} [\beta(1+g)^{1-\nu}]^t \frac{\tilde{c}_t^{1-\nu}}{1-\nu} \mid \tilde{a}_0, e_0 \right] \quad (8)$$

$$\text{s.t. } (1 + \hat{r})\tilde{a}_t + \hat{w}e_t + \mathcal{X} \geq \tilde{c}_t + (1 + g)\tilde{a}_{t+1} \quad (9)$$

$$\tilde{c}_t \geq 0, \tilde{a}_t \geq 0, t \geq 0$$

In response to the changes of government debt, consumer can decide how much of his time to work and how much to leisure to mitigate the impacts. Let ℓ_t denote per capita leisure at time t , η denote the relative share of consumption, thus the consumer's problem becomes

$$\max_{\{c_t, \ell_t, a_{t+1}\}} E \left[Y_0^{\eta(1-\nu)} \sum_{t=0}^{\infty} [\beta(1+g)^{\eta(1-\nu)}]^t \frac{(c_t^{\eta} \ell_t^{1-\eta})^{1-\nu}}{1-\nu} \mid \tilde{a}_0, e_0 \right] \quad (10)$$

$$\text{s.t. } (1 + \hat{r})\tilde{a}_t + \hat{w}e_t(1 - \ell_t) + \chi \geq \tilde{c}_t + (1 + g)\tilde{a}_{t+1} \quad (11)$$

$$\tilde{c}_t \geq 0, \ell_t \geq 0, \tilde{a}_t \geq 0, t \geq 0$$

We now describe government behavior. Let G_t denote per capita government consumption, B_t denote per capita government debt. Then the government budget constraint is given by

$$G_t + TR_t + (1 + \hat{r})B_t = B_{t+1} + \tau_y(Y_t - \delta K_t) \quad (12)$$

Let $\gamma = \frac{G_t}{Y_t}$, $\chi = \frac{TR_t}{Y_t}$, $b = \frac{B_t}{Y_t}$, and $\ell = \frac{K_t}{Y_t}$. Note that in a balanced growth equilibrium, γ , χ , b and ℓ will also be growing at a constant rate. Rewrite government budget constraint as

$$\gamma + \chi + (\hat{r} - g)b = \tau_y(1 - \delta\ell) \quad (13)$$

Assume the government is benevolent, and will act to maximize consumer welfare. Then, the consumer's optimum problem becomes

$$\max_{\{\tilde{c}_t, \ell_t, \tilde{a}_{t+1}\}} E \left[Y_0^{\eta(1-\nu)} \sum_{t=0}^{\infty} [\beta(1+g)^{\eta(1-\nu)}]^t \frac{(\tilde{c}_t^{\eta} \ell_t^{1-\eta})^{1-\nu}}{1-\nu} \mid \tilde{a}_0, e_0 \right]$$

$$\text{s.t. } \gamma + \chi + (\hat{r} - g)b = \tau_y(1 - \delta\ell)$$

$$\tilde{c}_t \geq 0, \ell_t \geq 0, t \geq 0$$

B. Parameterization of the Model

All of the parameter values that we used are taken from 2001 to 2011 period. Since the model period is specified to be one year, so we use arithmetic mean of the parameter values over the 2001-2011 period to parameterize the model. All values are obtained or are computed from Year Book of Population Statistics of Taiwan, Year Book of Multifactor Productivity Statistics of Taiwan, Year Book of Financial Statistics of Taiwan and Taiwan Economic Journal Database (TEJ). By so doing, the per capita growth rate g is set equal to 0.04 per year.

The relative share of total product accruing to labor $(1-\alpha)$ is set equal to 0.51, the capital share α is therefore 0.49. The capital output ratio ℓ is 2.65. The estimate of the depreciation rate δ is 0.167. In equilibrium, $\delta = \frac{\alpha}{\ell} - r$, it implies $r = 0.018$.

The ratio of government debt to GDP is $b = 0.347$. The ratio of government transfers to GDP \mathcal{X} is set equal to 0.06 and the ratio of government purchase γ is set equal to 0.138. The discount factor $\beta = \frac{1}{1+i}$, i is the rate of return for bond in Taiwan, the estimate of i is 1%, β is then equal to 0.99. For the risk aversion parameter ν we use a value of 1.09 which is in line with other estimates in the related literature of Taiwan. In this study, realistic working hours were used to stand for per capita labor input N . Employee's monthly working hours of Taiwan are taken from TEJ Database, it was then standardized by 720 hours to get value of N . In this study $N = 0.25$.

Individual's labor productivity e_t is i.i.d. and follows some Markov process over time. Let $\ln e_{t+1} = \rho \ln e_t + \eta_{t+1}$. ρ is the first order autocorrelation coefficient, η is a disturbance term and assume $\eta_{t+1} \sim N(0, \sigma^2)$. The estimates, $\hat{\alpha}_1$ and $\hat{\varepsilon}_t$ of the real wage rate (rwage) equation, $\ln rwage_t = \alpha_0 + \alpha_1 \ln rwage_{t-1} + \varepsilon_t$, are used as the proxies of ρ and σ^2 . Regression results show that $\rho = 0.55$, $\sigma = 0.02$.

With Cobb-Douglas type production function, we have

$$\alpha = (r + \delta)\ell \quad (14)$$

Given values of α , δ and ℓ , we get $r = 0.018$. We can rewrite (14) as $\delta = \frac{\alpha}{\ell} - r$, and plug into (12), we get

$$\tau_y(1 - \alpha) + \tau_y r \ell + gb = \gamma + \chi + (1 - \tau_y)rb \quad (15)$$

Given values of α , r , k , g , b , γ and X , the income tax rate τ_y is set equal to 0.338.

Since $N = E(e_t(1 - \ell_t))$. Normalized e_t to unity so that $E(e_t) = 1$, we have $N = 1 - E(e_t \ell_t)$. $E(e_t \ell_t)$ is set equal to 0.75, given $N = 0.25$.

As $\widehat{w}N = (1 - \tau_y)(1 - \alpha)$, \widehat{w} will be equal to 1.35 when values of N , τ_y and α are given. The resource constraint for the economy implies that per capita consumption is $E(\tilde{c}_t) = 1 - \gamma - (g + \delta)k$, $E(\tilde{c}_t)$ equals to 0.313. If we assume an interior solution, the first order condition for the optimum choice between labor and leisure is $\ell_t = (1 - \eta)\tilde{c}_t/\eta\widehat{w}e_t$, $E(e_t \ell_t) = (1 - \eta)E(\tilde{c}_t)/\eta\widehat{w}$. Using the derived values of $E(e_t \ell_t)$, $E(\tilde{c}_t)$, and \widehat{w} , the value of η is then 0.236.

III. RESULTS

The simulated effects of various debt/GDP ratio from 0.1 to 0.6 on interest rate, working hours, tax rate, per capita consumption, and welfare gain are shown in the Table I. The results are also reported in Fig. 2. Table I shows that the benchmark debt/GDP ratio, which the welfare gain is set to zero, is 0.347. As the public debt ratio increased beyond benchmark ratio, and reach 0.4, welfare gain will turn to negative. It can be seen from Table I, the welfare maximizing optimal debt/GDP ratio is 0.2. The optimum per capita consumption is approximately 0.431, the interest rate at the optimum is approximately 6.69%, with the income tax rate equal to 30.65%.

TABLE I
 EFFECTS OF DIFFERENT DEBT/GDP RATIO

	Debt/GDP Ratio						
	0.1	0.2	0.3	0.347	0.4	0.5	0.6
Interest rate	6.6	6.69	7.39	7.4	7.41	7.42	7.44
Work hours	0.2023	0.2019	0.198	0.1977	0.1975	0.1969	0.1964
Tax rate	30.55	30.65	30.5	30.58	30.66	30.82	30.98
Consumption	0.4313	0.4306	0.4389	0.4386	0.4382	0.4375	0.4368
Welfare gain	0.85974	0.85977	0.01235	0	0.00257	-0.01744	-0.02753

Remarks: The optimum debt/GDP ratio in this study is 0.2.

As the public debt ratio increases, the increase in debt crowds out private investment, and reduced households precautionary saving, resulting a lower capital stock, and hence slowing down economic growth. Form (14), decrease in the capital output ratio k will raise the interest rate, which then leads to the debt crisis, because the higher interest rates may make the government unable to pay back the debt. From (13), as a result, there is an increase in the tax rate, which creates another negative effect on economic growth.

Fig. 2 shows the graph of the interest rate, the aggregate hours, the income tax rate, the consumption, and the welfare gain versus the different debt/GDP ratio for Taiwan. The plot of welfare shows that the maximal welfare gain happens at the public debt ratio of 0.2, implies that the optimum debt/GDP ratio is 0.2 for Taiwan. Fig. 2 also shows that when the debt/GDP ratio rise, the interest rates and the tax rate are showing upward trend, whereas, household working hours and

consumptions are showing downward trend. This may be because the increase of public debt raise the interest rates, and hence decrease the bonds prices, households then increase their holdings of public debt, feel wealthier, and thus works less and leisure more. The upward trend or downward trend is more pronounced when the debt/GDP ratio rises above the benchmark level of 0.347. When government continues to finance public expenditures with debts, people no longer have fiscal illusion, thus reducing their consumption. Meanwhile, when the public debt ratio gradually increased, the self-insurance function of bonds getting weaker, and unable to effectively reduce liquidity constraints, household increase savings for precautionary motives and therefore reduce consumption.

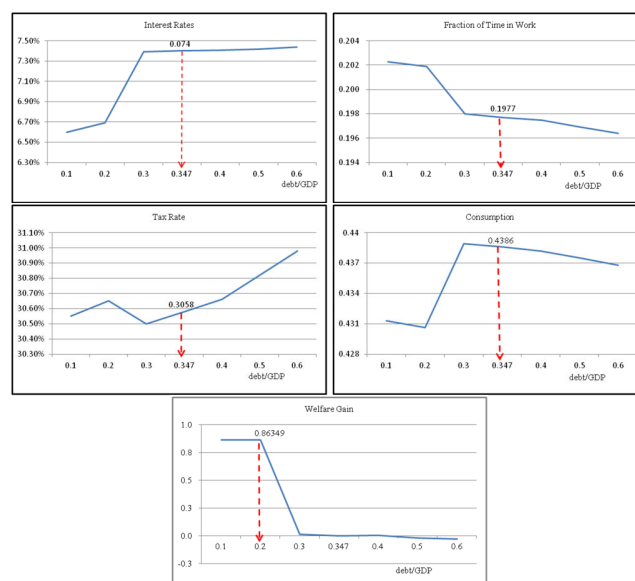


Fig. 2 Interest rate, work hours, tax rate, consumption, and welfare gain for the benchmark economy

IV. CONCLUSIONS

Due to increasing public debt, debt sustainability has become a significant topic of interest. If a country has a sustainable public debt level, she can continue to service debt at that public debt level while maintaining tax revenue and the public debt-to-GDP ratio.

Taiwan recently adjusted her debt ceiling from 48% of debt to GDP to 50% of debt to GDP, sparking heated debates among politicians and scholars on how to set up a suitable debt ceiling. This study investigates debt sustainability of Taiwan by using a dynamic stochastic general equilibrium framework. In this study, the optimal level of public debt is defined as the sustainable level of debt that can maximize the social welfare of the entire economy. We find that the negative role of public debt including crowding out private capital, distorting labor supply and precautionary saving decision through higher taxes will outweigh the positive role of enhancing liquidity that public debt plays, as the debt/GDP ratio of Taiwan reaches 40%. Based on our calculation the estimated optimal public debt ratio of Taiwan, by weighing benefits against costs of

public debt, is 20 percent. Our finding also shows that the benchmark debt to GDP ratio, which the welfare gain is set to zero, is 0.347. This benchmark ratio is much smaller than the current level of Taiwan, therefore, argues for raise the debt ratio in Taiwan may be misleading.

To ensure sustainability of public debt, Taiwan should pay more attention to the increase in the public debt ratio and be cautious of any increase of her public debt ratio as it cumulatively reaches more than 40%. To prevent this from happening, government authorities should not only refrain from adjusting her debt ceiling up to 50% of GDP, but also actively cut the debt ceiling down to 40% of GDP.

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