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Production Plan and Technological Variants Optimization by Goal Programming Methods

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***Abstract*—**In this paper, the goal programming methodology for solving multiple objective problem of the technological variants and production plan optimization has been applied. The optimization criteria are determined and the multiple objective linear programming model for solving a problem of the technological variants and production plan optimization is formed and solved. Then the obtained results are analysed. The obtained results point out to the possibility of efficient application of the goal programming methodology in solving the problem of the technological variants and production plan optimization. The paper points out on the advantages of the application of the goal programming methodology compare to the Surrogat Worth Trade-off method in solving this problem.

***Keywords*—**Goal programming, multi objective programming, production plan, SWT method, technological variants.

I. Introduction

PRODUCTION plan and technological variants optimization is one of the most important problems which are facing manufacturing companies. By its nature production plan and technological variants optimization is a multi-objective problem for which solving we should apply multi objective programming methods.

However, many methods for solving multi-objective programming problems are proposed. The proposed multi-objective programming methods differ in efficiency from the perspective of decision makers and analysts. Some methods are universal and some are designed for specific multi-objective programming problems solving. The first major review of multiple objective programming methods is given in [1].

Goal programming which was originally presented in [2] is one of the most important methods for solving multi-objective programming problems. This method has been improved in [3]-[5]. In addition there are many variants of goal programming methods which are all based on the abovementioned papers. References [6] and [7] propose an impressive list of papers which introduce or apply goal programming.

In this paper we apply goal programming methods in solving the problem of production plan and technological variants optimization.

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The paper besides an introduction, conclusion and list of literature contains three chapters: goal programming methodology, solving a specific problem of the production plan and technological variants optimization using goal programming and analysis of the optimal solutions.

II. Goal Programming Methodology

A. Multiple Objective Linear Programming Model

The multi-objective linear programming model can be presented as

$$\max \begin{pmatrix} z_1(x) = \sum_{j=1}^{n} c_{1j}x_j,\ z_2(x) = \sum_{j=1}^{n} c_{2j}x_j,\ \ldots, \\ z_K(x) = \sum_{j=1}^{n} c_{Kj}x_j \end{pmatrix} \tag{1}$$

$$\text{s.t.} \quad Ax \leq = \geq b, \tag{2}$$

$$x \geq 0, \tag{3}$$

where x is an n – dimensional vector of decision variables, z_1, z_2, …, z_K are linear objective functions, A is a m x n dimensional matrix of constraint coefficients, while b is a m – dimensional vector of constraint values.

By solving the model (1) - (3) one or more nondominated (efficient) solutions are obtained. Nondominated solution that is accepted by the decision maker is called the preferred solution.

B. Goal Programming Method

To solve the model (1)–(3) by the goal programming method we have to find marginal solutions for all the objective functions in the given constraints set with objective function values: $z_1^*, z_2^*, \ldots, z_k^*$. After that we form the goal programming model in one of the five possible ways [7]-[9]:

(i) The Min – Max Form

$$\text{Min max } g_k(n_k, p_k) \tag{4}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} c_{kj}x_j + n_k - p_k = \overline{Z}_k,\ k = 1,2,\ldots,K \tag{5}$$

$$\sum_{j=1}^{n} a_{ij}x_j \leq = \geq b_i,\ i = 1,2,\ldots,m \tag{6}$$

$$x_j \geq 0,\ n_k \geq 0,\ p_k \geq 0,\ n_k \cdot p_k = 0,\ k = 1, 2, \ldots, K. \tag{7}$$

Aspiration level $\overline{Z}$ is determined by the decision maker or is equal to Z^*.

(ii) The Minimization of the Sum of Deviations Form

$$\text{Min } \sum_{k=1}^{K} g_k(n_k, p_k) \tag{8}$$

$$\text{s.t.} \quad \text{constraints (5) – (7)} \tag{9}$$

(iii) The Minimization of the Weighted Sum of Deviations Form

$$\text{Min } \sum_{k=1}^{K} w_k g_k(n_k, p_k) \tag{10}$$

$$\text{s.t.} \quad \text{constraints (5) – (7),} \tag{11}$$

where w_k (k = 1, 2, …, K) are weights determined by the decision maker.

(iv) The Min-Max Weighted Form

$$\text{Min max } g_k(w_k n_k, w_k p_k) \tag{12}$$

$$\text{s.t. constraints (5) – (7),} \tag{13}$$

where w_k (k = 1, 2, …, K) are weights determined by the decision maker.

(v) The Preemptive Priority Form:

In this form the K objectives are rearranged according to decision maker's priority levels, the highest priority goal is considered first, then the second and so on. The general lexicographical goal program is:

$$\text{Min } \left\{ \sum_{k \in P_i} w_k g_k(n_k, p_k) : \ i = 1,2,...,I \right\} \tag{14}$$

$$\text{s.t.} \quad \text{constraints (5) – (7),} \tag{15}$$

where I is the number of priority levels and $k \in P_i$ means that the kth goal is in the ith priority level.

Models (i), (ii), (iii), (iv), and (v) are linear programming models which can be solved by the simplex method.

III. Production Plan and Technological Variants Problem

In the following sections we present problem of production program and technological variants determination in a metal processing company. The data are taken from [10].

A. Production Plan and Technological Variants Data

For the current year January – December period the company is to produce, in addition to single products made to order, 11 different products marked by numbers from 1 to 11.

Net sale price and net-profit per product made by a particular technological variant are shown in Table I.

Technological variants represent different ways to produce the same product. In our example the variant U-7 presents production by a special machine requiring high participation of labour, while by the variant NC-A the production is completely automated with the minimal participation of workforce.

TABLE I
NET SALE PRICE AND NET-PROFIT PER PRODUCT [10]

Product x_i	Net sale price c_{ig3}	Net-profit - c_{ig1}			
		Var. U-7	Var. U-11	Var. NC-P	Var. NC-A
1	12.80	0.35	-	0.77	0.95
2	78.00	3.5	-	4.68	-
3	14.50	0.45	0.57	0.87	1.11
4	10.80	0.50	0.65	-	-
5	9.81	0.33	0.41	0.59	0.71
6	13.60	0.51	0.62	0.82	0.99
7	15.80	0.88	0.95	-	-
8	20.30	1.19	1.22	-	-
9	19.80	0.88	0.95	1.19	-
10	13.45	0.57	0.68	0.81	0.99
11	218.50	9.12	10.5	13.11	-

TABLE II
DIRECT LABOUR REQUIRED AND EQUIVALENCY COEFFICIENTS [10]

Product x_i	Direct labour required in minutes and equivalency coefficients – c_{ig2}							
	Var. U-7	Equiv. coeff.	Var. U-11	Equiv. coeff.	Var. NC-P	Equiv. coeff.	Var. NC-A	Equiv. coeff.
1	18	1,28	-	0,00	9	1,80	7	1,40
2	132	9,43	-	0,00	66	13,2	-	0,0
3	14	1,00	10	1,00	5	1,00	5	1,0
4	15	1,07	9	0,90	-	0,00	-	0,0
5	9	0,64	8	0,80	5	1,00	3	0,6
6	13	0,93	12	1,20	6	1,20	5	1,0
7	22	1,57	20	2,00	-	0,00	-	0,0
8	30	2,14	25	2,50	-	0,00	-	0,0
9	18	1,28	14	1,40	11	2,20	-	0,0
10	14	1,00	11	1,10	7	1,40	5	1,0
11	250	17,8	200	20	145	29	-	0,0

TABLE III
LATHES OPERATION TIME [10]

Product x_i	Lathes operation time in minutes[1] - a_{ig}			
	Var. U-7	Var. U-11	Var.NC-P	Var. NC-A
1	10	-	6	5
2	80	-	40	-
3	8	8	4	4
4	5	5	-	-
5	4	5	3	2
6	6,5	6,5	3	3
7	6	5,5	-	-
8	25	21	-	-
9	6,5	5,8	5	-
10	12	10	7	5
11	47	39	30	-
Available capacity - b_g	405000	101000	130460	99490

[1] The time required to manufacture a product besides turning operation includes some other operations (cutting, milling, polishing, heat treatment, etc.) Some of these operations are carried out on the lathe depending on the type of product and type of lathe. Turning operation on different lathe types will be of different grade, which results in additional operations and consequently in additional time needed to finish the product. As in this process turning is the most important operation the selection of technological variants will be carried out with this operation in mind.

For production of different products the needed direct labour will be different in different technological variants. The direct labour required for particular products and variants, and equivalency coefficients obtained by division of the necessary working time for production of each product by different variants with the needed time to produce product 3 (freely chosen by the authors) by different variants are shown in Table II.

The turning plant consists of seven U-7 lathes, one U-11 lathe, two NC-P machines and one NC-A machine.

The time needed to manufacture one product on these machines is shown in Table III.

The available capacity of particular groups of machines is calculated according to the form (in minutes): $b_i = p_i \cdot m_i \cdot n_i \cdot s_i \cdot \eta_i - K_i$, where b_i = available capacity of the i–lathe group, p_i = number of lathes of the i–group, m_i = number of working days in the given period, n_i = number of working hours per shift s_i = number of shifts per day, η_i = utilization level of the i–lathe group (taking into account the waste of time resulting from technological, organisational, market and other factors). Based on the analysis of data from the previous period it is assumed that the utilization level of the U-7 lathe group will be 0.85, while U-11, NC-P i NC-A will be 0.95. K_i is the capacity of the i– lathe group intended for production to order (single units and small series). Thus:

$$b_1 = 6 \cdot 242 \cdot 7.5 \cdot 1 \cdot 0.85 \cdot 60 - 150390 = 405000$$
$$b_2 = 1 \cdot 242 \cdot 7.5 \cdot 1 \cdot 0.95 \cdot 60 - 2455 = 101000,$$
$$b_3 = 2 \cdot 242 \cdot 7.5 \cdot 1 \cdot 0.95 \cdot 60 - 76450 = 130460,$$
$$b_4 = 1 \cdot 242 \cdot 7.5 \cdot 1 \cdot 0.95 \cdot 60 - 3965 = 99490.$$

In the given period labour is not restricted because in the labour market there are a large number of workers with the required skills.

Consumption of the basic material in kilograms needed to manufacture single products on particular lathes is shown in Table IV.

TABLE IV
CONSUMPTION OF BASIC MATERIAL IN KILOGRAMS [10]

Product	Consumption of basic material in kilograms – q_{i1g}			
	Var. U-7	Var. U-11	Var. NC-P	Var. NC-A
1	0,41	-	0,40	0,40
2	1,75	-	1,70	-
3	0,35	0,35	0,34	0,34
4	0,40	0,40	-	-
5	0,49	0,47	0,45	0,44
6	0,36	0,35	0,35	0,33
7	0,71	0,69	-	-
8	0,66	0,65	-	-
9	0,42	0,41	0,41	-
10	0,33	0,31	0,30	0,30
11	3,15	3,10	3,05	-

In the given period the possibility to purchase the basic material is restricted to q_1 = 46000 kg.

There are no restrictions for the purchase of other materials.

As the products in question are specific and intended for limited market segments the company has restricted possibility of sale. Consequently, in the subsequent sales plan period the maximal sale will be 7500 units of the product 1 (u_1); 4500 units of the product 2 (u_2); 14500 units of the product 3 (u_3); 8000 units of the product 4 (u_4); 25500 units of the product 5 (u_5); 15500 units of the product 6(u_6); 9500 units of the product 7 (u_7); 4500 units of the product 8 (u_8); 8500 units of the product 9 (u_9); 8500 units of the product 10 (u_{10}) i 4500 units of the product 11 (u_{11}).

Based on the above data we will form the model containing three objective functions: net-profit, output, and revenues from exports. The stated objective functions have to be maximized.

B. Multi Objective Linear Programming Model

Technological variants are different ways of producing the same product. In our example the variant U-7 represents production on a special machine which requires high participation of labour in the manufacturing process, while in the variant NC-A the manufacturing process is wholly automated with minimal participation of labour.

Let x_{ig} = quantity of i–product produced by g - technological variant ($i = 1, \cdots, 11;\ g = 1, \cdots, 4$).

Objective functions [10]

Net-profit:

$$\max z_1 = \sum_{i=1}^{11} \sum_{g=1}^{4} c_{ig1} x_{ig}, \tag{16}$$

where c_{ig1} is net-profit from Table I.

Output:

$$\max z_2 = \sum_{i=1}^{11} \sum_{g=1}^{4} c_{ig2} x_{ig}, \tag{17}$$

where c_{ig2} are equivalency coefficients from Table II.

Revenues from exports:

$$\max z_3 = \sum_{i \in I} \sum_{g=1}^{4} c_{ig3} x_{ig} \tag{18}$$

where c_{ig3} are net sale prices from the Table I, where they are the same at any variant. As only some of the products are exported, I represents a set of indices of exported products or $I = \{1, 3, 4, 5, 10\}$.

It has to be pointed out that some products cannot be manufactured by all variants. Thus by the second variant U-11 the first and the second product cannot be produced, i.e. $x_{12} = x_{22} = 0$, which can be easily perceived from the Table I.

Constraints [10]

Lathes U-7, U-11, NC-P i NC-A

$$\sum_{i=1}^{11} a_{i1} x_{i1} \le 405000 \ , \qquad (19)$$

$$\sum_{i=1}^{11} a_2 x_{i2} \le 101000, \qquad (20)$$

$$\sum_{i=1}^{11} a_{i3} x_{i3} \le 130460, \qquad (21)$$

$$\sum_{i=1}^{11} a_{i4} x_{i4} \le 99490, \qquad (22)$$

where $a_{ig}, (g = 1,...,4)$ is the lathe operation time in a particular variant from Table III.

Material Capacity:

As only one material has a limited capacity (t = 1) this constraint is obtained from Table IV, where q_{i1g} are consumption indicators of the basic material from that table.

$$\sum_{i=1}^{11} \sum_{g=1}^{4} q_{i1g} x_{ig} \le 46000. \qquad (23)$$

Additional constraints result from market constraints depending on the possibility of sale, as explained above, and naturally from the non-negativity constraints.

$$l_i \le x_i \le u_i, \ (i = 1,...,11), \qquad (24)$$

where

$$x_i = \sum_{g=1}^{4} x_{ig} \ .$$

$$x_{ig} \ge 0 \qquad (i = 1,...,11; \ g = 1,...,4). \qquad (25)$$

C. Model Solving

Applying the Lingo software the optimal values of the objective functions are obtained. The obtained optimal values are presented in Table V.

From Table V it is obvious that by maximizing function z_1 the obtained maximal value for that function is significantly different from the value when functions z_2 and z_3 are maximized respectively. A significant difference in the value of particular functions also appears at maximization of the other two objective functions. Consequently, it is obvious that the application of linear programming is inadequate for determination of the optimal production program and selection of optimal technological variants, and that it is necessary to apply multi objective linear programming. Namely, the company that has to choose one solution for realisation is restricted only to optimal (marginal) solutions of one of the objective functions, which differ significantly, unless the MOLP methods are used. Solving the model by the MOLP methods will result in a compromising solution that will provide acceptable values for the objective functions.

TABLE V
OPTIMAL (MARGINAL) OBJECTIVE FUNCTION VALUES

Solution	Variable values	Objective function value		
		z_1	z_2	z_3
$\underline{x}^1$	$x_{2,1}$ = 2199, $x_{2,3}$ = 2074, $x_{3,3}$ = 11877, $x_{3,4}$ = 2622, $x_{4,1}$ = 8000, $x_{6,4}$ = 15500, $x_{7,1}$ = 7347, $x_{9,1}$ = 8500, $x_{10,4}$ = 8500, $x_{11,1}$ = 1910, $x_{11,2}$ = 2590	533344 (100%)	203499.76 (84% od z_2^*)	410960.50 (54% od z_3^*)
$\underline{x}^2$	$x_{1,4}$ =7500, $x_{2,1}$ = 4254, $x_{3,4}$ = 3872, $x_{4,1}$ = 8000, $x_{5,1}$ = 4369, $x_{6,4}$ = 15500, $x_{7,2}$ = 9500, $x_{9,1}$ = 1112, $x_{9,2}$ = 7388, $x_{11,2}$ = 151, $x_{11,3}$ = 4349	510011,59 (95% od z_1^*)	241245 (100%)	281403,89 (37% od z_3^*)
$\underline{x}^3$	x_{11} =7500, $x_{3,1}$ = 14500, $x_{4,1}$ = 8000, $x_{5,1}$ = 25500, $x_{10,1}$ = 6000, $x_{10,2}$ = 2500	181585 (34% od z_1^*)	156756 (24% od z_2^*)	757130 (100%)

For solving the above model we can use the numerous MOLP methods. Here we present the process of the problem solving by using goal programming methodology.

Model Solving by Goal Programming Methods

For solving the MOLP problem by the goal programming methodology we have to form a goal programming model. Considering the maximum value of the objective functions in Table V, as well as the preferences of the decision maker, we form the following goal programming model:

$$\min(d_1^- + d_2^- + d_3^-) \qquad (26)$$

$$\text{s.t.} \ \sum_{i=1}^{11} \sum_{g=1}^{4} c_{ig1} x_{ig} + d_1^- = 533344 \qquad (27)$$

$$\sum_{i=1}^{11} \sum_{g=1}^{4} c_{ig2} x_{ig} + d_2^- = 241245 \qquad (28)$$

$$\sum_{i=1} \sum_{g=1}^{4} c_{ig3} x_{ig} + d_3^- = 757130, \quad i \in \{1,3,4,5,10\} \qquad (29)$$

$$\sum_{i=1}^{11} a_{i1} x_{i1} \le 405000 \ , \ \sum_{i=1}^{11} a_2 x_{i2} \le 101000, \qquad (30)$$

$$\sum_{i=1}^{11} a_{i3} x_{i3} \le 130460, \ \sum_{i=1}^{11} a_{i4} x_{i4} \le 99490, \qquad (31)$$

$$\sum_{i=1}^{11} \sum_{g=1}^{4} q_{i1g} x_{ig} \le 46000, \ l_i \le x_i \le u_i, \ (i = 1,...,11) \qquad (32)$$

$$x_{ig} \geq 0 \quad (i = 1,...,11;\ g = 1,...,4). \tag{33}$$

Since the decision-maker could not provide information on the preferred value of the objective functions, we took the maximum value of the objective functions as preferred ones.

Model (26)-(33) is solved by using five different approaches of the goal programming methodology [8], [9]:

(i) The Min – Max Form:

Model (26)–(33) is solved by using Zimmermann's approach for linear programming problems solving [11], [12]:

$$\min \lambda \tag{34}$$

$$\text{s.t. } \lambda \geq d_1^- \tag{35}$$

$$\lambda \geq d_2^- \tag{36}$$

$$\lambda \geq d_3^- \tag{37}$$

$$\text{constraints (27) – (33)} \tag{38}$$

The obtained solution is presented in Table VI.

TABLE VI
THE SOLUTION OF THE MODEL (34) – (38)

Solution	Variable values	Objective function value		
		z_1	z_2	z_3
$x^{1,1}$	$x_{1,1} = 7500$, $x_{2,1} = 2862$, $x_{3,1} = 2838$, $x_{3,3} = 8505$, $x_{3,4} = 3157$, $x_{4,1} = 8000$, $x_{54} = 22574$, $x_{9,2} = 8500$, $x_{10,2} = 157$, $x_{10,4} = 8343$, $x_{11,2} = 1285$, $x_{11,3} = 3215$, $d_1^- = 28705,96$, $d_2^- = 28705,96$, $d_3^- = 28705,96$	499024.50	209714.80	728425.90

(ii) The Minimization of the Sum of Deviations Form:

$$\text{Min } (d_1^- + d_2^- + d_3^-) \tag{39}$$

$$\text{s.t. constraints (27) – (33)} \tag{40}$$

The obtained solution is shown in Table VII.

TABLE VII
THE SOLUTION OF THE MODEL (39) – (40)

Solution	Variable values	Objective function value		
		z_1	z_2	z_3
$x^{1,3}$	$x_{1,4} = 7500$ $x_{2,1} = 3254$, $x_{3,1} = 13086$, $x_{3,4} = 1414$, $x_{4,1} = 8000$, $x_{54} = 25500$, $x_{9,2} = 3580$, $x_{10,2} = 7433$, $x_{10,4} = 1067$, $x_{11,2} = 151$, $x_{11,3} = 4349$, $d_1^- = 37859.65$, $d_2^- = 18306.98$, $d_3^- = 0.00$	495484.90	222941.50	757130.00

(iii) The Minimization of the Weighted Sum of Deviations Form:

$$\text{Min } \sum_{k=1}^{K} \left(w_1 d_1^- + w_2 d_2^- + w_3 d_3^- \right) \tag{41}$$

$$\text{s.t. constraints (27) – (33),} \tag{42}$$

where w_k (k = 1, 2, …, K) are weights determined by the decision maker $\left(\sum_{k=1}^{K} w_k = 1 \right)$.

For $w_1 = 0.4$, $w_2 = 0.5$ and $w_3 = 0.1$ the following solution is obtained:

TABLE VIII
THE SOLUTION OF THE MODEL (41) – (42)

Solution	Variable values	Objective function value		
		z_1	z_2	z_3
$x^{1,4}$	$x_{1,1} = 156$, $x_{1,4} = 7344$ $x_{2,1} = 3093$, $x_{3,1} = 14500$, $x_{4,1} = 8000$, $x_{54} = 21585$, $x_{9,2} = 8500$, $x_{10,2} = 4580$, $x_{10,4} = 3920$, $x_{11,2} = 151$, $x_{11,3} = 4349$, $d_1^- = 26703.89$, $d_2^- = 15590.02$, $d_3^- = 38407.06$	495484.90	222941.50	757130.00

(iv) The Min – Max Weighting Form:

$$\text{Min } \lambda \tag{43}$$

$$\text{s.t. } \lambda \geq w_1 d_1^- \tag{44}$$

$$\lambda \geq w_2 d_2^- \tag{45}$$

$$\lambda \geq w_3 d_3^- \tag{46}$$

$$\text{constraints (27) – (33),} \tag{47}$$

where w_k (k = 1, 2, …, K) are weights determined by the decision maker $\left(\sum_{k=1}^{K} w_k = 1 \right)$.

For $w_1 = 0.4$, $w_2 = 0.5$ and $w_3 = 0.1$ the following solution has been obtained:

TABLE IX
THE SOLUTION OF THE MODEL (43) – (47)

Solution	Variable values	Objective function value		
		z_1	z_2	z_3
x^1	$x_{1,1} = 7500$, $x_{2,1} = 2798$, $x_{3,1} = 8268$, $x_{3,3} = 515$, $x_{3,4} = 5717$, $x_{4,1} = 8000$, $x_{5,4} = 17620$, $x_{6,4} = 6814$, $x_{9,2} = 8500$, $x_{10,2} = 4312$, $x_{10,4} = 4188$, $x_{11,2} = 220$, $x_{11,3} = 4280$, $d_1^- = 19236.25$, $d_2^- = 15461.00$, $d_3^- = 77305.01$, $\lambda = 7730.50$	514014.20	225782.30	679827.20

For $w_1 = 0.5$ $w_2 = 0.3$ and $w_3 = 0.2$ the following solution has been obtained:

TABLE X
THE SOLUTION OF THE MODEL (43) – (47)

Solution	Variable values	Objective function value		
		f_1	f_2	f_3
x^1	$x_{1,1} = 6270$, $x_{1,4} = 1230$, $x_{2,1} = 3552$, $x_{3,3} = 11707$, $x_{3,4} = 2793$, $x_{4,1} = 8000$, $x_{5,4} = 19834$, $x_{9,2} = 8500$, $x_{10,2} = 8500$, $x_{11,1} = 387$, $x_{11,2} = 1326$, $x_{11,3} = 2788$, $d_1^- = 22232.61$, $d_2^- = 28383.45$, $d_3^- = 55581.54$,	511170.80	212887.2	701546.5

For w_1 = 0.2 w_2 = 0.5 and w_3 = 0.3 the following solution has been obtained:

TABLE XI
THE SOLUTION OF THE MODEL (43) – (47)

Solution	Variable values	Objective function values		
		z_1	z_2	z_3
x^1	$x_{1,1}$ = 12, $x_{1,4}$ = 7488, $x_{2,1}$ = 3111, $x_{3,1}$ = 14500, $x_{4,1}$ = 8000, $x_{5,2}$ = 12983, $x_{5,4}$ = 9775, $x_{7.2}$ = 1533, $x_{9,2}$= 3751, $x_{10,4}$ = 8500, $x_{11,2}$ = 151, $x_{11,3}$ = 4349, d_1^- = 40360.30, d_2^- = 16144.12, d_3^- = 26906.86,	492986.5	225105.1	730231.0

(v) The Pre-Emptive Priority Form:

The decision maker has ranked the objectives according to their priority, then the lexicographical form of goal programming can be used. In this form the *K* objectives are rearranged according to their priority levels, the highest priority goal is considered first, then the second and so on [8]

In our case z_1 has level 1, z_2 level 2, and z_3 level 3.

First, the following model is solved:

$$\text{Max } z_1 = \sum_{i=1}^{11}\sum_{g=1}^{4} c_{ig1}x_{ig} \tag{48}$$

$$\text{s. t. constraints (30) – (33)} \tag{49}$$

The following solution has been obtained:

TABLE XII
THE SOLUTION OF THE MODEL (48) – (49)

Solution	Variable values	Objective function value		
		z_1	z_2	z_3
x^1	$x_{2,1}$ = 2199, $x_{2,3}$ = 2074, $x_{3,3}$ = 11877, $x_{3,4}$ = 2622, $x_{4,1}$ = 8000, $x_{6,4}$ = 15500, $x_{7,1}$ =7347, $x_{9,1}$ = 8500, $x_{10,4}$ =8500, $x_{11,1}$ = 1910, $x_{11,2}$ = 2590	533344.00	203499,76	410960,50

TABLE XIII
THE SOLUTION OF THE MODEL (50) – (52)

Solution	Variable values	Objective function value		
		z_1	z_2	z_3
x^1	$x_{1,4}$ = 7488 $x_{2,1}$ = 3467, $x_{3,1}$ = 10047, $x_{3,4}$ = 3888, $x_{4,1}$ =8000, $x_{6,4}$ =15500, $x_{7,2}$ = 9500, $x_{9,1}$ = 1112, $x_{9,2}$ = 7388, $x_{11,2}$ = 151, $x_{11,3}$ = 4349	520000.00	241079.60	384303.90

The decision maker has reduced the acceptable level of the objective function z_1 to allow an increase in the value of the functions z_2 and z_3. Now the acceptable level of the function z_1 is 510000.00. Then the following model is solved:

$$\max z_2 = \sum_{i=1}^{11}\sum_{g=1}^{4} c_{ig2}x_{ig} \tag{50}$$

$$\text{s. t. } \sum_{i=1}^{11}\sum_{g=1}^{4} c_{ig1}x_{ig} \geq 520000 \tag{51}$$

$$\text{constraints (30) – (33)} \tag{52}$$

The following solution has been obtained:

The decision maker has reduced the acceptable level of the objective function z_2 to allow an increase in the value of the function z_3. Now the acceptable level of the function z_2 is 230000. Then the following model is solved:

$$\max \sum_{i=1}\sum_{g=1}^{4} c_{ig3}x_{ig} \qquad i \in \{1,3,4,5,10\} \tag{53}$$

$$\text{s. t. } \sum_{i=1}^{11}\sum_{g=1}^{4} c_{ig1}x_{ig} \geq 520000 \tag{54}$$

$$\sum_{i=1}^{11}\sum_{g=1}^{4} c_{ig2}x_{ig} \geq 230000 \tag{55}$$

$$\text{constraints (30) – (33).} \tag{56}$$

The obtained solution is presented in Table XIV.

TABLE XIV
THE SOLUTION OF THE MODEL (53) – (56)

Solution	Variable values	Objective function value		
		z_1	z_2	z_3
x^1	$x_{1,1}$ = 5240, $x_{1,4}$ = 2260 $x_{2,1}$ = 2457, $x_{3,1}$ = 14500, $x_{4,1}$ =8000, $x_{5,4}$ = 11483, $x_{6,2}$ = 5635, $x_{7,2}$ = 1565, $x_{9,2}$ = 8500, $x_{10,4}$ = 8500, $x_{11,2}$ = 151, $x_{11,3}$ = 4349	520000.00	230000	619623.20

D. Analysis of the Obtained Solutions

From the previous tables we can see that generally goal programming techniques give more various solutions. Five different goal programming techniques are presented above. Which goal programming technique is appropriate for the decision maker? The answer to the question is not simple. It depends on the information which the decision maker can give to the analyst. If the decision maker cannot give any information about objective function acceptable values or their importance expressed in weights, then the min – max form is the best for him. However, if the decision maker can give information about acceptable values of objective function and/or information about relative importance of the objective functions, then he can use one of the rest goal programming techniques. We prefer the min – max weighting sum because it always gives solution which expresses the relative importance of the objective function for the decision maker. It is clear from the solutions presented in Tables VIII-X.

If the decision maker can order the objective functions by their importance, and give the information about acceptable value of the objective functions, then the decision maker can use the preemptive goal programming technique in determining his preferred solution.

Now we compare the goal programming approaches with Surrogat Worth Trade-off (SWT) method, which was applied in solving the same problem [10].

SWT method requires forming the SWT function w_{lj} which ensures interaction between the decision-maker and the model.

w_{lj} represents the worth of decision-maker's estimation of how much (on a scale of, say, from -10 to +10, with zero denoting equal preference) he/she prefers trading-off λ_{lj} percentages of marginal units of the l - objective function z_l for one percentage of the marginal unit of j - objective function z_j, whereby the worth of other objective functions is not changed. w_{lj} is defined as:

- $w_{ij} > 0$, when λ_{lj} marginal percentages of $z_l(\underline{x})$ are preferred to one marginal percentage of $z_j(x)$, whereby all objectives are satisfied on the level ε_j, $j = 1, \cdots, k$.
- $w_{ij} = 0$, when λ_{lj} marginal percentages of $z_l(\underline{x})$ are equivalent to one marginal percentage of $z_j(x)$, whereby all objectives are satisfied on the level of ε_j, $j = 1, \cdots, k$.
- $w_{ij} < 0$, when λ_{lj} marginal percentages of $z_l(\underline{x})$ are not preferred to one marginal percentage of $z_j(\underline{x})$, whereby all criteria are satisfied on the level of ε_j, $j = 1, \cdots, k$.

In order to find a set of indifferent nondominated solutions[2] the decision-maker is asked whether λ_{lj} percentages of objective function $z_l(\underline{x})$ are more, less, or equally preferred to one percentage of the objective function $z_j(\underline{x})$. The worth of λ_{lj}^* is selected so that $w_{lj}(\lambda_{lj}^*) = 0$.

The interaction with the decision-maker goes on until a single solution z^* is found for which all $w_{lj}(\lambda_{lj}^*)$ are equal to zero. This may not be realised in the first attempt [10].

Therefore SWT method is much more complicated compared to goal programming method. It requires more information from the decision maker. The required information can be difficult for the decision maker to answer. Sometimes the solution process can last long and be difficult for both the decision maker and the analyst. Unlike the SWT method, the goal programming method can give the solution without any information from the decision maker, and when the decision maker can give information on the relative importance of objective functions, the obtained solutions express the preferences of the decision maker. Because of that the proposed goal programming approaches are more appropriate compared to the SWT method.

IV. Conclusion

In this paper five different goal programming approaches for solving MOLPP has been presented. The applicability of the presented goal programming approaches has been tested on the concrete problem of the technological variants and production program optimization. The obtained results show the high level of applicability of the proposed approaches. The min-weighting sum approach is chosen as the best one for solving the problem of the technological variants and production plan optimization because it is simple for using for both the decision maker and the analyst, and the obtained results always reflect the preference of the decision maker expressed through the weights.

The goal programming approaches and the obtained results are compared with the SWT method that was applied in solving the same problem. The goal programming approaches have many advantages compared to the SWT method.

For the next research we propose further investigation of applicability the newest MOLP methods as MP method [13].

[2] Indifferent nondominated solution is the one for which $w_{lj}(\lambda_{lj}^*) = 0$.

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