Optimal Planning of Dispatchable Distributed Generators for Power Loss Reduction in Unbalanced Distribution Networks

Mahmoud M. Othman, Y. G. Hegazy, A. Y. Abdelaziz

Abstract—This paper proposes a novel heuristic algorithm that aims to determine the best size and location of distributed generators in unbalanced distribution networks. The proposed heuristic algorithm can deal with the planning cases where power loss is to be optimized without violating the system practical constraints. The distributed generation units in the proposed algorithm is modeled as voltage controlled node with the flexibility to be converted to constant power factor node in case of reactive power limit violation. The proposed algorithm is implemented in MATLAB and tested on the IEEE 37 -node feeder. The results obtained show the effectiveness of the proposed algorithm.

Keywords—Distributed generation, heuristic approach, Optimization, planning.

I. INTRODUCTION

INTEGRATION of distributed generators to the power delivery networks sparked broader interest in the last few years. Employing DG in a distribution network has several advantages as enhancement of system reliability and security, improvement of power quality by improving supply continuity, reliving transmission and distribution congestion, reduction in health care costs due to improved environment and finally DG use renewable sources of energy, reducing reserve requirements and the associated costs and hence it is a source of green power [1], [2].

The distribution system planning has been the subject for different publications due to the increasing penetration of DG into the distribution system [3]-[15]. In [3] a cost-benefit analysis approach for distribution network planning including DG was presented. The optimal DG size and location were obtained for different scenarios. The first scenario studied was Distribution Company has fixed bilateral contract for power purchase and the second scenario was Distribution Company operating as a competitive market player. Authors [4] compare between different scenarios for the distribution system planning either with or without DG. The first scenario compares between the utilization of DG and substation

expansion and the second scenario compares between the integration of DG in distribution network and purchasing power from inter-tie. In each scenario the feeder thermal capacity limit was taken into consideration. Authors [5] summarized the DG planning issues and the intelligent techniques used to deal with those issues. In addition this paper compares between the genetic algorithms, simulated annealing, tabu search and parallel tabu search as intelligent techniques used for DG planning. The DG planning issues discussed in this paper are DG investment and cost minimization, DG capacity and siting, power system islanding with DG and optimal placement of capacitors and FACTS devices in distribution system contain DG. The problem of optimal DG planning under uncertain factors as emission rate, renewable fraction and fuel consumption was discussed in [6]; analytical hierarchial process was used to solve the DG planning and to deal with uncertainties and other different objectives in a DG planning problem. A multi-period optimal power flow was solved using nonlinear programming in [7]. Genetic algorithm (GA) and optimal power flow were combined to solve the optimization problem in [8], and GA was applied to solve a DG optimization problem with reliability constraints in [9] and for maximizing the profit by the optimal placement of DGs in [10] and [11]. The DG optimal power was evaluated by the Tabu Search (TS) method for the case of uniformly distributed loads [12] and a continuous stochastic DG model optimal power was evaluated by a GA as well as by a combined TS and scatter search [13]. Authors in [14] proposed a method that integrates constant power factor DG units in balanced distribution networks for minimum power loss. However, the modeled DG power factor in the proposed technique is limited to four values only. In [15], an optimization approach that utilizes an artificial bee colony (ABC) algorithm to determine the optimal DG size, power factor, and location in order to minimize the total system real power loss was proposed. Authors [16] proposed a firefly based optimization algorithm for the optimal sizing and siting of dispatchable distributed generators for power loss reduction. A supervised Big Bang Big Crunch optimization method was proposed in [17] for the optimal sizing and siting of voltage controlled distributed generators for the sake of power loss as well as energy losses minimization.

This paper presents a novel heuristic algorithm that aims to determine the optimal location and size of voltage controlled DG units to satisfy predetermined power losses. The proposed algorithm starts with ranking the system nodes according to

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their sensitivity to power losses reduction. Then, the planned power loss can be achieved by determination of the magnitude of DG injection required. The reallocation of the DG unit if a better location is possible is done according to the calculated magnitude of DG injection. The proposed algorithm is implemented in MATLAB and tested on the IEEE 37-node feeder.

II. COMPONENT MODELING

The main components of the distribution network are lines, switches, capacitors and loads. Details of these components models can be found in [18].

Depending on the reactive power capability of DG it can operate in one of the following modes, to output power at specified power factor or to output power at a specified terminal voltage. Small capacity DG cannot supply sufficient power to control the output voltage, this means that the generation node in this as is represented as constant power (PQ) node or constant negative load with current injection into the node. Large capacity DG can supply required reactive power, hence the generator node in this case must be modeled as voltage controlled (PV) node. When modeled as PV node DG behaves as voltage dependent current source as the amount of reactive current injection depends on the difference between the voltage magnitude of the PV node and the scheduled value, steps for modeling DG when operating at specified terminal voltage are minutely discussed in [19] and [20] and can be summarized as follow:

1- Initially the generator real power and positive sequence terminal voltage are specified, the generator reactive power is initialized to zero and power flow is done after DG is incorporated.

2- The positive sequence voltage magnitude at all PV nodes is compared with the specified positive sequence voltage and positive sequence voltage magnitude mismatch for all PV nodes is calculated

$$\Delta v^i = \left| v_{spec}^i \right| - \left| v_{cal}^i \right| \le \varepsilon \tag{1}$$

where $i \in$ set of PV nodes, ε is the specified tolerance. $\left| v_{spec}^i \right|$ is the specified positive sequence voltage magnitude at node j and $\left| v_{cal}^i \right|$ is the calculated positive sequence voltage magnitude at node j.

3- If the voltage mismatch is higher than the specified tolerance at any PV node, then reactive power compensation generated by that PV node in order to maintain the voltage at specified value, the magnitude of the positive sequence reactive current injection is expresses as

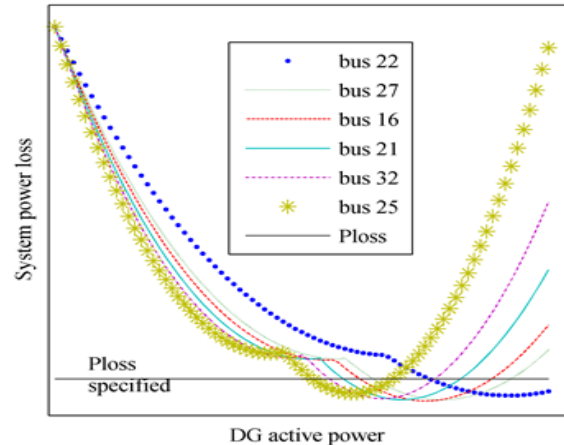$$[\Delta I_q] = [Z_v]^{-1} \times [\Delta v] \tag{2}$$

Fig. 1 System power loss variation Vs DG power

where Z_v is the PV node sensitivity matrix, the dimension of this square symmetrical matrix is equal to number of PV nodes, the main diagonal elements of $[Z_v]$ is equal to the modulus of the sum of positive sequence impedance of all line sections between PV node and substation bus, the off diagonal element Zij is equal to the modulus of the sum of positive sequence impedance of all line section on the common path between any two PV nodes i, j.

4- The reactive current injections are

$$\Delta I_{qa,b,c}^i = \left| \Delta I_q^i \right| e^{j(\pm 90 + \delta_{va,b,c}^i)} \tag{3}$$

where $\delta_{va,b,c}^i$ are the voltage angles of the converged voltage at ith PV node, $\pm$ is used to indicate either the reactive power is injected or absorbed, $\pm$ follows the sign of the voltage mismatch (Δv), if Δv is +ve then the reactive power is injected by the DG, if Δv is –ve then the reactive power is absorbed by the DG.

5- These currents are added to the currents calculated from the previous iterations at node i.

$$I_{qa,b,c}^i = I_{qa,b,c}^i + \Delta I_{qa,b,c}^i \tag{4}$$

6- The required reactive power generation Q_g^i for all PV nodes are calculated, this reactive power is a combination of the desired reactive power injection to compensate voltage magnitude and the load reactive power.

$$Q_g^i = \text{Im}[v_a^i I_a^{i*}] + \text{Im}[v_b^i I_b^{i*}] + \text{Im}[v_c^i I_c^{i*}] \tag{5}$$

7- Q_g^i is then compared with the reactive power generation limits, if during the computation the reactive power of any DG violates its limits, it is fixed at the limiting value and this node is treated as PQ node, the row and column in $[Z_v]$ corresponding to this node are removed.
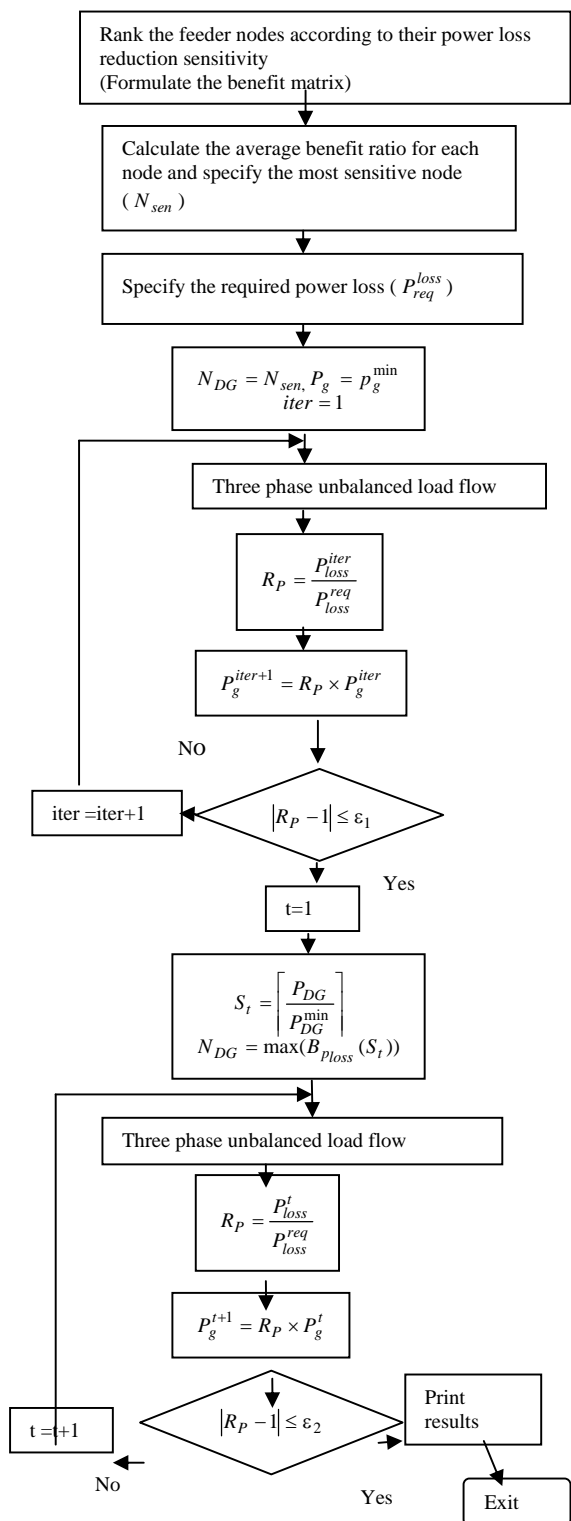
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corresponding minimum size required to achieve a certain planned power loss. As shown in Fig. 1, The power loss follows a U shape trajectory when varying versus the DG power, that's mean for every location there are two values of DG power that can achieve a certain planned power loss. It is required to select the optimal location and the minimum capacity of DGs for achieving a specified power loss. The optimization problem under study is defined as:
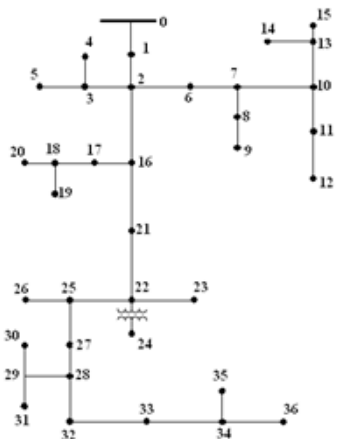
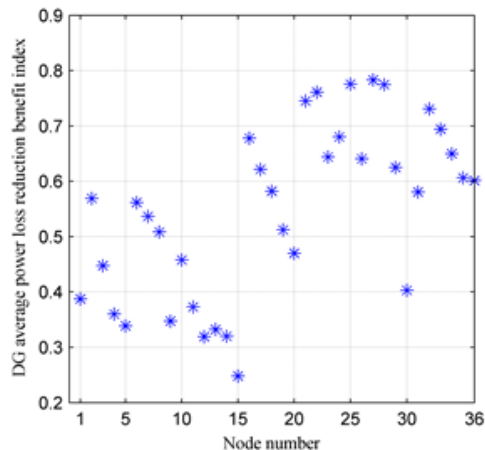Rank the feeder nodes according to their power loss reduction sensitivity (Formulate the benefit matrix)

Calculate the average benefit ratio for each node and specify the most sensitive node (N_{sen})

Specify the required power loss (P_{req}^{loss})

$N_{DG} = N_{sen}, P_g = p_g^{min}$
$iter = 1$

Three phase unbalanced load flow

$R_P = \dfrac{P_{loss}^{iter}}{P_{loss}^{req}}$

$P_g^{iter+1} = R_P \times P_g^{iter}$

NO

iter =iter+1

$|R_P - 1| \le \varepsilon_1$

Yes

t=1

$S_t = \left\lceil \dfrac{P_{DG}}{P_{DG}^{min}} \right\rceil$
$N_{DG} = \max(B_{P_{loss}}(S_t))$

Three phase unbalanced load flow

$R_P = \dfrac{P_{loss}^{t}}{P_{loss}^{req}}$

$P_g^{t+1} = R_P \times P_g^{t}$

t =t+1

$|R_P - 1| \le \varepsilon_2$

Print results

No

Yes

Exit

Fig. 2 Flow chart of the proposed heuristic technique

Fig. 3 Renumbered IEEE 37- node feeder

Fig. 4 DG average power loss reduction benefit
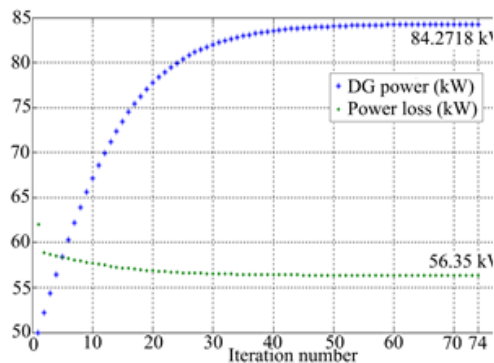
Fig. 5 Convergence characteristics of DG power and power loss

III. PROBLEM STATEMENT

One of the main orientations of the research issues concerning the DG placement and sizing is to minimize the active power loss. However, optimal placement of DG may face some geographical and technical obstacles. An alternative solution is to find the optimal DG location and the

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TABLE I
A SAMPLE OF THE BENEFIT INDEX MATRIX

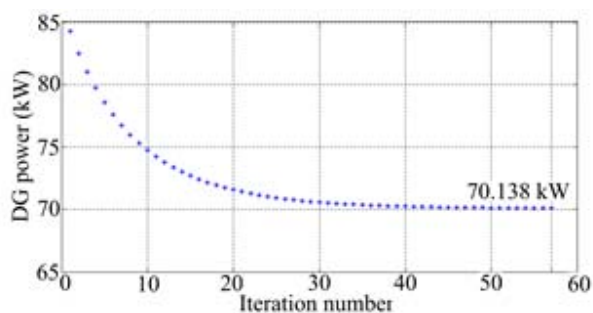| Location	$0 \leq P_{DG} < 50kW$ Injection step (1)	$50 \leq P_{DG} < 100kW$ Injection step (2)	$100 \leq P_{DG} < 150kW$ Injection step (3)	$150 \leq P_{DG} < 200kW$ Injection step (4)
1	0.0266	0.0262	0.0259	0.0255
2	0.0409	0.0402	0.0395	0.0389
3	0.0432	0.0419	0.0407	0.0395
4	0.0437	0.0421	0.0406	0.039
5	0.0439	0.0422	0.0406	0.0389
6	0.0447	0.0437	0.0428	0.0419
7	0.0492	0.0479	0.0466	0.0453
8	0.0495	0.0481	0.0466	0.0452
9	0.0504	0.0483	0.0461	0.044
10	0.0543	0.0524	0.0505	0.0486
11	0.0544	0.052	0.0497	0.0474
12	0.0545	0.0517	0.049	0.0463
13	0.0606	0.0573	0.0541	0.0509
14	0.0612	0.0578	0.0544	0.051
15	0.0607	0.0564	0.0521	0.0479
16	0.0528	0.0518	0.0507	0.0497
17	0.0551	0.0537	0.0523	0.0509
18	0.0562	0.0545	0.0529	0.0513
19	0.057	0.055	0.0531	0.0513
20	0.0562	0.0542	0.0522	0.0502
21	0.0635	0.062	0.0605	0.059
22	0.0668	0.0651	0.0634	0.0617
23	0.0674	0.0652	0.0631	0.061
24	0.0665	0.0646	0.0626	0.0607
25	0.0715	0.0695	0.0676	0.0656
26	0.0716	0.0692	0.0668	0.0644
27	0.0758	0.0736	0.0714	0.0692
28	0.0825	0.0798	0.0771	0.0745
29	0.0846	0.0811	0.0777	0.0743
30	0.0848	0.0795	0.0742	0.0691
31	0.0851	0.0813	0.0776	0.0739
32	0.0876	0.0844	0.0812	0.0781
33	0.0897	0.0861	0.0826	0.0792
34	0.0906	0.0867	0.0829	0.0791
35	0.091	0.0869	0.0828	0.0787
36	0.0906	0.0864	0.0823	0.0783

Fig. 6 Convergence characteristics of DG power at the reallocated location

TABLE II
SUMMARY OF 10% POWER LOSS REDUCTION USING THE HEURISTIC ALGORITHM

DG power before reallocation	84.2718 kW
DG location before reallocation	27
No. of iterations required for convergence	74
DG power after reallocation	70.138 kW
DG location before reallocation	35
No. of iterations required for convergence	54

TABLE III
SUMMARY OF DIFFERENT SPECIFIED POWER LOSS

Method	Specified power loss (KW)	24	30	35	40	45
The heuristic Algorithm	DG power before reallocation	1113	649	484	364	265
	Optimal DG location before reallocation	27	27	27	27	27
	DG injection step (S_t)	23	13	10	8	6
	Optimal DG location after reallocation	28	32	32	33	33
	DG power after reallocation (kW)	1044	613	433	318	228

A. Objective Function

It is required to select the optimal location and capacity of DGs for a required planned power loss using the following objective function

$$\sum_{f=1}^{Nf} P_{loss,f} = P_{loss}^{req} \qquad (6)$$

where f is feeder number, Nf is total number of feeders, $P_{loss,f}$ is the power loss at certain feeder f, P_{loss}^{req} is the system power loss required to be achieved.

B. Technical Constraints

- Voltage limits: Voltage at each bus should be within a permissible range usually:

$$0.95 \, p.u. \leq V \leq 1.05 \, p.u. \qquad (7)$$

- Lines thermal limit (line Ampacity): It represents the maximum current that the line can withstand at certain DG penetration, exceeding this value leads to melting of the line.

$$I_{flow} \leq I_{Thermal} \qquad (8)$$

where I_{flow} is the flowing branch current and $I_{Thermal}$ is the branch current thermal limit.

- Substation limit: this constraint represents the maximum apparent power that the substation can provide.

$$S_{substation,flow} \leq S_{substation,max.} \qquad (9)$$

where $S_{substation,flow}$ and $S_{substation,max}$ is the substation complex power and maximum complex power respectively.

- Power balance: The sum of input power should be equal to the sum of output active power in addition to the active power loss. The input power may include the DG active power and the active power supplied by the utility. The active output power is the sum of loads active power.

$$P_{substaion} + \Sigma P_{DG} = \Sigma P_{loads} + P_{loss} \qquad (10)$$

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- DG power limits: Active, reactive and complex powers of the DG unit are constrained between minimum and maximum value and this range should not be violated

$$P_g^{\min} \le P_g \le P_g^{\max} \qquad (11)$$

$$Q_g^{\min} \le Q_g \le Q_g^{\max} \qquad (12)$$

$$s_g^{\min} \le S_g \le \Sigma S_{load} \qquad (13)$$

where P_g is the DG active power, $P_g^{\min}, P_g^{\max}$ are the maximum and minimum values of the active DG power, Q_g is the DG reactive power, $Q_g^{\max}, Q_g^{\min}$ are the maximum and minimum values of the reactive DG power, S_g is the DG complex power, $s_g^{\min}$ is the minimum value of DG complex power and ΣS_{load} is the sum of feeder loads power.

IV. METHODOLOGY

A heuristic algorithm to determine the optimal location and size of DG units to satisfy predetermined power loss is proposed in this section. The proposed algorithm starts with ranking the system nodes according to their sensitivity to power loss reduction. Then, the planned power loss can be achieved by determination of the magnitude of DG injection required. The reallocation of the DG unit has been made if a better location is possible according to the calculated magnitude of DG injection.

The proposed heuristic algorithm is summarized in Fig. 3 and explained in the following steps:

1) The system nodes are ranked according to their sensitivity to power loss reduction by using the following procedure:

a) Calculate the base power loss (P_{loss}^{base}) using the developed unbalanced load flow.

b) Start two counters, *NDG* counter which represents the node at which DG is connected and *K* counter which represents an integer multiple of DG minimum power.

c) If the DG power is lower than DG maximum power perform load flow and calculate the elements of matrix of the benefit index in power loss reduction per kW of DG power ($B(P_{Loss})$) and the benefit index matrix consists of N rows and K columns and can be formulated using (14):

$$B_{(NDG,K)}(P^{loss}) = \frac{P_{loss}^{base} - P_{(NDG,k)}^{loss}}{P_g} \qquad (14)$$

d) Repeat step c until reach the maximum number of nodes for the feeder under test (N) and replace any negative element by zero.

e) Calculate the average benefit ratio index for each node by adding the elements of each row of the benefit index matrix.

2) Specify the required power loss and inject the minimum DG power at the most sensitive node (i.e. the node that achieve the highest average benefit ratio index) determined from step (1).

3) Solve the load flow problem to obtain the system power loss.

4) At each iteration *iter*, calculate the ratio between the system loss and the specified power loss (R_P) and then use it to calculate the DG power of the next iteration using (15):

$$P_g^{iter+1} = R_P \times P_g^{iter} \qquad (15)$$

5) Repeat steps (3) and (4) until the convergence is met. Convergence occurs if the magnitude of ($R_P - 1$) is less than a preset tolerance (ε_1)

6) Determine the DG injection step (S_t) by dividing the converged DG power obtained in the previous step over the minimum DG power, the result is then approximated to the higher integer.

7) Obtain the node of highest power loss reduction benefit ratio from the column corresponding to the calculated DG injection step in the benefit index matrix of power loss reduction ($B(P_{Loss})$).

8) Move the converged DG power obtained from step (5) to the new node obtained from (7)

9) Repeat steps (3) and (4) until the convergence is met at the new location. Convergence occurs if the magnitude of ($R_P - 1$) is less than a preset tolerance (ε_2)

V. TEST CASES AND RESULTS

The proposed algorithm is implemented in MATLAB and tested on the IEEE 37 node feeder presented in Fig. 3. to evaluate the optimal DG location and size required to achieve a certain specified power loss. The following test cases were performed.

A. Optimal Location and Power for 10 % Reduction of System Power Loss

Firstly, the proposed heuristic algorithm ranks the feeder nodes according to their power loss reduction benefit. The minimum DG power is chosen to be 50 kW; a sample of the calculated benefit matrix is presented in Table I. Fig. 4 shows the average DG benefit index for all locations, it is clear that node 27 is the most sensitive node at which DG is firstly connected before reallocating to the new location.

The proposed method is applied to the IEEE 37 node feeder. The initial power loss is 62.6124 kW, the specified system loss are set to make 10% reduction in system power loss. Fig. 5 shows the convergence characteristics of the DG power and the power loss when the DG is connected to highest average benefit index node (i.e. node 27). The DG injection step (S_t) is calculated by dividing the converged DG power obtained at node 27 which is 84.2718 kW over the minimum

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DG power (50 kW), the result is $S_t = 2$. Hence, the DG is reallocated to node 35 (most sensitive node in step 2). Fig. 6 shows the convergence characteristics when the DG is reallocated, Table II summarizes the results of the heuristic technique for 10% reduction in power loss. B. *Optimal location and power for different specified power loss*

In this section, the proposed heuristic technique is applied for achieving different specified power losses. The results are summarized in Table III.

VI. CONCLUSIONS

In this paper, a heuristic planning algorithm to achieve the best size and location of the DG unit is proposed. The proposed heuristic algorithm can deal with the planning cases where the active power loss is to be optimized. The algorithm consists of two steps; the first one is to rank the DG locations according to their sensitivity to power loss reduction. The second step is to determine the minimum magnitude of DG injection that should be applied to a certain node to achieve a certain specified power loss. The proposed algorithm is applied to the IEEE 37 nodes unbalanced distribution feeder and different test cases were conducted to prove the efficiency of the proposed algorithm.

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