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Evaluation of Static Modulus of Elasticity Depending on Concrete Compressive Strength

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***Abstract*—**The paper is focused on monitoring of dependencies of different composition concretes on elastic modulus values. To obtain a summary of elastic modulus development in dependence of concrete composition design variability was the objective of the experiment. Essential part of this work was initiated as a reaction to building practice when questions of elastic moduli arose at the same time and which mostly did not obtain the required and expected values from concrete constructions.

***Keywords*—**Concrete, Compressive strength, Modulus of elasticity, EuroCode 2.

I. Introduction

WITH growing interest in this theme the elastic modulus questions have been developing further. In terms of construction design the modulus of elasticity represents a basic feature essentially influencing the static behavior of concrete constructions.

From available sources it is not possible to rely on automatic determination of moduli of elasticity on the basis of concrete strength class (dis) any more. Majority of national standards relate the modulus of elasticity to compressive strength. In technical literature several empirical relations are published which express just the dependence between the concrete compressive strength and elasticity [1], [2].

The value of static elastic modulus depends to certain extent on how it is defined by the standard EN 1992-1-1 EuroCode 2: Design of concrete structures - Part 1-1: General rules and rules for buildings [3]-[5].

However, another question arises here, whether it is relevant to apply the term of compressive strength for the estimation of elastic modulus, namely at designing of constructions where final deflection is very important and further it must be taken into account that resulting real value of the elastic modulus may strongly differ from the stated guide values while the concrete class set on compressive strength basis is kept [3]. And this applies also to the assumption that there exists large variability of static modulus of elasticity within one strength class of concretes [6].

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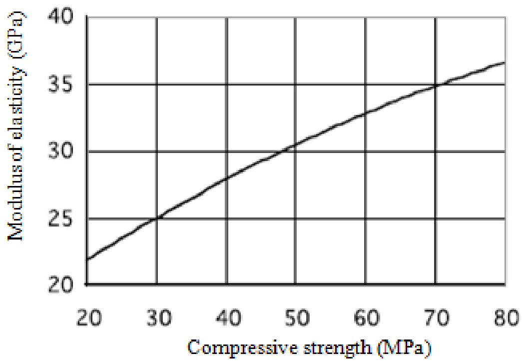

Fig. 1 Dependence between elasticity modulus and compressive strength based on empirical relationship according to A. M. Neville for common concrete [1]

The standard EuroCode 2 gives approximate values of guide module and following comment to the modulus of elasticity [3], [5]:

1) Concrete elastic deformations depend to great extent on its composition, namely on aggregates. The standard gives elastic modulus values considered as guide ones for general applications. However, if a construction is sensitive to any deviations from these general values, then the values should be set more precisely.
2) The modulus of elasticity depends on moduli of elasticity of its components.

TABLE I
Values According to EuroCode 2 [5]

Concrete strength class	Modulus of elasticity
C 12/15	27 000
C 16/20	29 000
C 20/25	30 000
C 25/30	31 000
C 30/37	33 000
C 35/45	34 000
C 40/50	35 000
C 45/55	36 000
C 50/60	37 000
C 55/67	38 000
C 60/75	39 000
C 70/85	41 000
C 80/95	42 000
C 90/105	44 000

TABLE II
EXAMPLES OF EMPIRICAL OBSERVATIONS OF MODULUS ELASTICITY [2], [7]-[11]

Designation	Equation	Validity
EN 1992-1-1	$E_c = 22(f_{cm}/10)^{0,3}$	EC 2
ACI 318	$E_c = 0.043 \cdot \rho_c^{1/5}\sqrt{f_c}$	USA
ACI 318-08	$E_c = 4700\sqrt{f_c}$	USA
ACI 363R-92	$E_c = 3\,320\sqrt{f_c} + 6\,900$	USA
CSA A23.3-04	$E_c = 4\,500\sqrt{f_c}$	Canada
NZS 3106:2006	$E_c = 3\,320\sqrt{f_c} + 6\,900$	New Zealand
TS 500	$E_c = 3\,250\sqrt{f_c} + 14\,000$	Turkey
AIJ	$E = 21\,000(\gamma/2\;300)^{1.5}(\sigma_B/20)^{1/2}$	Japan
BS 8110: part 2	$E_c = 9\,100 f_c^{0,33}$	Great Britain
NS 3473	$E_c = 9.5(f_c)^{0.3}$	Norway
IS 456-1979	$E_c = 5\,688\sqrt{f_c}$	India
A.M. Neville	$E_c = 4,73 \cdot f_c^{0.5}$	-
M. Collepardi	$E_c = K_E \cdot f_c^{1/2}$	-

In case of bigger differences in the composition, applied aggregates, cement type and dose, applied mineral admixtures this dependence can be markedly different [1]. The equation and to it similar ones have in fact informative character only. They consider only the dependence of the elastic modulus on compressive strength (therefore on water-cement ratio, treatment way and time and cement class). Nevertheless, the modulus of elasticity also depends on the dose of aggregates (modulus of elasticity approximately 70–90 kN/mm^2) and cement paste, the more transformative component (modulus of elasticity approximately 20–30 N/mm^2) [12].

TABLE III
MODULUS OF ELASTICITY CONCRETE AND COMPONENTS [7]

	Modulus of elasticity [N/mm^2]	
	Ordinary concrete	Lightweight concrete
Aggregate	70 000 - 140 000	14 000 - 35 000
Hardened cement aggregate	7 000 - 28 000	
Concrete	14 000 - 42 000	10 000 - 18 000

From the designers' point of view, to state the required elastic modulus value in concrete specification is not everyday practice. This trend is namely supported by EN 206 [14], because it does not state any requirements. However, when a designer or investor requires this value, the modulus of elasticity used to be mostly a value determined just on the basis of concrete compressive strength. Nevertheless, as proved by building practice, this is definitely improper way because it does not take into account the large variability of produced concretes.

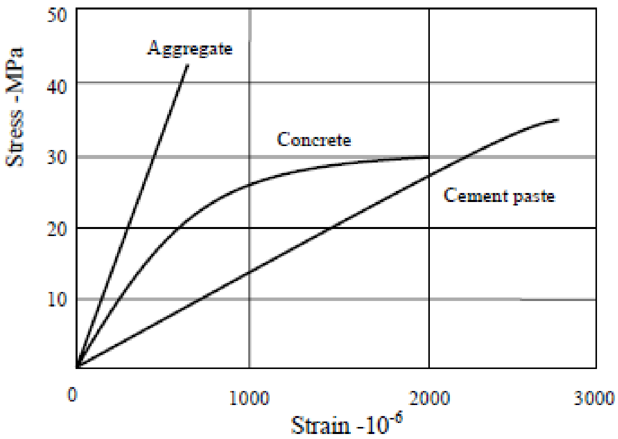

Fig. 2 Stress-strain relations for cement paste, aggregate and concrete (A.M.Neville, 1996) [13]

II. EXPERIMENTS

This part of the paper presents a summary of experimental results obtained at setting of compressive strength and modulus of elasticity with assessed concretes. Above all, the evaluation of real results of modulus of elasticity was the main objective.

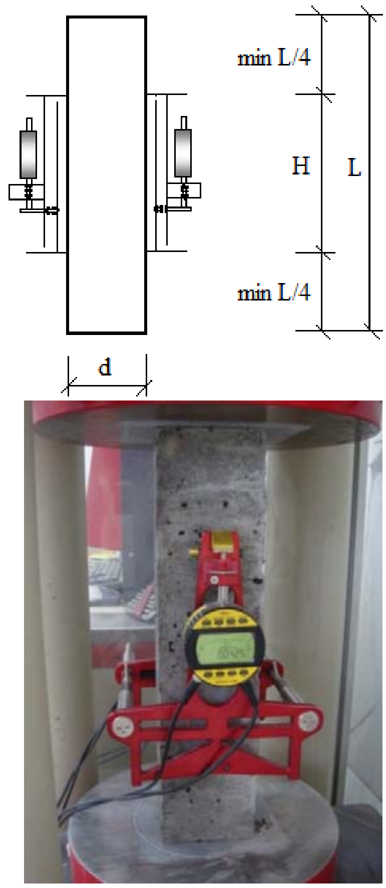

Fig. 3 Schema and practical measurement of modulus of elasticity [15]

Table IV shows the summary of results of measured modulus of elasticity to particular compressive strengths. Mean values of three, exceptionally two measurements are mentioned for each variable of 28-day matured concrete.

TABLE IV
SUMMARY OF 28-DAYS CONCRETE COMPRESSIVE STRENGTH AND VALUES OF ACTUALLY MEASURED MODULUS OF ELASTICITY

Compressive strength [N/mm²]	Static modulus of elasticity [N/mm²]	Compressive strength [N/mm²]	Static modulus of elasticity [N/mm²]
64.1	31 500	64.0	41 000
54.8	33 500	64.6	41 500
55.4	35 000	51.6	35 500
64.0	35 000	53.8	34 500
34.1	25 000	57.3	37 500
46.1	28 500	62.6	39 000
57.2	34 500	69.1	39 000
68.4	33 000	75.5	40 500
44.7	28 500	44.4	33 000
51.2	30 000	42.1	32 000
51.7	31 500	41.8	32 000
64.2	34 500	46.0	31 000
83.8	34 500	40.5	26 000
64.1	32 000	38.7	26 000
71.8	33 500	45.8	28 500
61.2	43 000	49.6	29 500
40.9	28 000	54.1	33 000
36.6	26 500	54.9	32 500
42.2	30 000	52.6	31 000
38.1	29 500	56.9	33 000
51.8	31 500	51.7	31 000
41.0	27 000	51.3	32 500
49.0	30 500	49.0	31 000
50.8	33 000	38.4	28 000

The set modulus of elasticity registered significant differences depending on compressive strength. The lowest mean value of elastic modulus was 26 000 N/mm^2 while maximum one was 43 000 N/mm^2. The difference of elastic moduli can be well seen for instance with concrete compressive strength moving on the limit of 64.0 N/mm^2. Set moduli of elasticity moved on limit of this compressive strength within values of 31 500 N/mm^2, 35 000 N/mm^2, 34 500 N/mm^2, 32 000 N/mm^2, 41 000 N/mm^2, 41 500 N/mm^2. From this clear dependence implies that the composition of concrete might not influence compressive strength only, but, on the other hand it may strongly influence the concrete modulus of elasticity.

Fig. 4 shows the dependence between concrete compressive strength and static modulus of elasticity based on the results of assessed concretes within experimental part of the work. At determination of the dependence between parameters of assessed concrete the determination index rate 0.5229 was shown. The value of compressive strength - modulus elasticity curve reached the limit 52 %, which does not mean there is higher immediate dependence between the monitored parameters.

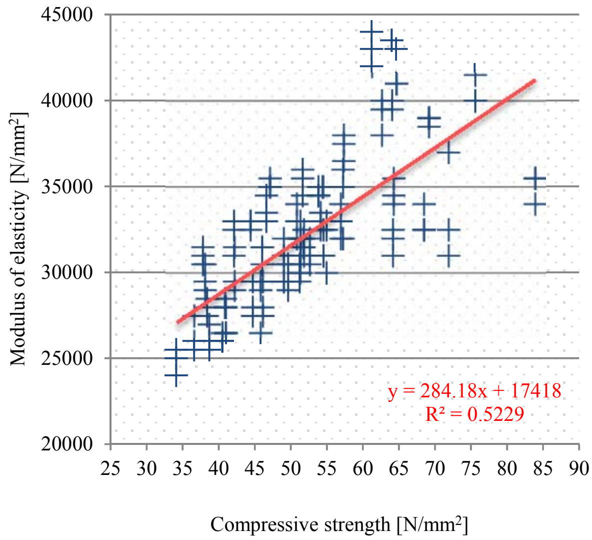

Fig. 4 Dependence between the static modulus of elasticity in compression and compressive strength of concrete at the age of 28 days [15]

III. CONCLUSION

To obtain summary of results of elastic moduli in dependence on different designs of concrete composition was primary objective of this experiment. In addition to the mentioned, the work was also focused on proposal of possible recommendations which would eliminate current condition of more difficult obtainment of concrete construction moduli of elasticity. In its major part this set of experimental results confirms the existing practice and widens thus the awareness of concrete elastic modulus questions.

Although the majority of hardened concrete characteristic properties is required in the age of 28 days, the knowledge of these parameters in early or on the contrary, in long term concrete age may be additional requirement. The results in 28-day age mostly declare the obtainment of about 90% final values. However, mainly when mineral admixtures are applied, the same trend cannot be expected. Strength properties in 28 days of age move around 75 % and it is generally known, that these concretes reach final values even at its age of 90 days.

Existing standard EN 206 [14] mentions the specification, features, production and conformity of concretes but it does not require specifying the elastic modulus value neither in basic nor in additional requirements of concrete specification. Many different factors enter in the concrete production process which more or less influence the modulus of elasticity and which were mentioned in previous parts of the work. One of possible solutions is to reassess the existing dependencies stated in EuroCode 2 [5], which would reflect the currently still more applied raw materials as blended cements, mineral admixtures and highly efficient plasticisers for cement production.

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