

# Sampled-Data Control for Fuel Cell Systems

H. Y. Jung, Ju H. Park, S. M. Lee

**Abstract**—Sampled-data controller is presented for solid oxide fuel cell systems which is expressed by a sector bounded nonlinear model. The proposed control law is obtained by solving a convex problem satisfying several linear matrix inequalities. Simulation results are given to show the effectiveness of the proposed design method.

**Keywords**—Sampled-data control, Sector bound, Solid oxide fuel cell, Time-delay.

## I. INTRODUCTION

SOLID OXIDE FUEL CELL (SOFC) has attracted considerable interest as it offers wide application ranges, flexibility in the choice of fuel, high system efficiency and rapid load following capability [1]-[4]. In this regard, during the last several years, many researches have been investigated the dynamic model for simulating transient behavior as well as designing model-based controllers of the SOFC systems. In [1], a mathematical dynamic model for SOFC stack is simply presented. Its detailed model with temperature dynamics is developed in [5]. It is important to establish control over the fuel cell voltage because of its heavily nonlinear behavior and dependence on disturbances such as load current and inlet temperatures. However, these mathematical models have a difficulty on the controller design due to its complexity. Thus, based on the experimental input-output data, numerical modeling techniques such as support vector machine, T-S fuzzy, and neural network have been proposed in [3], [4]. In [3], a model predictive control method for SOFC systems is presented by use of a simplified dynamic model which is obtained by subspace identification method.

The previous studies have been conducted by use of input-output data based model for control of SOFC systems. It should be pointed out that sampled-data control scheme is very useful since it is possible to handle digital controller and increasing research efforts have been devoted to sampled-data control systems with the development of modern high-speed computers, microelectronics, and communication networks. Recently, the control technique has been applied to the SOFC systems due to its slow dynamics and tight operating constraints [4], [5]. In this paper, we consider a sector bounded nonlinear model to handle SOFC systems and propose a

sampled-data control method for the systems. The sector bounded nonlinear systems, which have a feedback connection with a linear dynamical system and nonlinearity satisfying certain sector type constraints, have been extensively studied in control theory research area. Since the pioneer work of Lur'e was presented in 1940', the notion of absolute stability has played an important role in stability analysis and controller design problems [6].

To the best of authors' knowledge, there are no approaches considering sampled-data for SOFC systems. The proposed nonlinear control method guarantee closed-loop stability, a stabilization criterion is proposed in terms of LMIs which can be solved very efficiently by convex algorithms [8].

Finally, we demonstrate the effectiveness of the proposed approach via numerical simulations.

## II. PROBLEM STATEMENT

Consider the dynamics of an SOFC system [2] as shown in Fig. 1 and assume that all states are available. As a widely adopted dynamic model of the SOFC system, the average voltage magnitude of the fuel cell stack is determined by the partial pressure of hydrogen, oxide and water.

Table I contains the parameters of the SOFC model. Applying Nernst's equation, the output of the SOFC can be modeled as:

$$V_0 = N_0 \left[ E_0 + \frac{R_0 T_0}{2F_0} \ln \frac{p_{H_2} (p_{H_2}/101.325)^{\frac{1}{2}}}{p_{H_2O}} \right] \quad (1)$$

where  $p_{H_2}$ ,  $p_{O_2}$ ,  $p_{H_2O}$  are partial pressures of hydrogen, oxygen and water. Also, the dynamics of fuel processor and the stack voltage are:

$$\dot{q}_H = \frac{1}{\tau_H} (q_F - q_H) \quad (2)$$

$$\dot{V}_S = \frac{1}{\tau_C} (V_0 - V_S - rI_D) \quad (3)$$

where  $q_H$  is the hydrogen flow rate,  $q_F$  is inlet fuel flow rate,  $V_S$  is stack voltage and  $I_D$  is external current load that is assumed to be not changed.

The dynamic equation of the partial pressure inside the channel of hydrogen, oxygen and water are as:

$$\begin{aligned} \dot{p}_{H_2} &= \frac{1}{\tau_{H_2} K_{H_2}} (q_H - 2K_r I_s - p_{H_2} K_{H_2}), \\ \dot{p}_{O_2} &= \frac{1}{\tau_{O_2} K_{O_2}} \left( \frac{q_H}{\tau_{H-O}} - K_r I_s - p_{O_2} K_{O_2} \right) \\ \dot{p}_{H_2O} &= \frac{1}{\tau_{H_2O} K_{H_2O}} (2K_r I_s - p_{H_2O} K_{H_2O}), \end{aligned} \quad (4)$$

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Then, the nonlinear function  $f(v(t))$  and  $\dot{f}(v(t))$  can be expressed as:

$$f(v(t)) = \Lambda v(t), \quad \dot{f}(v(t)) = \bar{\Lambda} \dot{v}(t), \quad (11)$$

and the parameters  $\Lambda, \bar{\Lambda}$  belong to the following set:

$$\Phi := \{(\Lambda, \bar{\Lambda}) \mid \Lambda \in \text{Co}\{\Delta_1, \Delta_2\}, \bar{\Lambda} \in \text{Co}\{\bar{\Delta}_1, \bar{\Delta}_2\}\} \quad (12)$$

**Theorem 1.** For given positive scalars  $h$  and  $\delta$ , the system (7) with the sampled-data controller (9) is stable, if there exist symmetric positive definite matrices  $\mathcal{P} \in \mathcal{R}^{2n \times 2n}, Q \in \mathcal{R}^{2n \times 2n}$  any matrices  $\mathcal{S} \in \mathcal{R}^{2n \times 2n}$  symmetric matrices  $G \in \mathcal{R}^{n \times n}$  and appropriate dimension matrix  $T$  satisfying the following LMIs:

$$\sum_{i=1}^4 \hat{\Xi}_i - 2e_2 G^{-1} e_2^T + e_2 G^{-1} \Lambda D e_1^T + e_1 D^T \Lambda^T G^{-1} e_2^T - 2e_8 G^{-1} e_8^T + e_8 G^{-1} \bar{\Lambda} D e_7^T + e_7 D^T \bar{\Lambda}^T G^{-1} e_8^T (e_1 + \delta e_7) G^{-1} \Phi G^{-1} + G^{-1} \Phi^T G^{-1} (e_1 + \delta e_7)^T < 0 \quad (13)$$

$$\begin{bmatrix} \hat{\mathcal{R}} & \hat{\mathcal{S}} \\ * & \hat{\mathcal{R}} \end{bmatrix} \geq 0. \quad (14)$$

Further, the sampled-data controller gain matrix in (9) is given by:

$$K = T G^{-1}. \quad (15)$$

**Proof.** Consider the following Lyapunov function candidate:

$$V(t) = \sum_{i=1}^3 V_i \quad (16)$$

where

$$\begin{aligned} V_1 &= x_a^T(t) \mathcal{P} x_a(t), \\ V_2 &= \int_{t-h}^t x_a^T(s) \mathcal{Q} x_a(s) ds, \\ V_3 &= h \int_{-h}^0 \int_{t+\alpha}^t x_a^T(s) \mathcal{R} x_a(s) ds. \end{aligned}$$

The time-derivative of the Lyapunov function  $V(t)$  can be obtained as:

$$\dot{V}_1 = \dot{x}_a^T(t) \mathcal{P} x_a(t) + x_a^T(t) \mathcal{P} \dot{x}_a(t) = \xi^T(t) \Xi_1 \xi(t), \quad (17)$$

where

$$\begin{aligned} \Xi_1 &= [e_1 \ e_2] \mathcal{P} [e_7 \ e_8]^T + [e_7 \ e_8] \mathcal{P} [e_1 \ e_2]^T, \\ \dot{V}_2 &= x_a^T(t) \mathcal{Q} x_a(t) - x_a^T(t-h) \mathcal{Q} x_a(t-h) = \xi^T(t) \Xi_2 \xi(t) \end{aligned} \quad (18)$$

where

$$\begin{aligned} \Xi_1 &= [e_1 \ e_2] \mathcal{Q} [e_1 \ e_2]^T - [e_5 \ e_6] \mathcal{Q} [e_5 \ e_6]^T, \\ \dot{V}_3 &= h^2 \dot{x}_a^T(t) \mathcal{R} x_a(t) - h \int_{t-h}^t \dot{x}_a^T(s) \mathcal{R} x_a(s) ds, \end{aligned} \quad (19)$$

Since  $\begin{bmatrix} \mathcal{R} & \mathcal{S} \\ * & \mathcal{R} \end{bmatrix} \geq 0$ , by employing Jensen's inequality and the reciprocally convex combination technique [7], one can obtain:

$$-h \int_{t-h}^t \dot{x}_a^T(s) \mathcal{R} x_a(s) ds \leq \begin{bmatrix} \int_{t-h(t)}^t \dot{x}_a(s) ds \\ \int_{t-h}^{t-h(t)} \dot{x}_a(s) ds \end{bmatrix}^T \begin{bmatrix} \mathcal{R} & \mathcal{S} \\ * & \mathcal{R} \end{bmatrix} \begin{bmatrix} \int_{t-h(t)}^t \dot{x}_a(s) ds \\ \int_{t-h}^{t-h(t)} \dot{x}_a(s) ds \end{bmatrix} \quad (20).$$

Hence, from (19) and (20), we have

$$\dot{V}_3 \leq \xi^T(t) \Xi_3 \xi(t), \quad (21)$$

where

$$\begin{aligned} \Xi_3 &= h^2 [e_7 \ e_8] \mathcal{R} [e_7 \ e_8]^T - [e_1 - e_3 \ e_2 - e_4 \ e_3 - e_5 \ e_4 - e_6] \\ &\quad \mathcal{R} [e_1 - e_3 \ e_2 - e_4 \ e_3 - e_5 \ e_4 - e_6]^T, \end{aligned}$$

From (11) and (10), for any symmetric matrices  $G$ , the following equations are satisfied:

$$2f^T(v(t)) G^{-1} [\Lambda D \ -I \ 0 \ 0 \ 0 \ 0 \ 0 \ 0] \xi(t) = 0, \quad (22)$$

$$2\dot{f}^T(v(t)) G^{-1} [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ \bar{\Lambda} D \ -I] \xi(t) = 0, \quad (23)$$

$$\begin{aligned} 2[x^T(t) G^{-1} + \delta \dot{x}^T(t) G^{-1} [A \ F \ BK \ 0 \ 0 \ 0 \ -I]] \xi(t) \\ = \xi^T(t) ((e_1 + \delta e_7) G^{-1} \Phi G^{-1} \\ + G^{-1} \Phi^T G^{-1} (e_1 + \delta e_7)^T) \xi(t) = 0, \end{aligned} \quad (24)$$

An upper bound of the difference of  $V(t)$  is:

$$\begin{aligned} \dot{V}(t) \leq \xi^T(t) (\sum_{i=1}^3 \hat{\Xi}_i - 2e_2 G^{-1} e_2^T + e_2 G^{-1} \Lambda D e_1^T + e_1 D^T \Lambda^T G^{-1} e_2^T - 2e_8 G^{-1} e_8^T \\ + e_8 G^{-1} \bar{\Lambda} D e_7^T + e_7 D^T \bar{\Lambda}^T G^{-1} e_8^T (e_1 + \delta e_7) G^{-1} \Phi G^{-1} + G^{-1} \Phi^T G^{-1} (e_1 + \delta e_7)^T) \xi(t) \end{aligned} \quad (25)$$

Let us define

$$\begin{aligned} \hat{\mathcal{P}} &= \text{diag}\{G, G\}^T \mathcal{P} \text{diag}\{G, G\}, \\ \hat{\mathcal{Q}} &= \text{diag}\{G, G\}^T \mathcal{Q} \text{diag}\{G, G\}, \\ \hat{\mathcal{R}} &= \text{diag}\{G, G, G, G\}^T \mathcal{R} \text{diag}\{G, G, G, G\}, \\ \hat{\mathcal{S}} &= \text{diag}\{G, G\}^T \mathcal{S} \text{diag}\{G, G\}, \end{aligned}$$

then pre and post multiplying the matrix  $\text{diag}\{G, \dots, G\}^T$  and  $\text{diag}\{G, \dots, G\}$  in (25) leads to LMI (13). This completes the proof.

TABLE I  
 PARAMETERS IN THE SOFC SYSTEM [6]

Parameter	Value Unit	Representation
$T_0$	1279K	Absolute temperature
$F_0$	96,485Cmol <sup>-1</sup>	Faraday's constant
$R_0$	8.314Jmol <sup>-1</sup> K <sup>-1</sup>	Universal gas constant
$E_0$	0:9378 V	Ideal standard potential
$N_0$	384 -	Number of cells in series in the
$K_r$	0.995 × 10 <sup>-3</sup> mol s <sup>-1</sup> A <sup>-1</sup>	Constant, Kr = N0=4F0
$K_{H_2}$	8.32 × 10 <sup>-3</sup> mol s <sup>-1</sup> KPa <sup>-1</sup>	Valve molar constant for hydrogen
$K_{H_2O}$	2.77 × 10 <sup>-3</sup> mol s <sup>-1</sup> KPa <sup>-1</sup>	Valve molar constant for water
$K_{O_2}$	2.49 × 10 <sup>-3</sup> mol s <sup>-1</sup> KPa <sup>-1</sup>	Valve molar constant for oxygen
$\tau_{H_2}$	26.1 s	Response time of hydrogen flow
$\tau_{O_2}$	2.91 s	Response time of Oxygen flow
$\tau_{H-O}$	1.145	Ratio of hydrogen to oxygen
$\tau_f$	5 s	Time constant of the fuel processor
$r$	0.126Ω	Ohmic loss
$C$	10 F	Parallel Capacitance

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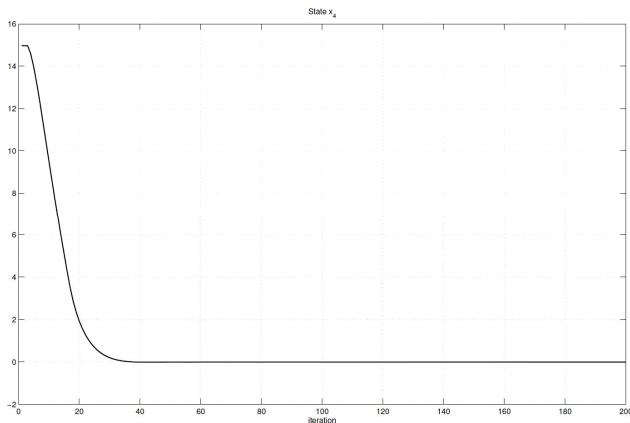


Fig. 2 Simulation result (Stack voltage  $x_4(t)$ )

IV. SIMULATION RESULT

Using the state transformation around the nominal operation point with  $q_F^* = 0.7023 \text{ mols}^{-1}$ ,  $I_D^* = 300A$ ,  $V_s^* = 240.7075V$ ,  $p_{H_2}^* = 12.6623$ ,  $p_{O_2}^* = 12.6466$  and Euler's first order approximation with a sampling time 1 sec for the derivative, we obtain a sector bounded system (1) with the following matrices.

$$\begin{aligned}
 A &= \begin{bmatrix} -0.0383 & 0 & 0.0383 & 0.0006 \\ 0 & -0.3436 & 0.3001 & 0.0027 \\ 0 & 0 & -0.2000 & 0 \\ 0 & 0 & 0 & -0.7937 \end{bmatrix}, \\
 B &= \begin{bmatrix} 0 \\ 0 \\ 0.2000 \\ 0 \end{bmatrix}, F = \begin{bmatrix} -0.0128 & -0.0064 \\ -0.0572 & -0.0286 \\ 0 & 0 \\ 16.7151 & 8.3576 \end{bmatrix}, \\
 C &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \\
 f(x(k)) &= \begin{bmatrix} \ln\left(\frac{x_1(t)+p_{H_2}^*K_{H_2}}{p_{H_2}^*K_{H_2}}\right) \\ \ln\left(\frac{x_2(t)+p_{O_2}^*K_{O_2}}{p_{O_2}^*K_{O_2}}\right) \end{bmatrix}, \\
 \Delta(k) &\in \left\{ \begin{bmatrix} 6.67 & 0 \\ 0 & 2.83 \end{bmatrix}, \begin{bmatrix} 9.07 & 0 \\ 0 & 3.52 \end{bmatrix} \right\}. \quad (26)
 \end{aligned}$$

It should be noted that lower and upper sector bounds of the nonlinear function vector  $f(x(t))$  are obtained as  $[6.67 \ 2.83]^T$  and  $[9.07 \ 3.52]^T$  in  $x_1 \in [0 \ 0.1]$ . The initial condition is  $x_0 = [0.0867 \ 0.0777 \ 0.0898 \ 14.9727]^T$ . Fig. 1 show that the proposed method with the case of only state feedback control law. This results show that the proposed method can be well applied to the set-point tracking.

V. CONCLUSION

In this paper, we proposed a sampled-data control algorithm for SOFC systems using a sector bounded nonlinear model. A stabilization condition by using the augmented vector feedback is expressed in the form of a finite number of LMIs. The proposed control method was applied to the SOFC system which is expressed as simplified mathematical model. The effectiveness of the proposed method was verified by numerical simulations.